ARITHMETIC,

DESIGNED FOR

ACADEMIES AND SCHOOLS;

UNITING THE INDUCTIVE REASONING OF THE FRENCH WITH THE PRACTICAL METHODS OF THE ENGLISH SYSTEM.

WITH FULL ILLUSTRATIONS OF THE

METHOD OF CANCELLATION.

BY

CHARLES DAVIES, LL.D.,

AUTHOR OF FIRST LESSONS IN ARITHMETIC; UNIVERSITY ARITHMETIC; ELEMENTARY ALGEBRA; ELEMENTARY GEOMETRY; ELEMENTS OF DRAWING AND MENSURATION; ELEMENTS OF SURVEYING; ELEMENTS OF ANALYTICAL GEOMETRY; DESCRIPTIVE GEOMETRY; SHADES, SHADOWS, AND LINEAR PERSPECTIVE; AND DIFFERENTIAL AND INTEGRAL CALCULUS.

[ENLARGED AND IMPROVED EDITION.]

NEW YORK:

PUBLISHED BY A. S. BARNES & CO.,

No. 51 JOHN-STREET.

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BOARD OF COMMISSIONERS OF PUBLIC SCHOOLS,
BALTIMORE, *August*, 1842.

At a meeting of the Board of Commissioners of Public Schools, Baltimore, to hear the report of the Book Committee, upon Davies' Elementary Series. The following resolution was offered, and adopted:—

Resolved,—That DAVIES' FIRST LESSONS IN ARITHMETIC, DAVIES' ARITHMETIC, DAVIES' ALGEBRA, DAVIES' PRACTICAL GEOMETRY, and DAVIES' ELEMENTARY GEOMETRY, be introduced into the Public Schools of Baltimore.

JAMES LUCAS,
MICHAEL TONER,
JOHN F. MONMONIER,
Commissioners

From the Minutes,
JOHN F. TILYARD, *Clerk*.

CHAMBER OF THE CONTROLLERS OF PUBLIC SCHOOLS,
FIRST SCHOOL DISTRICT OF PENNSYLVANIA.
Philadelphia, September 15, 1842.

At a meeting of the Board of Controllers of the Public Schools of the First School District of Pennsylvania, held at the Controllers' Chamber, on Tuesday afternoon, September 13, 1842, it was

Resolved,—That DAVIES' FIRST LESSONS IN ARITHMETIC, and DAVIES' ARITHMETIC, be introduced into the Public Schools of the District; and also, that DAVIES' ALGEBRA be introduced therein,—the latter under the Resolution of the 12th day of November, 1839.

From the Minutes,
THOMAS B. FLORENCE,
Secretary.

Stereotyped by
RICHARD C. VALENTINE,
New York.

C. A. ALVORD, Printer,
Corner of John and Dutch streets

PREFACE.

SCIENCE has been well defined to be knowledge reduced to order; that is, knowledge so classified and arranged as to be easily remembered, readily referred to, and advantageously applied.

ARITHMETIC is the science of numbers, and a correct and accurate knowledge of it is one of the most important elements of a liberal or practical education. It is the corner-stone of the exact and mixed sciences, and the first subject in a well-arranged course of instruction to which the reasoning powers of the mind are applied—yet in all that relates to the uses and applications of numbers, it is the guide and daily companion of the mechanic and man of business.

These two objects have been kept constantly in view in the preparation of this work—viz.:

I. To present to the young mind, unacquainted with the methods of exact reasoning, the elementary principles of arithmetic in their simplest form and combination.

II. To explain and illustrate the various applications of arithmetic in the transactions of business, and thus make known its great practical utility. To attain the first of these ends the following method has been adopted.

1. To present to the mind every new idea by a simple question, and then to express the idea in general terms under the form of a *definition* or *principle*.

2. When a sufficient number of ideas are thus fixed in the mind, they are combined, forming a proposition or rule; so that the separate elements are arranged in the order of exact reasoning, as fast as they are learned.

3. An entire system of Mental or Oral Arithmetic has been carried forward in connection with the text, by means of a connected series of questions placed at the bottom of each page; and if these, or their equivalents, are carefully put by the teacher, the

pupil will understand the reasoning in every process, and at the same time cultivate the powers of analysis and abstraction.

4. The better to attain these objects, the Arithmetic has been divided into paragraphs or sections, each containing a number of connected principles—and these paragraphs constitute a series of dependant propositions that make up the entire system of reasoning which the work develops. The Oral Arithmetic corresponds with these connected propositions. The subject of Fractions, will perhaps best illustrate the advantages of this method.

In regard to the second part, the table of contents, under the head of "APPLICATIONS TO BUSINESS," and "MENSURATION," shows how large a portion of the work has been given to what may justly be termed the PRACTICAL AND USEFUL.

The First Lessons in Arithmetic, the School Arithmetic, and the University Arithmetic, embrace a series of works on the subject of numbers which are designed to meet the wants of different classes of pupils.

The first work is for beginners. It treats of the simplest properties of numbers, and although but the alphabet of the science, yet that alphabet is the basis of all subsequent combinations. The School Arithmetic is an entire and complete treatise, and embraces all the subjects usually taught in academies and schools. The University Arithmetic is also a treatise of itself. It contains much that is found in the School Arithmetic, but the reasoning is more elaborate and the difficult and hidden properties of numbers more fully developed. The proof of the four ground rules by means of the Properties of the 9's, the subject of Circulating or Repeating Decimals,—of Coins, Currencies, and general Exchanges—of Book-keeping by Double Entry, are all treated in the larger work, which may be studied to great advantage by higher classes and all who desire to obtain a thorough and full knowledge of the wonderful properties of numbers and their numerous applications.

NEW YORK, *June*, 1847.

CONTENTS.

FIRST FIVE RULES.

OF DENOMINATE NUMBERS.

1*

EVOLUTION.

ARITHMETICAL PROGRESSION.

GEOMETRICAL PROGRESSION.

MENSURATION.

GAUGING, &c.

ARITHMETIC.

NUMERATION AND NOTATION.

1. A single thing is called	- - -	*One,*
One and one more are called	- -	*Two,*
Two and one more are called	- -	*Three,*
Three and one more are called	- -	*Four,*
Four and one more are called	- -	*Five,*
Five and one more are called	- -	*Six,*
Six and one more are called	- -	*Seven,*
Seven and one more are called	- -	*Eight,*
Eight and one more are called	- -	*Nine,*
Nine and one more are called	- -	*Ten,*
&c.	&c.	&c.

Each word, *one, two, three, four, five, six,* &c., points out how many things are spoken of. These words are called NUMBERS. Hence, NUMBERS *are expressions for one or more things of the same kind.*

2. The *unit* of a number is one of the equal things which the number expresses. Thus, if the number express six apples, one apple is the unit; if it express five pounds of tea, one pound of tea is the unit; if ten feet of length, one foot is the unit; if four hours of time, one hour is the unit.

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1. What is a single thing called? One and one? Two and one? Three and one? Four and one? Five and one? Six and one? Seven and one? &c. What are Numbers?

2. What is the unit of a number? What is the unit of the number six apples? Of the number five pounds of tea? Of the number ten feet? Of the number four hours?

3. ARITHMETIC treats of numbers. Numbers are expressed by certain characters, called figures. There are ten of these characters, and they are the alphabet of the arithmetical language. They are

0	which is called a cipher, or	Naught,
1	- - - - - -	One,
2	- - - - - -	Two,
3	- - - - - -	Three,
4	- - - - - -	Four,
5	- - - - - -	Five,
6	- - - - - -	Six,
7	- - - - - -	Seven,
8	- - - - - -	Eight,
9	- - - - - -	Nine.

We see from the language of figures, that

1	expresses	a single thing, or a *unit* of a number.		
2	-	two things of the same kind,		or two units.
3	-	three things	- -	or three units.
4	-	four things	- -	or four units.
5	-	five things	- -	or five units.
6	-	six things	- -	or six units.
7	-	seven things	- -	or seven units.
8	-	eight things	- -	or eight units.
9	-	nine things	- -	or nine units.

4. The character 0 is used to denote the absence of a thing. Thus, to express that there are no apples in a basket, we write, the number of apples is 0. The nine other figures are called, *significant figures*, or *Digits*.

5. We have no separate character for the number ten. If we wish to express it by figures, we must *combine* the characters already known. This we do by writing 0 on the right hand of the 1; thus, 10, which is read *ten*.

---

3. Of what does arithmetic treat? How are numbers expressed? How many figures are there? Name them. What may they be called? How many things does 1 express? How many things does 2 express? How many units in 3? In 4? In 5? In 6? In 7? In 8? In 9?

4. What does 0 express? What are the other nine figures called?

5. Have we a separate character for ten? How do we express ten? To how many units 1 is ten equal? May we consider it a single unit? Of what order?

This 10 is equal to *ten* of the units expressed by 1. It is, however, but a *single ten*, and in this sense may be regarded as a *unit*, the value of which is *ten times greater* than the unit expressed by 1. It is called a unit of the *second order*.

6. When two figures are written by the side of each other, the one on the right is called the *place of units*, and the one on the left, the *place of tens*, or *units of the second order*. *Each unit of the second order is equal to ten units of the first order*.

When units simply are named, *units of the first order are always meant*.

Two tens, or two units of the second order, are written	20
Three tens, or three units of the second order, are written	30
Four tens, or four units of the second order, are written	40
Five tens, or five units of the second order, are written	50
Six tens, or six units of the second order, are written	60
Seven tens, or seven units of the second order, are written	70
Eight tens, or eight units of the second order, are written	80
Nine tens, or nine units of the second order, are written	90

These figures are also read, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

The intermediate numbers between 10 and 20, between 20 and 30, &c., may be readily expressed by considering the tens and units of which they are composed. For example, the number twelve is made up of one unit of the second order and two of the first. It must therefore be written by setting 1 in the place of tens, and 2 in the place of units; thus, - - 12

Eighteen has 1 ten and 8 units, and is written - -	18
Twenty-five has 2 tens and 5 units, and is written -	25
Thirty-seven has 3 tens and 7 units, and is written -	37
Fifty-four has 5 tens and 4 units, and is written - -	54

---

6. When two figures are written by the side of each other, what is the place on the right called? The place on the left? When units simply are named, what units are meant? How many units of the second order in 20? In 30? In 40? In 50? In 60? In 70? In 80? In 90? Of what is the number 12 made up? Also, 18, 25, 37, 54?

Hence, any number greater than nine, and less than one hundred, may be expressed by two figures. The right hand figure will express units of the first order, and the other, units of the second order.

7. In order to express *ten units of the second order*, or *one hundred*, we have to form a new combination.

It is done thus, - - - - - 100

by writing two ciphers on the right of 1. This number is read, one hundred. Now this one hundred expresses 10 *units of the second order*, or 100 *units of the first order*. But the one hundred is but *an individual hundred*, and in this light may be regarded as a unit of the *third order*.

We can now express any number less than one thousand.

For example, in the number three hundred and seventy-five, there are 5 units, 7 tens, and 3 hundreds. Write, therefore, 5 units of the first order, 7 units of the second order, and 3 of the third; and read from the right, *units*, *tens*, *hundreds*.

huns.	tens.	units.
3	7	5

In the number eight hundred and ninety-nine, there are 9 units of the first order, 9 of the second, and 8 of the third, and it is read, *units*, *tens*, *hundreds*.

huns.	tens.	units.
8	9	9

In the number four hundred and six, there are 6 units of the first order, 0 of the second, and 4 of the third: and in a similar manner we may express, by three figures, any number greater than ninety-nine and less than one thousand. The right hand figure will express units of the first order, the next, units of the second order, and the other, units of the third order.

huns.	tens.	units.
4	0	6

---

7. How do you express one hundred? To how many units of the second order is it equal? To how many of the first order? May it be considered a single unit? Of what order is it? How many units of the third order in 200? In 300? In 400? In 500? In 600? In 700? In 800? In 900? Of what is the number 375 composed? The number 406? What numbers may be expressed by three figures? What order of units will each figure express?

8. To express *ten* units of the third order, or one thousand, we form a new combination by writing three ciphers on the right of 1; thus, - - - 1000

Now, although this thousand expresses one thousand units of the first order, it is, nevertheless, but *one single thousand*, and may be regarded as a unit of the fourth order.

Proceeding in this manner, we may form as many orders of units as we please: thus, a single unit of the first order is expressed by - - - - 1,
a unit of the second by 1 and a 0; thus, 10,
a unit of the third order by 1 and two 0's; thus, 100,
a unit of the fourth order by 1 and three 0's; thus, 1000,
a unit of the fifth order by 1 and four 0's; thus, 10000;
and so on, for units of higher orders.

9. We see, from the language of figures,

1st. *That the same figure expresses different values according to the place which it occupies.*

2d. *That units of the first order always occupy the place on the right: units of the second order the second place from the right: units of the third order the third place; and so on for places still to the left.*

3d. *That ten units of the first order make one of the second; ten of the second one of the third; ten of the third one of the fourth, and so on for the higher orders.*

4th. *That when figures are written by the side of each other, ten units in any one place make one unit of the place next to the left.*

---

8. To what are ten units of the third order equal? How do you express them? How is a single unit of the first order expressed? How do you express a unit of the second order? One of the third? One of the fourth? One of the fifth?

9. On what does the value of the same figure depend? What is the unit of the first place on the right? What is the unit of the second place? What is the unit of the third place? Of the fourth? Of the fifth? Sixth? How many units of the first order make one of the second? How many of the second one of the third? How many of the third one of the fourth? &c. When figures are written by the side of each other, how many units in any place make one unit of the place *next to the left*?

10. Expressing or writing numbers by figures, is called NOTATION. Reading the order of their places, correctly, when written, is called NUMERATION.

EXAMPLES IN WRITING THE ORDERS OF UNITS.

1. Write 3 tens. *Ans.* 30.
2. Write 8 units of the second order. *Ans.* 80.
3. Write 9 units of the first order. *Ans.* ——
4. Write 4 units of the first order, 5 of the second, 6 of the third, and 8 of the fourth. *Ans.* ——
5. Write 9 units of the fifth order, none of the fourth, 8 of the third, 7 of the second, and 6 of the first. *Ans.* 90876.
6. Write 1 unit of the sixth order, 5 of the fifth, 4 of the 4th, 9 of the third, 7 of the second, and none of the first. *Ans.* ——
7. Write 4 units of the 11th order.
8. Write forty units of the second order. *Ans.* 400.
9. Write 60 units of the third order, with four of the 2d, and 5 of the first.
10. Write 16 units of the 12th order, with 8 of the 9th, 4 of the 5th, 7 of the 2d, and 1 of the 1st.
11. Write 7 units of the ninth order, with 6 of the 7th, 9 of the third, 8 of the 2d, and 9 of the first.
12. Write 6 units of the 8th order, with 9 of the 6th, 4 of the 5th, 2 of the 3d, and 1 of the 1st.
13. Write 14 units of the 12th order, with 9 of the 10th, 6 of the 8th, 7 of the 6th, 6 of the 5th, 5 of the 3d, and 3 of the first.
14. Write 13 units of the 13th order, 8 of the 12th, 7 of the 9th, 6 of the 8th, 9 of the 7th, 7 of the 6th, 3 of the fourth, and 9 of the first.
15. Write 9 units of the 18th order, 7 of the 16th, 4 of the 15th, 8 of the 12th, 3 of the 11th, 2 of the 10th, 1 of the 9th, 0 of the 8th, 6 of the 7th, 2 of the third, and 1 of the 1st.

---

10. What is notation? What is numeration? Which way do *you numerate?*

6th Period, or period of Quadrillions. (Hundreds of Quadrillions, Tens of Quadrillions, Quadrillions)	5th Period, or period of Trillions. (Hundreds of Trillions, Tens of Trillions, Trillions)	4th Period, or period of Billions. (Hundreds of Billions, Tens of Billions, Billions)	3d Period, or period of Millions. (Hundreds of Millions, Tens of Millions, Millions)	2d Period, or period of Thousands. (Hundreds of Thousands, Tens of Thousands, Thousands)	1st Period, or period of Units. (Hundreds, Tens, Units)
-	-	-	-	-	6
-	-	-	-	-	75
-	-	-	-	-	879
-	-	-	-	6,	023
-	-	-	-	82,	301
-	-	-	-	123,	087
-	-	-	7,	628,	735
-	-	-	43,	210,	460
-	-	-	548,	721,	087
-	-	6,	245,	289,	421
-	-	72,	549,	136,	822
-	-	894,	602,	043,	288
-	7,	641,	248,	907,	456
-	84,	912,	876,	410,	285
-	912,	761,	257,	327,	826
6,	407,	212,	936,	876,	541
57,	289,	678,	541,	297,	313
920,	323,	842,	768,	319,	675

The words at the head of the numeration table, *units*, *tens*, *hundreds*, &c., are equally applicable to all numbers, and must be committed to memory; after which, the pupil may read the Table.

To make the reading of figures easy, they are often separated into periods of three figures each, counting from the *right hand*.

EXAMPLES IN EXPRESSING NUMBERS BY FIGURES.

1. Write four in figures. *Ans.* 4.
2. Write four tens or forty. *Ans.* ——
3. Write four hundred. *Ans.* 400.
4. Write four thousand. *Ans.* ——
5. Write forty thousand. *Ans.* 40,000.
6. Write four hundred thousand. *Ans.* ——
7. Write four millions. *Ans.* 4,000,000.

These examples show us very clearly that the same significant figure will have different values according to the place which it occupies.

8. Write six hundred and seventy-nine. *Ans.* 679.
9. Write six thousand and twenty-one.
10. Write two thousand and forty.
11. Write one hundred and five thousand and seven.
12. Write three billions.
13. Write ninety-five quadrillions.
14. Write one hundred and six trillions, four thousand and two.
15. Write fifty-nine trillions, fifty-nine billions, fifty-nine millions, fifty-nine thousands, fifty-nine hundreds, and fifty-nine.
16. Write eleven thousand, eleven hundred and eleven.
17. Write nine billions and sixty-five.
18. Write three hundred and four trillions, one million, three hundred and twenty-one thousand, nine hundred and forty-one.
19. Write nine trillions, six hundred and forty billions, with 7 units of the ninth order, 6 of the seventh order, 8 of the fifth, 2 of the third, 1 of the second, and 3 of the first.
20. Write three hundred and five trillions, one hundred and four billions, one million, with 4 units of the 5th order, 5 of the 4th, 7 of the 2d, and 4 of the first.
21. Write three hundred and one billions, six millions, four thousand, with 8 units of the 14th order, 6 of the 3d, and 2 of the second.
22. Write nine hundred and four trillions six hundred and six, with four units of the 18th order, five of the 16th, four of the 12th, seven of the 9th, and 6 of the 5th.

11. There is another method of expressing numbers, called the Roman. In this method the numbers are represented by letters. The letter I stands for *one;* V, five; X, ten; L, fifty; C, one hundred; D, five hundred, &c.

ROMAN TABLE.

I. - - - -	One	LXX. - - -	Seventy
II. - - - -	Two	LXXX. - -	Eighty
III. - - -	Three	XC. - - -	Ninety
IV. - - -	Four	C. - - - -	One hundred
V. - - - -	Five	CC. - - - -	Two hundred
VI. - - -	Six	CCC. - - -	Three hundred
VII. - - -	Seven	CCCC. - - -	Four hundred
VIII. - - -	Eight	D. - - - -	Five hundred
IX. - - -	Nine	DC. - - -	Six hundred
X. - - -	Ten	DCC. - - -	Seven hundred
XX. - - -	Twenty	DCCC. - -	Eight hundred
XXX. - - -	Thirty	DCCCC. - -	Nine hundred
XL. - - -	Forty	M. - - - -	One thousand
L. - - - -	Fifty	MM. - - -	Two thousand
LX. - - -	Sixty	MMD. - - -	2500.

12. We see, that there are three methods of expressing numbers: 1st, by words or common language; 2d, by figures, called the *Arabic* method; and 3d, by letters, called the Roman method.

EXAMPLES.

1. Write 1847 in common language: also in the Roman notation.
2. Write MDCCC in figures, and also in common language.
3. Write 2675 in common language: also in the Roman.
4. Write 98447096 in common language.
5. Write MMMDCCIV in common language, also in figures.

---

11. What characters are used in the Roman notation? What does X stand for? What does D stand for?

12. How many methods are there of expressing numbers? What are they? What is the one by means of figures called? The one by letters?

ADDITION OF SIMPLE NUMBERS.

13. John has three apples and Charles two: how many apples have both of them? Every boy will answer five.

Here a single apple is the unit, and the number five contains as many units as the two numbers three and two. The operation by which this result is obtained is called *Addition*. Hence,

ADDITION *is the process of uniting together two or more numbers, in such a way, that all the units which they contain may be expressed by a single number.*

Such single number is called the ***sum*** or ***sum total*** of the numbers added. Thus, five is the sum of the apples possessed by John and Charles.

What is the sum of 2 and 4? Of 3 and 5? Of 6 and 3? Of 4, 3 and 1? Of 2, 3 and 4? Of 1, 2, 3, and 4? Of 5 and 7? How many units in 4 and 6?

OF THE SIGNS.

14. The sign +, is called *plus*, which signifies more. When placed between two numbers it denotes that they are to be added together.

The sign =, is called the sign of equality. When placed between two numbers it denotes that they are equal to each other; that is, that they contain the same number of units. Thus, $3+2=5$.

When the numbers are small we generally read them, by saying, 3 and 2 are 5.

Before adding large numbers the pupil should be able to add, in his mind, any two of the ten figures. Let him commit to memory the following table, which is read, two and 0 are two; two and one are three; &c.

---

13. What is addition? What is the single number called which expresses all the units of the numbers added? How many units in 3 and 2? What is 5 called?

14. What is the sign of addition? What is it called? What does it signify? Express the sign of equality. When placed between two numbers what does it show? When are two numbers equal to each other? Give an example.

ADDITION TABLE.

2+0= 2	3+0= 3	4+0= 4	5+0= 5
2+1= 3	3+1= 4	4+1= 5	5+1= 6
2+2= 4	3+2= 5	4+2= 6	5+2= 7
2+3= 5	3+3= 6	4+3= 7	5+3= 8
2+4= 6	3+4= 7	4+4= 8	5+4= 9
2+5= 7	3+5= 8	4+5= 9	5+5=10
2+6= 8	3+6= 9	4+6=10	5+6=11
2+7= 9	3+7=10	4+7=11	5+7=12
2+8=10	3+8=11	4+8=12	5+8=13
2+9=11	3+9=12	4+9=13	5+9=14
6+0= 6	7+0= 7	8+0= 8	9+0= 9
6+1= 7	7+1= 8	8+1= 9	9+1=10
6+2= 8	7+2= 9	8+2=10	9+2=11
6+3= 9	7+3=10	8+3=11	9+3=12
6+4=10	7+4=11	8+4=12	9+4=13
6+5=11	7+5=12	8+5=13	9+5=14
6+6=12	7+6=13	8+6=14	9+6=15
6+7=13	7+7=14	8+7=15	9+7=16
6+8=14	7+8=15	8+8=16	9+8=17
6+9=15	7+9=16	8+9=17	9+9=18

2+3= how many?
1+2+4= how many?
2+3+5+1= how many?
6+7+2+3= how many?
1+6+7+2+3= how many?
1+2+3+4+5+6+7+8+9= how many?

1. What is the sum of 3 and 3 tens? *Ans.* ——
2. What is the sum of 8 tens and 9? *Ans.* 89.
3. What is the sum of 4, 5, and 4 tens? *Ans.* ——
4. What is the sum of 1, 2, 3, 4, and 9 tens?
5. What is the sum of 1, 2, 3, 4, 5, and 6 tens?
6. What is the sum of 1, 4, 9, and 5 tens? *Ans.* 64.
7. What is the sum of 4, 8, 3, and 7 tens?
8. What is the sum of 1, 2, 4, and one hundred?
9. What is the sum of 1, 3, 4, and 4 units of the second order?

10. What is the sum of 4 and 5, and 4 units of the third order?

11. What is the sum of 6 and 2, and 5 units of the third order?

12. James has 14 cents, and John gives him 21: how many will he then have?

Having written the numbers, as at the right of the page, draw a line beneath them.

14
21
35

The first number contains 4 units and 1 ten, the second 1 unit and 2 tens. We write the *units* under the *units*, and the *tens* under the *tens*.

We then begin at the right hand, and say 1 and 4 are 5, which we set down below the line in the units' place. We then proceed to the next column, and add the tens, by saying 2 and 1 are 3, which we write in the tens' place. Hence, the sum is 35: that is, James will have 35 cents.

13. John has 24 cents, and William 62: how many have both of them?

We write the numbers as before, and draw a line beneath them. We then add the units to the units, and the tens to the tens.

24
62
86

14. A farmer has 160 sheep, 20 cows, and 16 young cattle: how many in all?

We write the numbers so that units shall stand under units, tens under tens, and hundreds under hundreds. By adding, we find the sum of the units to be 6, the sum of the tens 9, and the sum of the hundreds 1: and the entire sum 196.

160
20
16
196

Add together the following numbers:

(1)	(2)	(3)	(4)
328	304	891	3607
171	273	104	4082
499			7689

Also the following:

(5)	(6)	(7)
30704	398463	7430673
47192	401536	2569326
77896		

15. A farmer bought 25 cows, 4 horses, 70 hogs, and 200 sheep: how many did he buy in all? *Ans.* ——

16. Add 5 units, 6 tens, and 7 hundreds.

We set down the 5 units in the place of units, the 6 tens in the place of tens, and the 7 hundreds in the place of hundreds. We then add them up, and find the sum to be 765. We must observe that in all cases, *units of the same order fall under each other.*

hundreds	tens	units
		5
	6	
7		
7	6	5

17. What is the sum of 3 units, 8 tens, and 4 thousands? *Ans.* 4083.

18. What is the sum of 8 hundreds, 4 tens, 6 units, and 6 thousands? *Ans.* 6846.

19. What is the sum of 3 units, 5 units, 6 tens, 3 tens, 4 hundreds, 3 hundreds, 5 thousands, and 4 thousands?

20. What is the sum of five units of the 4th order, one of the 3d, 3 of the 4th, five of the 3d, and one of the 1st?

21. What is the sum of six units of the 2d order, five of the third, six of the 4th, three of the 2d, four of the 3d, two of the 1st, and four of the 2d?

22. If a top costs 6 cents, a knife 25 cents, a slate pencil 1 cent, and a slate 12 cents, what does the whole amount to? *Ans.* 44 *cts.*

23. John gives 30 cents for a bunch of quills, 18 cents for an inkstand, 25 cents for a quire of paper: what did they all cost him? *Ans.* 73 *cts.*

Thus far, the amount of any one column, when added up, has not exceeded 9; and therefore its sum could be expressed by a single figure. But the sum of a single column will often exceed 9, and we will now show what is to be done in that case.

EXAMPLES ILLUSTRATING THE PROCESS OF CARRYING.

1. Add together 894 and 637.

	OPERATION.
Write the numbers thus - - - - - -	894
	637
And draw a line beneath them - - -	——
Sum of the column of units - - - - -	11
Sum of the column of tens - - - - -	12
Sum of the column of hundreds - -	14
Sum total - -	1531

In this example, the sum of the units of the first order is 11. But 11 units are equal to 1 ten and 1 unit; therefore, we set down 1 in the place of units, and 1 in the place of tens. The sum of the units of the second order is 12. But 12 tens are equal to 1 hundred, and 2 tens; so that 1 is set down in the hundreds' place, and 2 in the tens' place. The sum of the units of the third order is 14. The 14 hundreds are equal to 1 thousand, and 4 hundreds; so that 4 is set down in the place of hundreds, and 1 in the place of thousands. The sum of these numbers, viz. 1531, is the sum sought.

The example may be done in another way, thus: Having set down the numbers, as before, we say, 7 and 4 are 11: we set down 1 in the units' place, and write the 1 ten under the 3 in the column of tens. We then say, 1 to 3 is four and 9 are 13. We set down the 3 in the tens' place, and write the 1 hundred under the 6 in the column of hundreds. We then add the 1, 6, and 8 together, for the hundreds, and find the entire sum, 1531, as before.

OPERATION.
894
637
11
1531

When the sum in any one of the columns cannot be expressed by a single figure, write down the excess over exact tens, and then add to the next left hand column as many units of its own order as there were tens in the sum.

This is called *carrying to the next column.* The number to be carried may be written under the column or *remembered* and added in the mind.

14. Hence, for the addition of simple numbers, we have the following

RULE.

I. *Write down the numbers to be added, so that units of the same order shall fall directly under each other, and draw a line beneath them.*

II. *Beginning at the foot of the units' column, add up each column in succession, and write the sum under the column when it can be expressed by a single figure.*

III. *When the sum in any column cannot be expressed by a single figure, write down the excess over exact tens, and then add to the next left hand column as many units of its own order as there are tens in the sum; observing to set down the entire sum of the last column.*

EXAMPLES.

1. What is the sum of the numbers 375, 6321 and 598?

The small figure placed under the 4, shows how many are to be carried from the first column to the second, and the small figure under the 9, how many are to be carried from the second column to the third.

OPERATION.
375
6321
598
7294
11

Also, in the examples below, the small figure under each column, shows how many are to be carried to the next column to the left. Beginners should set down the numbers to be carried as in the examples.

	(2.)		(3.)		(4.)
	96972		9841672		81325
	3741		793159		6784
	9299		888923		2130
Sum	110012	Sum	11523754	Sum	90239
	2221		221111		1110

14. How do you set down the numbers for addition? Where do you begin to add? If the sum of any column can be expressed by a single figure, what do you do with it? When it cannot, what do you write down? What do you then add to the next column? When you add to the next column, what is it called? What do you set down when you come to the last column?

PROOF OF ADDITION.

15. Begin at the right hand figure of the upper line, and add all the columns downwards, carrying from one column to the other, as before. If the two results agree, the work is supposed right.

SECOND PROOF.

Draw a line under the upper number. Add the lower numbers together, and then add their sum to the upper number. If the last sum is the same as the sum total, first found, the work may be regarded as right.

EXAMPLES.

	(1.)		(2.)		(3.)
	34578		22345		23456
	3750		67890		78901
	87		8752		23456
	328		340		78901
	17		350		23456
	327		78		78901
Sum	39087	Sum	99755	Sum	307071
	4509		77410		
Proof	39087	Proof	99755	Proof	307071

(4.)	(5.)	(6.)
672981043	91278976	8416785413
67126459	7654301	6915123460
39412767	876120	31810213
7891234	723456	7367985
109126	31309	654321
84172	4871	37853
72120	978	2685

15. How do you prove addition by the first method? How do you prove addition by the second method?

7. Add 8635, 2194, 7421, 5063, 2196, and 1245 together. *Ans*. 26754.

8. Add 246034, 298765, 47321, 58653, 64218, 5376, 9821, and 340 together. *Ans*. 730528.

9. Add 27104, 32547, 10758, 6256, 704321, 730491, 2787316, and 2749104 together. *Ans*. 7047897.

10. Add 1, 37, 39504, 6790312, 18757421, and 265 together. *Ans*. 25587540.

11. Add 562163, 21964, 56321, 18536, 4340, 279, and 83 together. *Ans*. ——

12. What is the sum of the following numbers: viz., seventy-five; one thousand and ninety-five; six thousand four hundred and thirty-five; two hundred and sixty-seven thousand; one thousand four hundred and fifty-five; twenty-seven millions and eighteen; two hundred and seventy millions and twenty-seven thousand?

Ans. 297303078.

13. Add together fifty-eight billions, nine hundred and eighty-two millions, four hundred and eighty-seven thousand, six hundred and fifty-four; seven hundred and forty billions, three hundred and fifty millions, five hundred and forty thousand, seven hundred and sixty; four hundred and twenty-five billions, seven hundred and three millions, four hundred and two thousand six hundred and three; thirty-four billions, twenty millions, forty thousand and twenty; five hundred and sixty billions, eight hundred millions, seven hundred thousand and five hundred.

APPLICATIONS.

1. How many days are there in the twelve calendar months? January has 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31.

Ans. 365 *days*.

2. What is the total weight of seven casks of merchandise; viz. No. 1, weighing 960 pounds, No. 2, 725 pounds, No. 3, 830 pounds, No. 4, 798 pounds, No. 5, 698 pounds, No. 6, 569 pounds, No. 7, 987 pounds?

Ans. 5567 *pounds*.

3. A merchant on settling his accounts finds that he owes A 60 dollars, B 150 dollars, C 240 dollars, and to D 100 dollars. How much does he owe in all?

Ans. 550 *dollars.*

4. A man borrowed a sum of money and paid in part 267 dollars, and afterwards paid the remainder, 325 dollars: how much did he borrow? *Ans.* 592 *dollars.*

5. At the Custom House, on the first day of June, there were entered 1800 yards of linen; on the 10th 2500 yards; on the 25th, 600 yards; on the day following, 7500 yards; and the three last days of the month, 1325 yards each day: what was the whole amount entered during the month? *Ans.* ——

6. A farmer has his live-stock distributed in the following manner: in pasture No. 1, there are 5 horses, 14 cows, 8 oxen, and 6 colts; in pasture No. 2, 3 horses, 4 colts, 6 cows, 20 calves, and 12 head of young cattle; in pasture No. 3, 320 sheep, 16 calves, 2 colts, and 5 head of young cattle. How much live-stock had he of each kind, and how many head had he altogether?

Ans. 8 *horses,* 20 *cows,* 8 *oxen,* 12 *colts,* 36 *calves,* 17 *head of young cattle, and* 320 *sheep.* Total live-stock, 421 *head.*

7. What is the interval of time between a transaction which happened 125 years ago, and one that will happen 267 years hence? *Ans.* 392 *years.*

8. A merchant paying his debts pays to one man 763 dollars; to another 4663; to another 37; to several others 49763; to another 6178, and to another 671: how much did he pay in all? *Ans.* 62075.

9. A government paid for repairing and building vessel of war as follows: at one time 4550 dollars; at anothe 247000; at another 936; and at another 7000700: ho much did it pay in all? *Ans.* 725318

10. A debtor pays his creditor at one time 4638 d lars; at another 216; at another 8329; at another 121 how much does he pay in all? *Ans.* 143

11. A bookseller had books on five rows of shelv on one row 5221; on another 7540; on a third 16 on a fourth 5648; on a fifth 7300: how many on al

12. A grocer bought at one time 6214 pounds of raisins; at another 2403 pounds; at another 590; at another 8732; at another 1217, and at another 2464: how many pounds in all? *Ans.* 21620.

13. A man has 7420 hats in one store; in another 612; in another 2541; in another 9103; in another 430; in another 1000: how many in all?

14. A lot of Sicily oranges came in boxes as follows: 1st box 3750; 2d box 216; 3d box 8481; 4th box 275; 5th box 8610; 6th box 2541: how many oranges in all? *Ans.* 23873.

15. A bookbinder bound books on different days, as follows: 423, 315, 531, 414, 612, 234, 621, 414, 711, 144, 621 and 918: how many did he bind in all? *Ans.* 5958.

16. A paper dealer bought paper at different times as follows: 5674 reams, 2004, 8601, 3430, 47, 1101, 7, 24651, 90, and 314: how many reams did he buy in all? *Ans.* 45919.

17. In ten cities and villages the number of houses is as follows: 1728, 26510, 35, 100, 3261, 9, 245, 16408, 6733, and 40000: how many in all? *Ans.* 95029.

18. There are 60 seconds in a minute, 3600 in an hour, 86400 in a day, 604800 in a week, 2419200 in a month, and 31557600 in a year: how many seconds in the time named above? *Ans.* 34671660.

19. Suppose a merchant to buy the following parcels of cloth: 3912 yards, 1856, 2011, 4540, 937, 6338, 3603, 1586, 2044, 2951, 4228, 1345, 1011, 6138, 960, 607, 5150, 13886, 617, 7513, 4079, 743, 612, 2519, 1238, and 2445 yards: how many yards in all?

20. Nine different countries contained inhabitants as follows: 1st country 11260555; 2d, 717103; 3d, 2092014; 4th, 6846949; 5th, 310000; 6th, 40981; 7th, 20827; 8th, 2860; 9th, 2614: how many inhabitants in all the countries? *Ans.* 21293903.

21. A flour merchant bought barrels of flour as follows: in 1st lot 4000; 2d, 570; 3d, 99; 4th, 54; 5th, 273; 6th, 69073; 7th, 4000; 8th, 61998; and in the 9th, 752: how many in all?

22. Suppose a man was born on the 1st of January, 1795: when will he be 85 years old? *Ans.* In 1880.

23. If one man raises 24031 bushels of wheat, another 1320, another 40214, and another 34314; how many bushels are raised in all? *Ans.* 99879.

24. What is the sum of two millions bushels of corn, five hundred and thirty-one thousand bushels, one hundred and twenty bushels, fourteen thousand bushels, thirty thousand and twenty-four bushels, five hundred and sixty bushels, and seven hundred and two bushels?

Ans. 2576406.

25. A merchant bought three bales of cloth; the first contains 61297 yards; the 2d, 100038; the 3d, 289163, yards: how many yards in all? *Ans.* 450498.

26. An army consists of 4000 foot-soldiers, 4006 cavalry or horse, 3093 artillery-men, 1224 riflemen, 1400 pioneers, and 200 miners: what is the whole number of men in the army? *Ans.* 13923.

27. The mail route from Albany to New York is 144 miles, from New York to Philadelphia 90 miles, from Philadelphia to Baltimore 98 miles, and from Baltimore to Washington City 38 miles: what is the distance from Albany to Washington? *Ans.* ——

28. If you travel on one journey 6243 miles; on another 4123 miles; on another 9401, and on another 130 miles; how far do you travel in all?

Ans. 19897 *miles.*

29. A man dying leaves his only daughter nine hundred and ninety-nine dollars, and to each of three sons two hundred dollars more than he left the daughter. What was each son's portion, and what the amount of the whole estate? *Ans.* { Each son's part *dollars.* Whole estate *dollars.* }

30. There are 243 boys in one town; 5021 in another; 7628 in a third; 9207 in a fourth; 64 in a fifth; 5823 in a sixth; 742 in a seventh; 796 in an eighth; 5009 in a ninth; 325 in a tenth; 7426 in an eleventh; 31186 in a twelfth; 987 in a thirteenth; 6954 in a fourteenth; and 2748 in a fifteenth: how many in all?

Ans. 84159.

31. The number of acres of the public lands sold in 1834 was 4658218; in 1835, 12564478; in 1836, 25167833. The number sold in 1840 was 2236889; in 1841, 1164796; in 1842, 1129217. How many acres were sold in the first three, and how many in the last three years?

Ans. { 1st 3*yrs.* 42390529. last " 4530902. }

32. What was the population of the British provinces in North America in 1834, the population of Lower Canada being stated at 549005, of Upper Canada 336461, of New Brunswick 152156, of Nova Scotia and Cape Breton 142548, of Prince Edward's Island 32292, of Newfoundland 75000? *Ans.* 1287462.

33. The library of Alexandria contained 700000 volumes; that of Rome contains about the same; that of Paris 1200000; the other libraries of France about 3000000; those of Germany and St. Petersburgh 2600000; the Vatican library at Rome 360000, and 40000 MSS.; the other libraries of Italy about 700000; the Bodleian library at Oxford, England, about 500000; the library of the British Museum 240000; and the public libraries in the United States about 150000 volumes: how many volumes in the libraries enumerated?

Ans. ——

34. What was the whole number of inhabitants in the United States in 1840; there being in Maine 501,793; New Hampshire 284,574; Vermont 291,948; Massachusetts 737,699; Rhode Island 108,830; Connecticut 309,978; New York 2,428,921; New Jersey 373,306; Pennsylvania 1,724,033; Delaware 78,085; Maryland 469,232; Virginia 1,239,797; North Carolina 753,419; South Carolina 594,398; Georgia 691,392; Alabama 590,756; Mississippi 375,651; Louisiana 352,411; Tennessee 829,210; Kentucky 779,828; Ohio 1,519,467; Indiana 685,866; Illinois 476,183; Missouri 383,702; Arkansas 97,574; Michigan 212,267; Florida 54,477; Wisconsin Territory 30,945; Iowa Territory 43,112; District of Columbia 43,712; Naval Service 6,100?

Ans. 17,068,666.

SUBTRACTION OF SIMPLE NUMBERS.

16. John has 6 apples and Charles has 4: how many more apples has John than Charles? *Ans.* 2.

Two is called the *difference* between the number of apples which John has, and the number of apples which Charles has.

SUBTRACTION *is the process of finding the difference between two numbers.*

The larger of the two numbers is called the *minuend*, the less is called the *subtrahend*, and their difference is called the *remainder*.

OF THE SIGNS.

17. The sign —, is called *minus*, a term signifying less. When placed between two numbers it denotes that the one on the right is to be taken from the one on the left.

Thus, $6-4=2$, denotes that 4 is to be taken from 6. Here 6 is the minuend, 4 the subtrahend, and 2 the remainder.

When the numbers are small, their difference is apparent, and instead of saying, 6 minus 4 equals 2, we say, 4 from 6 leaves 2.

1. John has 9 cents and spends 6: how many has he left?
2. James has 7 marbles and gives 5 to William: how many has he left?
3. Isaac has 10 quills and lends Thomas 4: how many has he left?

---

16. What is Subtraction? What is the larger number called? What is the smaller number called? What is the difference called? In the first example, which number was the minuend? Which the subtrahend? Which the remainder?

17. What is the sign of subtraction? What is it called? What does the term signify? When placed between two numbers what does it denote? When the numbers are small how do you read them, as 6—4?

18. The following table should be committed to memory, and read, two from two, naught remains; two from three, one remains, &c.

SUBTRACTION TABLE.

2−2=0	3−3=0	4−4=0	5−5=0
3−2=1	4−3=1	5−4=1	6−5=1
4−2=2	5−3=2	6−4=2	7−5=2
5−2=3	6−3=3	7−4=3	8−5=3
6−2=4	7−3=4	8−4=4	9−5=4
7−2=5	8−3=5	9−4=5	10−5=5
8−2=6	9−3=6	10−4=6	11−5=6
9−2=7	10−3=7	11−4=7	12−5=7
10−2=8	11−3=8	12−4=8	13−5=8
11−2=9	12−3=9	13−4=9	14−5=9
12−2=10	13−3=10	14−4=10	15−5=10

6−6=0	7−7=0	8−8=0	9−9=0
7−6=1	8−7=1	9−8=1	10−9=1
8−6=2	9−7=2	10−8=2	11−9=2
9−6=3	10−7=3	11−8=3	12−9=3
10−6=4	11−7=4	12−8=4	13−9=4
11−6=5	12−7=5	13−8=5	14−9=5
12−6=6	13−7=6	14−8=6	15−9=6
13−6=7	14−7=7	15−8=7	16−9=7
14−6=8	15−7=8	16−8=8	17−9=8
15−6=9	16−7=9	17−8=9	18−9=9
16−6=10	17−7=10	18−8=10	19−9=10

12−2=10	17−7= how many?
12−3= how many?	16−8= how many?
16−4= how many?	19−9= how many?
11−6= how many?	20−4= how many?
18−9= how many?	13−7= how many?
25−8= how many?	14−2= how many?

When the numbers are small, as in the above examples, they may be subtracted in the mind, without writing them down. When the numbers are large, we write one number under the other, and then make the subtraction by parts.

EXAMPLES.

1. James has 27 apples, and gives 14 to John: how many will he have left?

The 27 is made up of 7 units and 2 tens; and the 14, of 4 units and 1 ten. If then we subtract the units from each other, 3 units will remain, and if we subtract the tens also, one ten will remain. Hence, the remainder is 13.

27	Minuend.
14	Subtrahend
13	

2. What are the remainders in the following examples:

	(1.)	(2.)	(3.)	(4.)
Minuends,	874	972	999	8497
Subtrahends,	642	631	367	7487
Remainders,	232			1010

3. A farmer had 378 sheep, and sold 256: how many has he left?

We first write the number 378, and then the 256 under it, so that units of the same order shall fall under each other. We then take 6 units from 8, 5 tens from 7 tens, and 2 hundreds from 3 hundreds, leaving for the remainder 122.

378
256
122

4. A merchant has 578 dollars in cash, and pays 475 dollars for goods: how much has he left?

Ans. 103 *dollars*.

5. What are the remainders in the following examples:

(1.)	(2.)	(3.)
62843	278846	894862
51720	167504	170641
11123		

6. What is the difference between 4 units of the first order and 1 of the second?

We write the ten, and then place the 4 under the units' place. Then we say, 4 *from* ten leaves 6.

10
4
6

7. What is the difference between two units of the 2d order and 6 of the first?

Here we may say, 6 from 20 leaves 14.

20
6
14

8. What is the difference between 14 and 8?

Having written the 8 under the 14, so that units shall stand under units, we see that 8 cannot be subtracted from 4, but we can say, 8 from 14 leaves 6.

14
8
6

9. What is the difference between 3 units of the 2d order and 7 of the first? What is the difference between 4 tens and 2 tens? What is the difference between 5 tens and 1 ten?

10. What is the difference between 9 units of the second order and 6 of the first?

90
6
84

11. What is the difference between a unit of the second order and a unit of the first order?

10
1
9

12. What is the difference between a unit of the third order and a unit of the second order?

100
10
90

13. What is the difference between four units of the third order and three tens?

400
30
370

14. What is the difference between four units of the 4th order and 2 of the third?

4000
200
3800

15. What is the difference between eight units of the 6th order and 5 of the 4th?

800000
5000
795000

EXAMPLES ILLUSTRATING PRINCIPLES.

19. We are now going to explain the process of making the subtraction when a figure of the subtrahend is greater than that of the minuend directly over it.

Beginning with the units of the lowest order, we say, 2 from 3 leaves 1, which we write down in the units' place. At the next step we meet a difficulty, for we cannot subtract 6 from 4. If, now, we add 10 to the 4, (which is written in small figures above,) and 10 also to the 6 directly under it, it is plain that the difference will not be affected, since both the numbers will thus be equally increased. But adding 10 to 6 is the same thing as adding 1 to the figure next to the left: hence,

OPERATION.

$$\begin{array}{r} {}^{10} \\ 843 \\ 562 \\ {}^{1} \\ \hline 281 \end{array}$$

We may consider 10 *to be added to any figure of the minuend, provided we add* 1 *to the next figure of the subtrahend to the left.*

20. Hence, to find the difference between two numbers, we have the following

RULE.

I. *Set down the less number under the greater, so that units of the same order shall fall under each other, and beginning with the simple units, subtract each figure from the one directly over it.*

II. *When any figure of the minuend is less than the one directly under it, suppose ten to be added and then make the subtraction; after which, add one to the next figure of the subtrahend and subtract as before.*

---

19. Can you subtract a greater number from a less? When the upper figure is the least, how do you proceed? Does this change the difference between the numbers? What then may we always do?

20. How do you set down the numbers for subtraction? Where do you begin to subtract? How do you subtract? Give the rule. How do you prove subtraction?

PROOF.

Add the remainder to the subtrahend. If the sum is equal to the minuend the work may be regarded as right.

EXAMPLES.

	(1.)	(2.)	(3.)
Minuends,	8592678	67942139	219067803
Subtrahends,	1078953	9756783	104202196
Remainders,	7513725		
Proofs,	8592678	67942139	219067803

	(4.)	(5.)	(6.)	(7.)	(8.)
	10000	30000	67987	100000	87000
	4	9999	40000	1	1009
Remainders,	9996				85991

9. From 2637804 take 2376982. *Ans.* 260822.
10. From 3762162 take 826541. *Ans.* 2935621.
11. From 78213609 take 27821890. *Ans.* ——
12. From thirty thousand and ninety-seven, take one thousand six hundred and fifty-four. *Ans.* 28443.
13. From one hundred million two hundred and forty-seven thousand, take one million four hundred and nine. *Ans.* 99246591.
14. Subtract one from one million. *Ans.* ——

APPLICATIONS.

1. Suppose John were born in eighteen hundred and fifteen, and James in eighteen hundred and twenty-five: what is the difference of their ages? *Ans.* 10 *years.*

2. A man was born in 1785: what was his age in 1830? *Ans.* 45 *years.*

3. Suppose I lend a man 1565 dollars, and he dies, owing me 450 dollars: how much had he paid me? *Ans.* 1115 *dollars.*

4. In five bags are different sums of money to the amount in all of 1000 dollars. In the first there are 100 dollars; in the second, 314 dollars; in the third, 143

dollars; and in the fourth, 209 dollars: how many dollars does the fifth contain? *Ans.* 234 *dollars.*

5. America was discovered by Christopher Columbus in the year 1492. What number of years has since elapsed?

6. George Washington was born in the year 1732, and died in 1799: how old was he at the time of his death? *Ans.* ——

7. The declaration of independence was published July 4th, 1776: how many years to July 4th, 1838?
Ans. 62 *years.*

8. By the census of 1840, it appeared, that the white population of the United States was 14,189,108, and the number of blacks 2,873,458: how much did the white population exceed the black? *Ans.* 11,315,650.

9. In 1840 there were in the State of New York 2,428,921 inhabitants, and in the State of Pennsylvania 1,724,033 inhabitants: how many more inhabitants were there in New York than in Pennsylvania? *Ans.* 704888.

10. The revolutionary war began in 1775; the late war in 1812: what time elapsed between their commencements? *Ans.* 37 *years.*

11. In 1840 there were in New York, which is the largest city in the United States, 312,710 inhabitants, and in Philadelphia, the next largest city, 258,037: how many more inhabitants were there in New York, than in Philadelphia? *Ans.* 54,673.

12. A man dies worth 1200 dollars; he leaves 504 to his daughter, and the remainder to his son: what was the son's portion? *Ans.* ——

13. Suppose a gentleman has an income of 3090 dollars a year, and pays for taxes 150 dollars, and expends besides 253 dollars: how much does he lay up?
Ans. 2687 *dollars.*

14. A merchant bought 500 barrels of flour for 3500 dollars; he sold 250 barrels for 2000 dollars: how many barrels remained on hand, and how much must he sell them for, that he may lose nothing?
Ans. 250 *barrels* remained, and he must sell for 1500 *dollars.*

15. The tune of Yankee Doodle was composed by a doctor of the British army to ridicule the Americans in 1755: how many years to the present time?

16. Lord Cornwallis surrendered at Yorktown, and marched into the American lines in 1781 to the tune of Yankee Doodle: how many years was it after the tune was composed? *Ans.* 26.

17. There are 4338472 children in the United States, between the ages of 5 and 15, of this number 2477667 are in schools: how many are out of schools?
Ans. 1860805.

18. The circulation of the blood was discovered in 1616: how many years to 1839? *Ans.* 223.

19. Henry Hudson sailed up the Hudson river in 1609: how many years since?

20. Pliny the historian died 17 years after Christ: how many years before the declaration of independence?
Ans. 1759.

21. Potatoes were carried to Ireland from America in 1565: how many years was it before the settlement of Plymouth, in 1620? *Ans.* 55.

22. If you have 35720 dollars and lose 9100: how many will you have left? *Ans.* 26620.

23. If you buy 8150 penknives, and lose 1634: how many will you have left? *Ans.* 6516.

24. The Mariner's Compass was discovered in England in the year 1302: how many years was this before the discovery of America in 1492? How many years to the present time? *Ans.* 190.

25. If you buy 1853 pounds of raisins, and give away 1370: how many pounds will you have left?
Ans. 483.

26. Subtract 4261 from 705684.

27. From 8473 take 1528. *Ans.* 6945.

28. Subtract 90462372 from 905106392.

29. A merchant bought 1675 yards of cloth, for which he paid 5025 dollars; he then sold 335 yards for 1005 dollars: how much had he left, and what did it cost him?

APPLICATIONS IN ADDITION AND SUBTRACTION.

1. A merchant buys 19576 yards of cloth of one person, 27580 yards of another, and 375 yards of a third; he sells 1050 yards to one customer, 6974 yards to another, and 10462 yards to a third: how many yards has he remaining? *Ans*. 29045.

2. A person borrowed of his neighbor at one time 355 dollars, at another time 637 dollars, and 403 dollars at another time: he then paid him 977 dollars. How much did he owe him? *Ans*. 418.

3. I have a fortune of 2543 dollars to divide among my four sons, James, John, Henry, and Charles. I give James 504 dollars, John 600 dollars, and Henry 725: how much remains for Charles? *Ans*. ——

4. I have a yearly income of ten thousand dollars. I pay 275 for rent, 220 dollars for fuel, 35 dollars to the doctor, and 3675 dollars for all my other expenses: how much have I left at the end of the year? *Ans*. 5795.

5. A man pays 300 dollars for 100 sheep, 95 dollars for a pair of oxen, 60 dollars for a horse, and 125 dollars for a chaise. He gives in return 100 bushels of wheat worth 125 dollars, a cow worth 25 dollars, a colt worth 40 dollars, and pays the rest in cash: what amount of money does he pay? *Ans*. 390 *dollars*.

6. A man owing 3456 dollars, paid at one time 350, at another 456, at another 675: how much did he still owe? *Ans*. 1975 *dollars*.

7. There are 12000 dollars in 6 boxes: the first contains 1240; 2d, 1346; 3d, 2012; 4th, 2101; 5th, 346: how much is contained in the 6th?

8. Tell me the difference between 10000 and 44. *Ans*. 9956.

9. The amount of the school fund in Virginia is 1551857 dollars, and in Connecticut 2027402: what is the difference? *Ans*. 475545 *dollars*.

10. There are about 805000000 inhabitants in the world. Of this number Asia contains 450000000, Europe 233500000, America 46500000, Oceanica 18000000: *how many* in Africa? *Ans*. 57000000.

11. Suppose you gain 34568 dollars, then 12456; a second time you gain 2467 and lose 2365; and a third time you gain 41210 and lose 39300: how much will you have left?

12. A merchant owes 450120 dollars, and has property as follows: bank stock 350000 dollars, western lands valued at 225100, furniture worth 4000 dollars, and a store of goods worth 96000: how much is he worth?

Ans. 224980 *dollars.*

13. How much greater is 1200, than 365 and 721 added together?

14. What other number with these four, viz., 2100, 3200, 1600 and 1200, will make 10000? *Ans.* 1900.

15. If a man's income is 3467 dollars a year, and he spends 269 dollars for clothing, 467 for house rent, 879 for provision, and 146 for travelling, how much will he have left at the end of the year? *Ans.* 1706.

16. A man gains 367 dollars, then loses 423; a second time he gains 875 and loses 912; he then gains 1012 dollars: how much has he gained in all?

Ans. 919 *dollars.*

17. A merchant paid 39246 dollars for a cargo of molasses, and after selling it found that he had cleared 2406 dollars: for what did he sell it? *Ans.* 41652.

18. If I agree to pay a man 36 dollars for ploughing 25 acres of land, 200 dollars for fencing it, and 150 for cultivating it, how much shall I owe him after paying 331 dollars? *Ans.* 55 *dollars.*

19. A merchant bought 85 hogsheads of sugar for 28675 dollars, paid 1231 dollars freight, and then sold it for 1683 dollars less than it cost him: how much did he receive for it?

20. There are 31769000 inhabitants in North America, 14731000 in South America, and in Europe 233500000: how many more in Europe than in the two Americas?

Ans. 187000000.

21. If I buy 489 oranges for 912 cents, and sell 125 for 186 cents, and then sell 134 for 199 cents, how many will be left, and how much will they have cost me?

MULTIPLICATION OF SIMPLE NUMBERS.

21. If Charles gives 2 cents apiece for two oranges: how much do they cost him? *Ans.* 4 *cents.*

If Charles gives 2 cents apiece for 3 oranges: how much do they cost him? *Ans.* ——

If he gives 2 cents apiece for 4 oranges: how much do they cost him? *Ans.* 8 *cents.*

The cost in each case, may be obtained by adding the price of the separate oranges; thus,

$2+2=4$ cents, the cost of 2 oranges,
$2+2+2=6$ cents, the cost of 3 oranges,
$2+2+2+2=8$ cents, the cost of 4 oranges.

In the first case 2 is taken *two times*, in the second case it is taken *three times*, in the third, *four times;* and in a similar manner any number may be taken as often as we please by adding it continually to itself.

MULTIPLICATION *is a short method of taking one number as many times as there are units in another.*

The number to be taken is called the *multiplicand.*

The number denoting how many times the multiplicand is to be taken, is called the *multiplier.*

The number arising from taking the multiplicand as many times as there are units in the multiplier, is called the *product.*

The multiplicand and multiplier are called *factors*, or *producers* of the *product.*

The sign $\times$, placed between two numbers, denotes that they are to be multiplied together. It is called, the *sign of multiplication.*

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21. What is multiplication? What is the number called which is to be taken? What does the multiplier denote? What is the product? In the case of the two oranges, which is the multiplicand? Which is the multiplier? Which is the product? In the case of three oranges, which is the multiplicand, which the multiplier, and which the product? What are the multiplicand and multiplier called? How do you denote that two numbers are to be multiplied together? What is the sign called?

MULTIPLICATION TABLE.

1 times 0 is 0	4 times 0 are 0	7 times 0 are 0
1 times 1 is 1	4 times 1 are 4	7 times 1 are 7
1 times 2 is 2	4 times 2 are 8	7 times 2 are 14
1 times 3 is 3	4 times 3 are 12	7 times 3 are 21
1 times 4 is 4	4 times 4 are 16	7 times 4 are 28
1 times 5 is 5	4 times 5 are 20	7 times 5 are 35
1 times 6 is 6	4 times 6 are 24	7 times 6 are 42
1 times 7 is 7	4 times 7 are 28	7 times 7 are 49
1 times 8 is 8	4 times 8 are 32	7 times 8 are 56
1 times 9 is 9	4 times 9 are 36	7 times 9 are 63
1 times 10 is 10	4 times 10 are 40	7 times 10 are 70
1 times 11 is 11	4 times 11 are 44	7 times 11 are 77
1 times 12 is 12	4 times 12 are 48	7 times 12 are 84
2 times 0 are 0	5 times 0 are 0	8 times 0 are 0
2 times 1 are 2	5 times 1 are 5	8 times 1 are 8
2 times 2 are 4	5 times 2 are 10	8 times 2 are 16
2 times 3 are 6	5 times 3 are 15	8 times 3 are 24
2 times 4 are 8	5 times 4 are 20	8 times 4 are 32
2 times 5 are 10	5 times 5 are 25	8 times 5 are 40
2 times 6 are 12	5 times 6 are 30	8 times 6 are 48
2 times 7 are 14	5 times 7 are 35	8 times 7 are 56
2 times 8 are 16	5 times 8 are 40	8 times 8 are 64
2 times 9 are 18	5 times 9 are 45	8 times 9 are 72
2 times 10 are 20	5 times 10 are 50	8 times 10 are 80
2 times 11 are 22	5 times 11 are 55	8 times 11 are 88
2 times 12 are 24	5 times 12 are 60	8 times 12 are 96
3 times 0 are 0	6 times 0 are 0	9 times 0 are 0
3 times 1 are 3	6 times 1 are 6	9 times 1 are 9
3 times 2 are 6	6 times 2 are 12	9 times 2 are 18
3 times 3 are 9	6 times 3 are 18	9 times 3 are 27
3 times 4 are 12	6 times 4 are 24	9 times 4 are 36
3 times 5 are 15	6 times 5 are 30	9 times 5 are 45
3 times 6 are 18	6 times 6 are 36	9 times 6 are 54
3 times 7 are 21	6 times 7 are 42	9 times 7 are 63
3 times 8 are 24	6 times 8 are 48	9 times 8 are 72
3 times 9 are 27	6 times 9 are 54	9 times 9 are 81
3 times 10 are 30	6 times 10 are 60	9 times 10 are 90
3 times 11 are 33	6 times 11 are 66	9 times 11 are 99
3 times 12 are 36	6 times 12 are 72	9 times 12 are 108

10 times 0 are 0	11 times 0 are 0	12 times 0 are 0
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

QUESTIONS SHOWING THE USE OF THE TABLE.

$9\times 8=$ how many?	$7\times 8=$ how many?
$1\times 2\times 3=$ how many?	$1\times 6\times 9=$ how many?
$1\times 4\times 5=$ how many?	$1\times 9\times 12=$ how many?
$2\times 6\times 5=$ how many?	$5\times 2\times 11=$ how many?
$3\times 4\times 9=$ how many?	$7\times 1\times 12=$ how many?
$4\times 3\times 11=$ how many?	$9\times 1\times 9=$ how many?
$5\times 2\times 9=$ how many?	$11\times 1\times 7=$ how many?
$6\times 2\times 5=$ how many?	$12\times 1\times 5=$ how many?

1. What is the cost of 7 pounds of butter at 12 cents a pound?

2. What is the cost of 12 pounds of tea at 6 shillings a pound?

3. What is the cost of 12 pounds of coffee at 9 cents a pound?

4. What is the cost of 11 yards of cloth at 6 dollars a yard?

5. What is the cost of 9 books at 11 cents each?

6. What is the cost of 12 pencils at 8 cents apiece?

7. What is the cost of 10 pairs of shoes at 2 dollars a pair?

8. What is the cost of 12 pairs of stockings at 3 shillings a pair?

9. What is the cost of 11 hens at 12 cents apiece?

10. What is the cost of 12 inkstands at 12 cents apiece?

11. What is the cost of 9 pairs of shoes at 10 shillings a pair?

12. What is the cost of 9 chairs at 6 dollars apiece?

13. What is the cost of 7 yards of cloth at 8 dollars a yard?

14. What is the cost of 9 handkerchiefs at 7 shillings apiece?

15. How many dollars must I pay for 9 yards of cloth at 8 dollars a yard?

16. If 5 bushels of wheat make one barrel of flour, how many bushels will it take to make 8 barrels?

17. There are nine rows of apple-trees in a field, and 11 apple-trees in each row: how many apple-trees are there in the field?

18. If I can earn 5 dollars in one month, how much can I earn in 3 months? In 4? In 5? In 9? In 11? In 12?

19. If a father gathers 12 bushels of corn in a day, and his son 8, how much would each gather in 3 days? In 5? In 7? In 9? In 10? In 12?

20. A man hires a horse and sleigh; he pays 12 cents a mile for the horse, and 7 cents for the sleigh: after riding 12 miles how much must he pay for them both?

21. If a man eats 11 ounces of bread in a day, and his wife 9, how much will each eat in 11 days? How much will they both eat in that time?

22. Jane bought 12 yards of riband at 8 cents a yard, and Mary 11 yards at 7 cents a yard: how much did each pay, and how much was paid by both of them?

23. A merchant bought 7 pieces of cloth, each containing 12 yards; and 9 other pieces, each containing 11 yards; how many yards in each lot, and how many yards in all?

24. What is the product of 3 tens multiplied by 5? Of 6 tens multiplied by 2?

25. What is the product of one unit of the third order multiplied by 2? By 3? By 4?

26. A man buys 12 sheep at 3 dollars apiece, and sells them at 2 dollars apiece: how much does he lose?

27. John bought 6 tops at 8 cents apiece, 3 knives at 30 cents each, and 11 quills at a penny a piece: what did he pay for all?

28. A merchant bought 8 packages of penknives, each package containing 12 knives; 9 other packages, each containing 11 knives, and 7 other packages, each containing 5 knives: how many knives in each package, and how many in all of them?

EXAMPLES ILLUSTRATING PRINCIPLES.

22. Let it be required to multiply 4 by 3, and also to multiply 5 by 3.

OPERATION.

Multiplicand. Multiplier.

$$4 \times 3 = \begin{cases} 4 \\ 4 \\ 4 \\ \overline{12} \text{ Product.} \end{cases}$$

OPERATION.

Multiplicand. Multiplier.

$$5 \times 3 = \begin{cases} 5 \\ 5 \\ 5 \\ \overline{15} \text{ Product.} \end{cases}$$

From these examples we see, that the product of 4 multiplied by 3 is 12, the number which arises from adding three 4's together; and that the product of 5 by 3 is equal to 15, the number which arises from adding three 5's together.

We see from the above examples, that *any product may be found by setting down the multiplicand as many times as there are units in the multiplier, and adding all the numbers together.*

MULTIPLICATION *is therefore a short method of addition.*

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22. How may any product be found? How then may multipli*cation be considered?*

23. Let it be required to multiply 236 by 4.

It is required to take 236, 4 times. Now 236 is made up of 6 units, 3 tens, and 2 hundreds, and the units of each order must be taken 4 times.

First, set down the 236; then write the 4 under the units' place and draw a line beneath it. Next, multiply the 6 by 4:—the product is 24 units; set them down. Then multiply the 3 tens by 4: the product is 12 tens; set down the 2 in the tens' place, leaving the 1 to the left, in the place of hundreds. Next multiply the 2 by 4: the product is 8, which being hundreds, is set down under the 1. The sum of these numbers, 944, is the entire product.

OPERATION.

236	
4	
24	units.
12	tens.
8	hundreds.
944	Product.

The product can also be found thus: say 4 times 6 are 24: set down the 4, and then say, 4 times 3 are 12 and 2 to carry are 14: set down the 4, and then say, 4 times 2 are 8 and 1 to carry are 9. Set down the 9, and the product is 944 as before.

OPERATION.

$$\begin{array}{r} 236 \\ 4 \\ \hline 944 \end{array}$$

Hence, we see that, *if units of any order be multiplied by simple units, the unit of the product will be of the same order as that expressed by the figure of the multiplicand.*

24. Let it be required to multiply 506 by 302.

In this example, we say, 2 times 6 are 12: then set down the 2, and say, 2 times 0 are 0 and 1 to carry make 1. Then say, 2 times 5 are 10. Then beginning with the 0, we say, 0 times 6 is 0: set down the 0. Then say, 0 times 0 is 0, and 0 times 5 is 0. Then multiply by the 3 hundreds, and set down the first figure 8 in the place of hundreds, and place the other figures to the left.

OPERATION.

$$\begin{array}{r} 506 \\ 302 \\ \hline 1012 \\ 000 \\ 1518 \\ \hline 152812 \end{array}$$

23. If the number 236 be multiplied by 4, how many times is it to be taken? How many times is each order of units to be taken? If units of any order be multiplied by simple units, what is the order of the units of the product?

When a 0 appears in the multiplier, we need not multiply by it, since each of the products is 0; but when we multiply by the next figure to the left, we must observe to set the first figure of the product directly under its multiplier.

CASE I.

25. When the multiplier does not exceed 12.

RULE.

I. *Set down the multiplicand and under it set the multiplier, so that units of the same order shall fall under each other, and draw a line beneath.*

II. *Multiply every figure of the multiplicand by the multiplier, setting down and carrying as in addition.*

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
867901	278904	678741	3021945
1	2	3	4
867901			12087780

(5.)	(6.)	(7.)	(8.)
28432	82798	6789	49604
8	9	11	12
227456			595248

(9.)	(10.)	(11.)	(12.)
84763	67046	470362	189104
5	11	9	10

13. A merchant sold 104 yards of cotton sheeting at 9 cents a yard: what did he receive for it?

14. A farmer sold 309 sheep at four dollars apiece: how much did he receive?

15. Mrs. Simpkins purchased 149 yards of table linen at two dollars a yard: how much did she pay for it?

24. When a 0 is found in the multiplier need you multiply by it? When you multiply by the next figure to the left, where do you place the first figure of the product?

25. When the multiplier does not exceed 12, how do you set it down? How do you multiply by it?

CASE II.

26. When the multiplier exceeds 12.

RULE.

I. *Set down the multiplier under the multiplicand, so that units of the same order shall fall under each other, and draw a line beneath.*

II. *Begin with the right hand figure, and multiply all the figures of the multiplicand by each figure of the multiplier, setting down and carrying to the next product as in addition: observing also to write the first figure of each product directly under its multiplier.*

III. *Add up the several products and their sum will be the product sought.*

NOTE 1. There are three numbers in every multiplication. First, the multiplicand: second, the multiplier: and third, the product.

NOTE 2. *In multiplication either of the factors may be used as the multiplier without altering the product.*

Let it be required to multiply 5 by 3.

OPERATION.

	5				
3	1	1	1	1	1
	1	1	1	1	1
	1	1	1	1	1

Place as many ones in a horizontal row as there are units in the multiplicand, and make as many rows as there are units in the multiplier: the product will then be equal to one row taken as many times as there are rows; that is, to the whole number of ones: viz., 15. But if we consider the number of ones in a vertical row to be the multiplicand, and the number of vertical rows the multiplier, the product will then be equal to a vertical row taken as many times as there are vertical rows; that is, it will be equal to the whole number of ones: viz., 15. Hence,

26. When the multiplier exceeds 12, how do you set it down? How do you multiply by it? How do you add up? How many numbers are there in every multiplication? Name them? Is the product of two numbers altered by changing the multiplicand into the multiplier? Is 7 multiplied by 8 the same as 8 multiplied by 7?

The product of two numbers is the same whichever factor is used as the multiplier.

EXAMPLES.

$3 \times 7 = 7 \times 3 = 21$: also, $6 \times 3 = 3 \times 6 = 18$.
$9 \times 5 = 5 \times 9 = 45$; also, $8 \times 6 = 6 \times 8 = 48$.
and, $8 \times 7 = 7 \times 8 = 56$: also, $5 \times 7 = 7 \times 5 = 35$.

PROOF OF MULTIPLICATION.

27. Write the multiplicand in the place of the multiplier and find the product as before: if the two products are the same, the work is supposed right.

EXAMPLES IN MULTIPLICATION.

1. Multiply 365 by 84: also, 37864 by 209.

	(1.)	(2.)	(3.)	(4.)
Multiplicand	365	37864	34293	47042
Multiplier	84	209	74	91
	1460			
	2920			
Product	30660			4280822

(5.)	(6.)	(7.)	(8.)
46834	679084	1098731	8971432
406	126	1987	10471
19014604			

9. Multiply 12345678 by 32. *Ans.* 395061696.
10. Multiply 9378964 by 42. *Ans.* 393916488.
11. Multiply 1345894 by 49. *Ans.* 65948806.
12. Multiply 576784 by 64. *Ans.* 36914176.
13. Multiply 596875 by 144. *Ans.* 85950000.
14. Multiply 46123101 by 72. *Ans.* ——
15. Multiply 61835720 by 132. *Ans.* 8162315040.
16. Multiply 718328 by 96. *Ans.* 68959488.
17. Multiply 7128368 by 1440. *Ans.* 10264849920.

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27. How do you prove multiplication?

18. Multiply 6795634 by 918546.
Ans. 6242102428164.

19. Multiply 86972 by 1208. *Ans.* ——

20. Multiply 1055054 by 570. *Ans.* 601380780.

21. Multiply 538362 by 9258. *Ans.* 4984155396.

22. Multiply 50406 by 8050. *Ans.* 405768300.

23. Multiply 523972 by 15276. *Ans.* ——

24. Multiply 760184 by 16150. *Ans.* 12276971600.

25. Multiply 1055070 by 31456. *Ans.* 33188281920.

26. Multiply 91874163 by 27498765.
Ans. 2526426017908695.

CASE III.

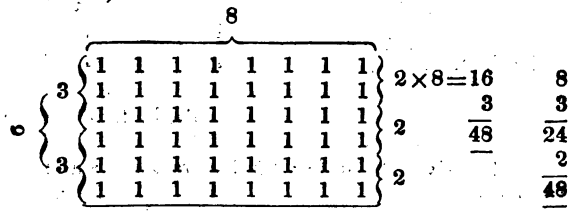
28. A composite number is one that may be produced by the multiplication of two or more numbers, which numbers are called the *components* or *factors*.

Thus, $2 \times 3 = 6$. Here 6 is the composite number, and 2 and 3 are the factors or components.

The number $16 = 8 \times 2$: here 16 is a composite number, and 8 and 2 are the factors; and since $4 \times 4 = 16$, we may also regard 4 and 4 as factors or components of 16. Sixteen has three factors: for $2 \times 2 \times 4 = 16$. It also has four: for $2 \times 2 \times 2 \times 2 = 16$.

EXAMPLES ILLUSTRATING PRINCIPLES.

Let it be required to multiply 8 by the composite number 6, in which the factors are 2 and 3.



28. What is a composite number? Is 6 a composite number? What are its components or factors? What are the factors of the composite number 16? What are the factors of the composite number 12? How do you multiply when the multiplier is a composite number?

If we write 6 horizontal lines with 8 units in each, it is evident that the product of $8\times6=48$, is the number of units in all the lines.

But let us first connect the lines in sets of two each, as on the right; the number in each set will then be expressed by $8\times2=16$. But there are 3 sets; hence, the number of units in all the sets is $16\times3=48$.

If we divide all the lines into sets of 3 each, as on the left, the number of units in each set will be equal to $8\times3=24$, and there being two sets, the whole number of units will be expressed by $24\times2=48$.

Hence, when the multiplier is a composite number, we have the following

RULE.

Multiply by each of the factors in succession, and the last product will be the entire product sought.

EXAMPLES.

1. Multiply 327 by 12.

The factors of 12, are 2 and 6, or they are 3 and 4, or they are 3, 2 and 2: for, $2\times6=12$, $3\times4=12$, and $3\times2\times2=12$.

	327	327		327
	6	3		3
	1962	981		981
	2	4		2
Product,	3924	3924		1962
				2
			Product,	3924

2. Multiply 5709 by 48; the factors being 8 and 6, or 16 and 3. *Ans.* —
3. Multiply 342516 by 56. *Ans.* 1918089
4. Multiply 209402 by 72. *Ans.* —
5. Multiply 937387 by 54. *Ans.* 506188
6. Multiply 91738 by 81. *Ans.* —
7. Multiply 3842 by 144. *Ans.* 5532

CASE IV.

29. When the multiplier is 1 and any number of ciphers after it, as 10, 100, 1000, &c.

Placing a cipher on the right of a number changes the units' place into tens, the tens into hundreds, the hundreds into thousands, &c., and *therefore increases the number ten times.*

Thus, 5 is increased ten times by making it 50. The addition of two ciphers increases a number *one hundred times*; the addition of three ciphers, *a thousand times, &c.* Thus, 6 is increased a hundred times by making it 600, and a thousand times, by making it 6000.

Hence, to multiply by 1 and any number of ciphers we have the following

RULE.

Annex, or place on the right of the multiplicand as many ciphers as there are in the multiplier, and the number so formed will be the required product.

EXAMPLES.

1. Multiply 254 by 10. *Ans.* 2540.
2. Multiply 648 by 100. *Ans.* ——
3. Multiply 7987 by 1000. *Ans.* 7987000.
4. Multiply 9840 by 10000. *Ans.* ——
5. Multiply 3750 by 100. *Ans.* 375000.

CASE V.

30. When there are ciphers on the right hand of one or both of the factors. In this case each number so formed may be regarded as a composite number, of which the significant figures will be one factor, and 1 with the requisite number of ciphers annexed, the other. Hence, we have the following

---

29. If you place one cipher on the right of a number, what effect has it on its value? If you place two, what effect has it? If you place three? And for any number of ciphers, how much will each increase it? How do you multiply by 10, 100, 1000, &c.?

30. When there are ciphers on the right hand of one or both the factors, how do you multiply?

RULE.

Neglect the ciphers and multiply the significant figures; then place as many ciphers to the right hand of the product, as there are in both of the factors.

EXAMPLES.

(1.)	(2.)	(3.)
76400	7532000	416000
24	580	357000
1833600		148512000000

4. 4871000×270000. *Ans.* 1315170000000.
5. 296200×875000. *Ans.* ——
6. 3456789×567090. *Ans.* 1960310474010.
7. 21200×70. *Ans.* ——
8. 359260×304000. *Ans.* 109215040000.
9. 7496430×695000. *Ans.* 5210018850000.

APPLICATIONS.

1. There are ten bags of coffee, each containing 48 pounds: how much coffee is there in all the bags? *Ans.* 480 *lbs.*

2. There are 20 pieces of cloth, each containing 37 yards, and 49 other pieces, each containing 75 yards: how many yards of cloth are there in all the pieces? *Ans.* 4415 *yards.*

3. There are 24 hours in a day, and 7 days in a week: how many hours in a week? *Ans.* 168 *hours.*

4. A merchant buys a piece of cloth containing 97 yards, at 3 dollars a yard: what does the piece cost him? *Ans.* ——

5. A farmer bought a farm containing 10 fields; three of the fields contained 9 acres each; three other of the fields 12 acres each; and the remaining 4 fields, each 15 acres: how many acres were there in the farm, and how much did the whole cost at 18 dollars an acre?

Ans. { The farm contained 123 *acres.*
It cost 2214 *dollars.* }

6. Suppose a man were to travel 32 miles a day: how far would he travel in 365 days? *Ans.* 11680 *miles.*

7. A merchant bought 49 hogsheads of molasses, each containing 63 gallons: how many gallons of molasses were there in the parcel? *Ans.* 3087 *gallons.*

8. In a certain city, there are 3751 houses. If each house on an average contains 5 persons, how many inhabitants are there in the city?

Ans. 18755 *inhabitants.*

9. If a regiment of soldiers contains 1128 men, how many men are there in an army of 106 regiments?

Ans. 119568.

10. If 786 yards of cloth can be made in one day, how many yards can be made in 1252 days?

11. If 30009 cents are paid for one man's services, how many cents would be paid to 814 men, each man receiving the same wages? *Ans.* 24427326.

12. If in one granary there are 375296321 kernels, how many kernels would there be in 79024 granaries?

13. If one farm costs 2730 dollars, how much would 8 farms cost? *Ans.* 21840.

14. Multiply 123456789 by 987654321.

15. What will 397 barrels of flour cost at 7 dollars a barrel? *Ans.* 2779 *dollars*

16. What will 596*hhd.* of sugar cost at 74 dollars a *hhd.*?

17. If a vessel sails 169 miles a day, how many miles will she sail in 576 days? *Ans.* 97344.

18. What will 687 yoke of oxen cost at 73 dollars a yoke?

19. If one man travels 74 miles in one day, how many miles will he travel in 365 days? *Ans.* 27010.

20. What will 698 heifers cost at 14 dollars per head?

21. Multiply 6176777 by 22222.

Ans. 137260338494.

22. What will 616783 yards of calico cost at 36 cents per yard?

23. There are 320 rods in a mile; how many rods are there in the distance from St. Louis to New Orleans, which is 1092 miles? *Ans.* 349440.

24. What will 46000 bushels of potatoes cost at 34 cents per bushel?

25. Suppose a book to contain 470 pages, 45 lines on each page, and 50 letters in each line: how many letters in the book? *Ans.* 1057500.

26. Supposing a crew of 250 men to have provisions for 30 days, allowing each man 20 ounces a day: how many ounces have they?

27. There are 350 rows of trees in a large orchard, 125 trees in each row, and 3000 apples on each tree: how many apples in the orchard? *Ans.* 131250000.

31. When a person sells goods he generally gives with them a bill, showing the amount charged for them, and acknowledging the receipt of the money paid; such bills are usually called *Bills of Parcels.*

BILLS OF PARCELS.

New York, Oct. 1, 1846.

James Johnson *Bought of W. Smith.*

4 Chests of tea, of 45 pounds each, at 1 doll. a pound.
3 Firkins of butter at 17 dolls. per firkin - - -
4 Boxes of raisins at 3 dolls. per box - - - -
36 Bags of coffee at 16 dolls. each - - - - -
14 Hogsheads of Molasses at 28 dolls. each - - -

Amount 1211 *dollars.*

Received the amount in full,

W. Smith.

Hartford, Nov. 1, 1846.

James Hughes *Bought of W. Jones.*

27 Bags of coffee at 14 dollars per bag - - - -
18 Chests of tea at 25 dolls. per chest - - - - -
75 Barrels of shad at 9 dolls. per barrel - - - - -
87 Barrels of mackerel at 8 dolls. per barrel - -
67 Cheeses at 2 dolls. each - - - - - - - -
59 Hogsheads of molasses at 29 dolls. per hogshead

Amount 4044 *dollars.*

Received the amount in full, for W. Jones,
per *James Cross.*

---

31. What are bills of parcels?

DIVISION OF SIMPLE NUMBERS.

32. Charles has 12 apples, and wishes to divide them equally between his four brothers.

He gives one to each, which takes 4. Subtracting 4 from 12, 8 remains. He then gives another to each, which takes 4 more. He then gives one more to each, which takes all his apples. He has thus divided them equally, and found that 12 *contains* 4, three times.

OPERATION.

```
12
 4
--
 8  1st remain.
 4
--
 4  2d remain.
 4
--
 0  3d remain.
--
```

We can arrive at the same result by a shorter method, called *Division*.

DIVISION *is the process of finding how many times one number is contained in another. It is a short method of subtraction.*

The number by which we divide is called the *divisor*.

The number to be divided is called the *dividend*.

The number expressing how many times the dividend contains the divisor, is called the *quotient*.

If there is a number left, it is called the *remainder*, which is always less than the *divisor*.

33. There are three signs used to denote division. They are the following:

$18 \div 4$ expresses that 18 is to be divided by 4.

$\frac{18}{4}$ expresses that 18 is to be divided by 4.

4)18 expresses that 18 is to be divided by 4.

When the last sign is used, a curved line is also drawn on the right of the dividend to separate it from the quotient, which is generally set down on the right.

---

32. When Charles divides 12 apples equally, among his four brothers, how many does he give to each? How many times does 12 contain 4? Which number is the dividend? Which the divisor? Which the quotient? What is division? What is the number to be divided called? What is the number called by which we divide? What is the answer called? What is the number called which is left? Is it greater or less than the divisor?

33. How many signs are there in division? Make them?

DIVISION TABLE.

1 in 1 1 time	5 in 5 1 time	9 in 9 1 time
1 in 2 2 times	5 in 10 2 times	9 in 18 2 times
1 in 3 3 times	5 in 15 3 times	9 in 27 3 times
1 in 4 4 times	5 in 20 4 times	9 in 36 4 times
1 in 5 5 times	5 in 25 5 times	9 in 45 5 times
1 in 6 6 times	5 in 30 6 times	9 in 54 6 times
1 in 7 7 times	5 in 35 7 times	9 in 63 7 times
1 in 8 8 times	5 in 40 8 times	9 in 72 8 times
1 in 9 9 times	5 in 45 9 times	9 in 81 9 times
2 in 2 1 time	6 in 6 1 time	10 in 10 1 time
2 in 4 2 times	6 in 12 2 times	10 in 20 2 times
2 in 6 3 times	6 in 18 3 times	10 in 30 3 times
2 in 8 4 times	6 in 24 4 times	10 in 40 4 times
2 in 10 5 times	6 in 30 5 times	10 in 50 5 times
2 in 12 6 times	6 in 36 6 times	10 in 60 6 times
2 in 14 7 times	6 in 42 7 times	10 in 70 7 times
2 in 16 8 times	6 in 48 8 times	10 in 80 8 times
2 in 18 9 times	6 in 54 9 times	10 in 90 9 times
3 in 3 1 time	7 in 7 1 time	11 in 11 1 time
3 in 6 2 times	7 in 14 2 times	11 in 22 2 times
3 in 9 3 times	7 in 21 3 times	11 in 33 3 times
3 in 12 4 times	7 in 28 4 times	11 in 44 4 times
3 in 15 5 times	7 in 35 5 times	11 in 55 5 times
3 in 18 6 times	7 in 42 6 times	11 in 66 6 times
3 in 21 7 times	7 in 49 7 times	11 in 77 7 times
3 in 24 8 times	7 in 56 8 times	11 in 88 8 times
3 in 27 9 times	7 in 63 9 times	11 in 99 9 times
4 in 4 1 time	8 in 8 1 time	12 in 12 1 time
4 in 8 2 times	8 in 16 2 times	12 in 24 2 times
4 in 12 3 times	8 in 24 3 times	12 in 36 3 times
4 in 16 4 times	8 in 32 4 times	12 in 48 4 times
4 in 20 5 times	8 in 40 5 times	12 in 60 5 times
4 in 24 6 times	8 in 48 6 times	12 in 72 6 times
4 in 28 7 times	8 in 56 7 times	12 in 84 7 times
4 in 32 8 times	8 in 64 8 times	12 in 96 8 times
4 in 36 9 times	8 in 72 9 times	12 in 108 9 times

34. There are three numbers in every division, and sometimes four: 1st, the dividend; 2d, the divisor; 3d, the quotient; and 4th, the remainder.

QUESTIONS.

1. If 12 apples be equally divided among 4 boys, how many will each have? How many times is 4 contained in 12?

2. If 24 peaches be equally divided among 6 boys, how many will each have? How many times is 6 contained in 24?

3. A man has 32 miles to walk, and can travel 4 miles an hour, how many hours will it take him?

4. A farmer receives 28 dollars for 7 sheep: how much is that apiece?

5. How many lead pencils could you buy for 42 cents, if they cost 6 cents apiece?

6. How many oranges could you buy for 72 cents, if they cost 6 cents apiece?

7. A trader wishes to pack 64 hats in boxes, and can put but 8 hats in a box: how many boxes does he want?

8. If a man could build 7 rods of fence in a day, how long will it take him to build 77 rods?

9. If a man pays 56 dollars for seven yards of cloth, how much is that a yard?

10. Twelve men receive 108 dollars for doing a piece of work, how much does each one receive?

11. A merchant has 144 dollars with which he is going to buy cloth at 12 dollars a yard: how many yards can he purchase?

12. James is to learn forty-two verses of Scripture in a week: how much must he learn each day?

13. How many times is 4 contained in 50, and how many over?

14. A man has 96 pounds of butter, and wishes to put 12 pounds in a box: how many boxes does he want?

15. James goes to school for 12 weeks, and receives 132 credit marks: how many does he get each week?

---

34. How many numbers are there in every division? Name them.

16. Four men bought a pair of oxen for one hundred dollars, and sold them again for eighty-four dollars: how much did each one lose?

17. How many times is 8 contained in 100, and how many over?

18. How many times is 11 contained in 60, and how many over?

19. How many times is 9 contained in 86, and how many over?

20. How many times is 7 contained in 79, and how many over?

21. How many times is 5 contained in 59, and how many over?

22. A mother wishes to distribute 30 apples among 8 children: how many will each have, and how many will be left?

23. How many times 4 in 23, and how many over? In 27? In 40? In 46? In 35? In 39?

24. How many times is 6 contained in 70, and how many over? In 65? In 61?

25. How many times 7 in 43, and how many over?

26. How many times 9 in 50, and how many over? In 54 how many times? In 80 how many times, and what over? In 86? In 94?

27. How many times 8 in 57, and how many over? In 65? In 87? In 100?

28. How many times 11 in 54, and how many over? In 67? In 85? In 97? In 62? In 59?

29. How many times 12 in 100, and how many over? In 120? In 87? In 77? In 66? In 130?

30. James bought 9 tops for 72 cents: what did he pay apiece? 12 in 90, how many times, and how many over?

31. A man paid thirty-six dollars for twelve sheep, and fifteen dollars apiece for two cows: how much more did he pay apiece for the cows than the sheep?

32. A drover has eight calves for which he paid forty dollars, and nine sheep for which he paid twenty-seven dollars: how much more did he pay apiece for the calves than for the sheep?

EXAMPLES ILLUSTRATING PRINCIPLES.

35. Let it be required to divide 86 by 2.

OPERATION.

Divisor. Dividend.
2)86
43 quotient.

Place the divisor on the left of the dividend, draw a curved line between them, and a straight line under the dividend.

Now, there are 8 tens and 6 units to be divided by 2. We say, 2 in 8, 4 times, which being 4 tens we write the 4 under the tens. We then say, 2 in 6, 3 times, which are three units, and must be written under the 6. The quotient, therefore, is 4 tens and 3 units, or 43.

1. Let it be required to divide 729 by 3.

OPERATION.

3)729
243

In this example there are 7 hundreds, 2 tens, and 9 units, all to be divided by 3. Now we say, 3 in 7, 2 times and 1 over. Set down the 2, which are hundreds, under the 7. But of the 7 hundreds there is 1 hundred or 10 tens not yet divided. We put the 10 tens with the 2 tens, making 12 tens, and then say, 3 in 12, 4 times, and write the 4 in the quotient, in the tens' place; then say, 3 in 9, 3 times. The quotient, therefore, is 243.

2. Let it be required to divide 729 by 9.

OPERATION.

9)729
81

In this example, we say, 9 in 7 we cannot, but 9 in 72, 8 times, which are 8 tens: then, 9 in 9, 1 time. The quotient is therefore 81.

---

35. When you divide 8 tens by 2, is the quotient tens or units? When 6 units are divided by 2, what is the quotient?

Ex. 1. When the 7 hundreds are divided by 3, of what denomination is the quotient? To how many tens is the undivided hundred equal? When the 12 tens are divided by 3, what is the quotient? When the 9 units are divided by 3, what is the quotient?

3. Let it be required to divide 36548 by 5.

OPERATION.

```
5)36548
  7309—3 remain.
```

The 36 thousands being divided by 5 gives 7 thousands for the first figure of the quotient. We then divide the 15 hundreds, giving a quotient figure of 3; then 4 tens, giving 0; and then 48 units, giving 9 and a remainder 3. The division of the 3 must be expressed by writing 5 under the 3, thus, $\frac{3}{5}$. The true quotient, therefore, is $7309\frac{3}{5}$; which is read, seven thousand three hundred and nine, and *three divided by five*. Therefore,

When there is a remainder after division, it must be written after the quotient, and the divisor placed under it.

CASE I.

36. Short Division, or when the divisor does not exceed 12.

RULE.

I. *Set down the divisor on the left of the dividend, draw a curved line between them, and a straight line under the dividend.*

II. *Begin with the left-hand figure of the dividend and divide each figure, in succession, by the divisor, when such division is possible, and write each quotient figure directly under the figure divided.*

III. *When any figure of the dividend is not divisible by the divisor, write 0 for the quotient figure, and consider the remainder as tens, to which add the next figure of the dividend, regarded as units, and divide this sum for the next figure of the quotient.*

Ex. 3. When there is a remainder after division, what do you do with it? If there be a remainder after division, how must it be written?

36. What is short division? How do you set down the numbers to be divided? How do you divide? Repeat the rule.

EXAMPLES.

(1.)	(2.)	(3.)
8)75890496	7)3505614	6)95040522
9486312		15840087
(4.)	(5.)	(6.)
3)7210463	9)674189904	11)1404967214
(7.)	(8.)	(9.)
12)167484329	10)27478041	7)4987494162

10. Divide 6794108 by 3. *Ans.* $2264702\frac{2}{3}$.
11. Divide 21090431 by 9. *Ans.* ——
12. Divide 2345678964 by 6. *Ans.* ——
13. Divide 570196382 by 12. *Ans.* $47516365\frac{2}{12}$.
14. Divide 67897634 by 9. *Ans.* ———5 remain.
15. Divide 75436298 by 12. *Ans.* ———2 remain.

CASE II.

37. Long Division, or when the divisor contains several figures.

RULE.

I. *Set down the divisor on the left of the dividend, draw a curved line between them, and also a curved line on the right of the dividend.*

II. *Note the fewest figures of the dividend, counted from the left hand, that will contain the divisor; find how often they contain it, and set the figure in the quotient.*

III. *Multiply the whole divisor by this figure; set the product under the first figures of the dividend, and subtract it from them.*

IV. *To the remainder annex the next figure of the dividend, then find how often the divisor is contained in this new number, and set the figure in the quotient.*

---

37. How do you set down the numbers for division? What do you do next? What do you do next? What is the next step? How many operations are there in division? Name them.

V. *Multiply the whole divisor by the last figure of the quotient, and subtract the product from the last number containing the divisor. To the remainder annex the next figure of the dividend, and find the figures of the quotient in the same way, till all the figures of the dividend are brought down.*

NOTE. There are five operations in division. First, to write down the numbers; second, find how many times; third, multiply; fourth, subtract; fifth, bring down.

EXAMPLES ILLUSTRATING PRINCIPLES.

38. Let it be required to divide 11772 by 327.

OPERATION.

```
Divisor. Dividend. Quotient.
327)11772(36
     981
     ----
     1962
     1962
     ----
     0000
```

Having set down the divisor on the left of the dividend, it is seen that 327 is not contained in 117; but by observing that 3 is contained in 11, 3 times and something over, we conclude that the divisor is contained at least 3 times in the first *four* figures of the dividend.

Set down the 3 in the quotient, and multiply the divisor by it; we thus get 981, which being less than 1177, the quotient figure is not too great: we subtract 981 from the first four figures of the dividend, and find a remainder 196, which being less than the divisor, the quotient figure is not too small.

Annex to this remainder the next figure 2, of the dividend.

As 3 is contained in 19, 6 times, we conclude that the

38. If any one of the products is too large, what do you do? If any one of the remainders is greater than the divisor, what do you do? What will be the order of units expressed by any figure of the quotient? When the divisor is contained in simple units, what units will the quotient figure express? When the divisor is contained in tens, what units will the quotient figure express? When it is contained in hundreds? In thousands?

divisor is contained in 1962 as many as 6 times. Setting down 6 in the quotient and multiplying the divisor by it, we find the product to be 1962. Therefore the entire quotient is 36, or the divisor is contained 36 times in the dividend.

NOTE 1. After multiplying by the quotient figure, if any one of the products is greater than the number supposed to contain the divisor, the quotient figure is too large, and must be diminished.

NOTE 2. When any one of the remainders is greater than the divisor, the quotient figure is too small, and must be increased by at least 1.

NOTE 3. The lowest order of units in the first dividend 1177, being tens, the first figure of the quotient will be tens; and generally,

Any figure of the quotient will express units of the same order as that expressed by the right-hand figure of its corresponding dividend.

39. Let it be required to divide 2756 by 26.

OPERATION.

```
26)2756(106
   26
   ---
    156
    156
    ---
```

We first say, 26 in 27 once, and place 1 in the quotient. Multiplying by 1, subtracting, and bringing down the 5, we say, 26 in 15, 0 times, and place the 0 in the quotient. Bringing down the 6, we find that the divisor is contained in 156, 6 times.

NOTE. If after having annexed the figure from the dividend to any one of the remainders, the number is less than the divisor, the quotient figure is 0, which being written in the quotient, annex the next figure of the dividend and divide as before.

PROOF OF DIVISION.

40. Multiply the divisor by the quotient and add in the remainder, when there is one: the sum should be equal to the dividend.

---

39. When any dividend is less than the divisor, what is the quotient figure? How do you then proceed?

40. How do you prove division?

DEMONSTRATION OF THE RULE OF DIVISION.

41. Let us suppose, as an example, that it were required to divide 11772 by 327.

We first consider, as we have a right to do, that 11772 is made up of 1177 tens and 2 units. We then divide the tens by the divisor 327, and find 3 tens for the quotient, by which we multiply the divisor and subtract the product from 1177, leaving a remainder of 196 tens. To this number we bring down the 2 units, making 1962 units. This number contains the divisor 6 times; that is, 6 units' times.

OPERATION.

```
327)11772(36
    981
    ----
    1962
    1962
    ----
```

When the unit of the first number which contains the divisor is of the 3d order, or 100, there will be 3 figures in the quotient; when it is of the 4th order there will be 4, &c.

Hence, the quotient found according to the rule, expresses the number of times which the dividend contains the divisor, and consequently is the true quotient.

EXAMPLES ILLUSTRATING PRINCIPLES.

1. Let it be required to divide 67289 by 261.

In this example, we find a quotient of 257 and a remainder of 212, which being less than the divisor will not contain it.

OPERATION.

```
261)67289(257
    522
    ----
    1508
    1305
    ----
     2039
     1827
     ----
      212 rem.
```

PROOF.

```
   261 divisor.
   257 quotient.
  ----
  1827
 1305
522
   212 remainder.
 -----
 67289=the dividend: hence, the work is right.
```

2. Let it be required to divide 119836687 by 39407.

```
OPERATION.                        PROOF.
39407)119836687(3041              39407 Divisor.
      118221                       3041 Quotient.
      ------                      -----
        161568                    39407
        157628                   157628
        ------                  118221
          39407                 --------
          39407                 119836687 Dividend.
          -----                 ---------
```

PROOF OF MULTIPLICATION.

42. When two numbers are multiplied together the multiplicand and multiplier are both factors of the product; and if the product be divided by one of the factors, the quotient will be the other factor. Hence,

If the product of two numbers be divided by the multiplicand, the quotient will be the multiplier; or, if it be divided by the multiplier, the quotient will be the multiplicand.

EXAMPLES.

```
   3679 Multiplicand.          3679)1203033(327
    327 Multiplier.                 11037
   ----                             -----
  25753                              9933
  7358                               7358
 11037                               -----
 -------                             25753
 1203033 Product.                    25753
 -------                             -----
                                     00000
```

2. The multiplicand is 61835720, and the product 8162315040: what is the multiplier? *Ans.* 132.

3. The multiplier is 270000: now if the product be 1315170000000, what will be the multiplicand?

4. The product is 68959488, the multiplier 96: what is the multiplicand? *Ans.* 718328.

---

42. If two numbers are multiplied together, what are the factors of the product? If the product be divided by one of the factors, what will the quotient be? *How do you prove multiplication?*

5. The multiplier is 1440, the product 10264849920: what is the multiplicand? *Ans.* ——

6. The product is 6242102428164, the multiplicand 6795634: what is the multiplier? *Ans.* ——

EXAMPLES IN DIVISION.

1. Divide 7210473 by 37. *Ans.* 24 rem.
2. Divide 147735 by 45. *Ans.* 3283.
3. Divide 937387 by 54. *Ans.* 1 rem.
4. Divide 145260 by 108. *Ans.* 1345.
5. Divide 79165238 by 238. *Ans.* ——
6. Divide 62015735 by 7803. *Ans.* $7947\frac{5294}{7803}$.
7. Divide 74855092410 by 949998. *Ans.* ——
8. Divide 47254149 by 4674. *Ans.* ——9 rem.
9. Divide 119184669 by 38473. *Ans.* ——33788 rem.
10. Divide 280208122081 by 912314. *Ans.* ——121 rem.
11. Divide 293839455936 by 8405. *Ans.* ——346 rem.
12. Divide 4637064283 by 57606. *Ans.* ——11707 rem.
13. Divide 352107193214 by 210472. *Ans.* ——165534 rem.
14. Divide 558001172606176724 by 2708630425. *Ans.* ——24 rem.
15. Divide 17143471493347 by 57143. Rem. 6347.
16. Divide 6754371495671594 by 678957. Rem. 81605.
17. Divide 71900715708 by 57149. Rem. 15785.
18. Divide 571943007145 by 37149. Rem. 12214.
19. Divide 671493471549375 by 47143. Rem. 35411.
20. Divide 121932631112635269 by 987654321.
21. Divide 571943007645 by 37149. Rem. 12714.
22. Divide 171493715947143 by 57007.

CONTRACTIONS IN DIVISION.

CASE I.

43. When the divisor is a composite number.

RULE.

Divide the dividend by one of the factors of the divisor; then divide the quotient thus arising by a second factor, and so on, till every factor has been used as a divisor: the last quotient will be the one sought.

EXAMPLES.

Let it be required to divide 1407 dollars equally among 21 men. Here the factors of the divisor are 7 and 3.

Let the 1407 dollars be first divided equally into 7 piles. Each pile will contain 201 dollars. Let *each* pile be now divided into 3 equal parts. Each one of the equal parts will be 67 dollars, and the whole number of parts will be 21: hence, the true quotient is found by dividing continually by the factors.

OPERATION.

```
7)1407
3)201  1st quotient.
   67  quotient sought.
```

2. Divide 18576 by $48=4\times12$. *Ans.* 387.
3. Divide 9576 by $72=9\times8$. *Ans.* 133.
4. Divide 19296 by $96=12\times8$. *Ans.* ——
5. Divide 55728 by $4\times9\times4=144$. *Ans.* ——
6. Divide 92880 by $2\times2\times3\times2\times2$. *Ans.* 1935.
7. Divide 57888 by $4\times2\times2\times2$. *Ans.* ——
8. Divide 154368 by $3\times2\times2$. *Ans.* 12864.

44. It sometimes happens that there are remainders after division, which are to be treated according to the following

---

43. What is a composite number? How do you divide when the divisor is a composite number? Repeat the rule.

44. When there are remainders, how do you find the true remainder? Give the *rule*.

RULE.

I. *The first remainder, if there be one, is a part of the true remainder.*

II. *Multiply the second remainder, if there be one, by the first divisor, and this product will also form a part of the true remainder.*

III. *If there be a remainder in any of the after divisions, multiply it by all the preceding divisors, and the final product will be a part of the true remainder: the sum of the several results will be the true remainder sought.*

EXAMPLE ILLUSTRATING PRINCIPLES.

1. What is the quotient of 751 grapes, divided by 16?

```
         ( 4)751
4×4=16  {  4)187 . . . . 3
         (    46 . . . . 3×4=12
                              3
                             15 the true remainder.
```

Ans. $46\frac{15}{16}$.

DEMONSTRATION OF THE RULE.

In 751 grapes there are 187 sets, (say bunches,) with 4 grapes or units in each bunch, and 3 units over. In the 187 bunches there are 46 piles, 4 bunches in a pile, and 3 bunches over. But there are 4 grapes in each bunch; therefore, the number of grapes in the 3 bunches is equal to $4\times3=12$, to which add 3, the grapes of the first remainder, and we have the entire remainder 15.

EXAMPLES.

1. Let it be required to divide 4967 by 32.

```
         ( 4)4967
4×8=32  {  8)1241 . . . 3, 1st remainder.
         (    155 . . . 1×4+3=7, the true remainder.
```

Ans. $155\frac{7}{32}$.

2. Divide 956789 by $7\times8=56$. *Ans.* ——

3. Divide 4870029 by $8\times9=72$. *Ans.* $67639\frac{21}{72}$.
4. Divide 674201 by $10\times11=110$. *Ans.* ——
5. Divide 445767 by $12\times12=144$. *Ans.* $3095\frac{87}{144}$.
6. Divide 1913578 by $7\times2\times2\times2=56$. *Ans.* $34171\frac{2}{56}$.
7. Divide 14610087 by $3\times3\times2\times2\times2=72$. *Ans.* $202917\frac{63}{72}$.
8. Divide 2696804 by $5\times2\times11=110$. *Ans.* $24516\frac{44}{110}$.
9. Divide 936496 by $3\times4\times2\times5\times6$. *Ans.* ——

CASE II.

45. When the divisor is 10, 100, 1000, &c.

RULE.

I. *Cut off from the right hand of the dividend as many figures as there are 0's in the divisor.*

II. *The left-hand figures of the dividend will express the quotient, and the figures cut off the remainder.*

EXAMPLE ILLUSTRATING PRINCIPLES.

Divide 3256 by 100.

	OPERATION.
In this example there are two 0's in the divisor, therefore, there are two figures cut off from the right hand of the dividend, and the quotient is 32, and 56÷100.	100)32|56 *Ans.* $32\frac{56}{100}$.

DEMONSTRATION OF THE RULE.

The quotient ought to be 10, 100, 1000, &c., times less than the dividend. But the same figure expresses a number 10, 100, 1000, &c., times greater or less in value, according to its distance from the units' place. By cutting off figures from the right hand, the units'

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45. How do you divide when the divisor is 1 with any number of naughts? *Give the reason of the rule.*

place is removed to the left, and consequently the dividend is diminished 10, 100, 1000, &c., times, according as you cut off 1, 2, 3, &c., figures.

EXAMPLES.

1. Divide 49763 by 10. *Ans.* $4976\frac{3}{10}$.
2. Divide 7641200 by 100. *Ans.* 76412.
3. Divide 496321 by 1000. *Ans.* $496\frac{321}{1000}$.
4. Divide 6497804 by 10000. *Ans.* $649\frac{7804}{10000}$.
5. Divide 16789742 by 100000. *Ans.* $167\frac{89742}{100000}$.

CASE III.

46. When there are ciphers on the right of the divisor.

RULE.

I. *Cut off the ciphers by a line, and cut off the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the significant figures of the divisor, and annex to the remainder, if there be one, the figures cut off from the dividend: this will form the true remainder.*

EXAMPLE ILLUSTRATING PRINCIPLES.

Divide 67389 by 700.

OPERATION.

```
7|00)673|89
     96...1 remain.
       189 true remain.
```

Ans. $96\frac{189}{700}$.

In this example we may regard the divisor as a composite number, of which the factors are 7 and 100. Hence, we strike off the 89, and then find that 7 is contained in the remaining figures, 96 times, with a remainder of 1; this we multiply by 100, and then add 89, forming the remainder 189: to the quotient 96 we annex 189 divided by 700 for the entire quotient.

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46. How do you divide when there are ciphers on the right of the divisor? How do you form the true quotient?

EXAMPLES.

1. Divide 8749632 by 37000.

```
37|000)8749|632(236
       74
       ---
       134
       111
       ---
        239
        222
        ---
         17
```

Ans. $236\frac{17632}{37000}$.

2. Divide 986327 by 210000. *Ans.* $4\frac{146327}{210000}$.
3. Divide 876000 by 6000. *Ans.* ——
4. Divide 36599503 by 400700. *Ans.* $91\frac{135803}{400700}$.
5. Divide 571436490075 by 36500. Rem. 9075.
6. Divide 194718490700 by 73000. Rem. 42700.
7. Divide 1495070807149 by 31500. Rem. 9649.
8. Divide 7149374947194715 by 1749000.

APPLICATIONS IN DIVISION.

1. Divide 80 dollars equally among four men.

Here the 80 dollars is to be divided into 4 equal parts, and the quotient 20 dollars expresses the value of one of the equal parts.

OPERATION.

```
4)80
  20 dollars.
```

2. Four persons buy a lottery ticket; it draws a prize of 10000 dollars: what is each one's share? *Ans.* ——

3. A person dying leaves an estate of 4500 dollars to be divided equally among 5 children: what is each one's share? *Ans.* 900 *dollars.*

4. There are 1560 eggs to be packed in 24 baskets: how many eggs will be put in each basket? *Ans.* ——

5. What number must be multiplied by 124 to produce 40796? *Ans.* 329.

6. How many times can 24 be subtracted from 1416? *Ans.* ——

7. The sum of 19125 dollars is to be distributed among a certain number of men; each is to receive 425 dollars: how many men are to receive the money? *Ans.* ——

8. By the census of 1840 the whole population of the 26 States was 16,890,320: if each one had contained an equal number of inhabitants, how many would there have been in each State? *Ans.* 649,627$\frac{18}{26}$.

9. If a man walks 12775 miles in a year, or 365 days, how far does he walk each day? *Ans.* *miles.*

10. A farmer sells a drove of sheep for 2 dollars a head, and receives 1250 dollars: how many sheep did he sell? *Ans.* 625.

11. It is computed that the distance to the sun is 95,000,000 of miles, and that light is 8 minutes travelling from the sun to the earth: how many miles does it travel per minute? *Ans.* ——

12. By the census of 1840 it appeared that the city of New York contained 312710 inhabitants; allowing 5 to each house, how many houses were there in that city at that time? *Ans.* 62,542.

13. A merchant has 5100 pounds of tea, and wishes to pack it in 60 chests: how many pounds must he put in each chest? *Ans.* ——

14. A person goes to a store and buys a piece of cloth containing 36 yards, for which he pays 288 dollars: how much does he pay per yard? *Ans.* ——

15. There are 7 days in a week: how many weeks in a year of 365? *Ans.* 52 *weeks* and 1 *day* over.

16. There are 24 hours in a day: how many days in 2040 hours? *Ans.* ——

17. Twenty-three persons dined together, their bill was 92 dollars: how much had each one to pay? *Ans.* 4 *dollars.*

18. In one pound there are 16 ounces: how many pounds are there in 223360 ounces? *Ans.* 13960.

19. If there are 160 square rods in an acre, how many acres are there in 2172480 square rods?

20. If you pay 94 cents a yard for cloth, how many yards can you buy for 6716394 cents?

21. If you pay 145 dollars for a pipe of wine, how many pipes can you buy for 10875 dollars? *Ans.* 75.

22. If a man travels 47 miles a day, how many day. would it take him to travel 1222 miles?

23. If a garrison of 987 men are supplied with 351372 pounds of beef, how much will that be for each man? *Ans.* 356 *lbs.*

24. How many times does 3942 go in 21587211936?

25. If a ship sails 142 miles a day, how many days will it take her to sail 48564 miles?

26. Divide 887124 dollars equally among 236 men. *Ans.* 3759 *dollars.*

27. Divide 1491 dollars among 7 men.

28. Divide 74400 dollars among 620 men. *Ans.* 120 *dollars.*

29. Suppose 60 men catch 295200 fish: how many would each have caught?

30. If 17028 dollars is to be divided among a ship's crew, and each man to receive 44 dollars: how many men compose the crew? *Ans.* 387.

31. Suppose 68959488 to be a dividend, and 718328 the quotient: what is the divisor?

32. Suppose 34310 pounds of pork to be equally divided among a body of soldiers, each receiving 47 pounds: what is the number of soldiers? *Ans.* 730.

33. What is the divisor of 761858465, if 90001 be the quotient?

34. If the dividend is 761858465, and divisor 8465, what will be the quotient? *Ans.* 90001.

35. There are 1893312 inhabitants in 912 villages; if each village contains the same number, what is the population of each?

36. If 61 rods of railroad cost 28609 dollars, how much will one rod cost? *Ans.* 469 *dollars.*

37. If 472 rods of the same railroad cost 251104 dollars, how much was it a rod? *Ans.* 532 *dollars.*

38. If 26537009535 dollars be equally divided among 27856 men, how many dollars will each have?

APPLICATIONS IN THE PRECEDING RULES.

1. A farmer sells a yoke of oxen for 90 dollars, 3 cows for 25 dollars each, 9 calves for 4 dollars each, and 65 sheep at 3 dollars a head. How much did he receive for them all? *Ans.* dollars.

2. The sum of two numbers is 365, one of the numbers is 221; what is the other number? *Ans.* 144.

3. The difference of two numbers is 95, the less number 327; what is the greater number? *Ans.* ——

4. A farmer sells 4 tons of hay at 12 dollars per ton, 80 bushels of wheat at 1 dollar per bushel, and takes in part payment a horse worth 65 dollars, a wagon worth 40 dollars, and the rest in cash. How much money did he receive? *Ans.* 23 *dollars.*

5. A farmer has 14 calves worth 4 dollars each, 40 sheep worth 3 dollars each; he gives them all for a horse worth 150 dollars: does he make or lose by the bargain? *Ans.* He loses *dollars.*

6. The product of two numbers is 51679680, and one of the factors is 615: what is the other factor? *Ans.* 84032.

7. When the divisor is 67941, and the quotient 30620, what is the dividend? *Ans.* 2080353420.

8. When the dividend is 1213193, the quotient 37, what is the divisor? *Ans.* ——

9. A piece of cloth containing 65 yards costs 455 dollars: what does it cost per yard? *Ans.* *dollars.*

10. A man has 6 children, all of whom are married, and each has four children; two of these grandchildren are married, and each has one child: how many children, grandchildren, and great-grandchildren are there? *Ans.* ——

11. The distance around the earth is computed to be about 25000 miles: how long would it take a man to travel it, supposing him to travel at the rate of 35 miles a day? *Ans.* $714\frac{10}{35}$ *days.*

12. The earth moves around the sun at the rate of 68000 miles an hour: how many miles does it travel in a day, and how many in a year?

Ans. { 1632000 *in a day.* / 595680000 *in a year.* }

13. A farmer purchased a farm for which he paid 18050 dollars. He sold 50 acres for 60 dollars an acre, and the remainder stood him in 50 dollars an acre: how much land did he purchase? *Ans.* 351 *acres.*

14. What number multiplied by 3456, will produce 3411072? *Ans.* 987.

15. If 1974 men are supplied with 175686 pounds of pork, how much will each man have?

16. In 1830, the national debt of the United States was 48565406 dollars; in 1836, the debt was cancelled: had the payments been equal, how much would have been paid in each year? *Ans.* 8094234$\frac{2}{6}$ *dollars.*

17. What would 1800 miles of railroad cost at 14000 dollars a mile?

18. The diameter of the earth is 7912 miles, and the diameter of the sun 112 times greater: what is the diameter of the sun, and how much greater is it than the diameter of the earth?

Ans. { 886144 *in diameter; and* 878232 *miles greater than the diameter of the earth.*

19. The volcano in the Island of Bourbon, in 1796, threw out 45000000 cubic feet of lava: how long would it take 25 carts to carry it off, if each cart carried 12 loads a day, and 40 cubic feet at each load?

20. In 1787 it threw out 60000000 cubic feet: how long would it take for 25 carts to remove it?

Ans. 5000 *days.*

21. The income of the Bishop of Durham, in England, is 292 dollars a day: how many clergymen would this support on a salary of 730 dollars per annum?

22. There are 31173 verses in the Bible: how many verses must be read each day, that it may be read through in a year? *Ans.* 85+.

23. A man's income is 2698 dollars a year, and his expenses 6 dollars a day: how much will he lay up?

24. The population of Europe in 1837 was estimated at 233884800, and the number of square miles at 3708871. How many inhabitants would this give to each square mile? *Ans.* 63. Rem. 225927.

25. In 1843, there were 3173 public schools in Massachusetts, which were attended during the winter by 119989 scholars. How many would this allow in each school? *Ans.* 37. Rem. 2588.

GENERAL PRINCIPLES FROM THE FOREGOING RULES.

47. Numeration, Addition, Subtraction, Multiplication, and Division, are called the five ground rules of Arithmetic.

48. When the cost of each of several things is given, their entire cost is found by adding the costs of the several things together.

Ex. What is the entire cost of a bag of coffee at 6 dollars, a chest of tea at 4 dollars, a box of raisins at 2 dollars, and a barrel of sugar at 12 dollars?

Ans. 24 *dollars.*

2. What is the entire cost of six sheep at 12 dollars, a cow at twenty dollars, a horse at one hundred and fifty dollars, and a yoke of oxen at eighty dollars?

3. What is the entire cost of 6 calves at 14 dollars, 25 sheep at 50 dollars, and one cow at 26 dollars?

49. The difference between two numbers is found by subtracting the less from the greater: or, if they are equal, either may be subtracted from the other.

Ex. What is the difference between 37 and 23? Also between 40 and 40?

50. If the subtrahend and remainder are known, the miuuend may be found by adding the remainder to the subtrahend. Hence, the following principles:

1st. *If the sum of two numbers be diminished by one of them, the remainder will be the other number.*

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47. How many principal rules are there in Arithmetic? Wha are they? Can multiplication be performed by addition? Can division be performed by subtraction? By how many rules, then may all the operations in Arithmetic be performed?

48. When the cost of each of several things is given, how do you find their entire cost?

49. How do you find the difference between two unequal numbers? Between two equal numbers?

50. How do you find the minuend, when the subtrahend and remainder are given? If the sum of two numbers be diminished by one of them, what will the remainder be? If the less of two numbers be added to their difference, what will the sum be?

2d. *The less of two numbers, added to their difference, will give the greater.*

Ex. 1. The sum of two numbers is 56, one of the numbers is 12: what is the other? *Ans.* 44.

2. The less of two numbers is 25, and their difference 30: what is the greater? *Ans.* ——

3. The less of two numbers is 35, and their difference 35: what is the greater? *Ans.* 70.

51. Knowing the price of a single thing, the cost of any number of things may be found by multiplying the number of things by the price of one of them.

Ex. 1. What is the cost of 35 pears at 2 cents each? What is the cost of 45 yards of cloth at 2 dollars a yard? Of 65 yards of riband at 30 cents a yard?

52. When the multiplier is 1 the product will be equal to the multiplicand. When the multiplier is greater than 1, the product will be as many times greater than the multiplicand as the multiplier is greater than unity.

Ex. 1. If 65 be multiplied by 1, what will the product be? If it be multiplied by 2 what will the product be? How many times greater than 65?

2. If 17 be multiplied by 3, what will the product be? How many times greater than 17?

53. When we know the number of things and their entire cost, the cost of a single thing may be found by dividing the entire cost by the number of things.

Ex. 1. If 6 oranges cost 36 cents, how much do they cost apiece?

2. If 4 yards of cloth cost 20 dollars, how much is it a yard?

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51. Knowing the price of a single thing, how do you find the cost of several things of the same kind?

52. When the multiplier is 1, how will the product compare with the multiplicand? When it is greater than 1, how will they compare with each other?

53. When you know the number of things and their entire cost, how will you find the cost of a single thing?

3. If 6 yards of cloth cost 42 dollars, how much will it cost a yard?

54. If the divisor is equal to the dividend the quotient will be 1, and the quotient will be as many times greater than 1, as the dividend is greater than the divisor.

Ex. 1. If 15 be divided by 15, what will the quotient be? If 18 be divided by 18, what is the quotient?

2. If 30 be divided by 6, what will the quotient be? How many times is 30 greater than 6?

55. A prime number is one which cannot be exactly divided by any number except itself and unity. Thus, 1, 2, 3, 5, 7, 11, 13, 17, &c., are prime numbers.

56. The product of two or more prime numbers will be exactly divisible only by one or other of the factors.

57. If an even number be added to itself any number of times, the sum will be even: hence, if one of the factors of a product be an even number, the product will be even.

58. An odd number is not divisible by an even number, nor is a less number exactly divisible by a greater.

59. Any number is divisible by 2, if the last significant figure is even; and is divisible by 4 if the last two significant figures are divisible by 4.

60. Any number whose last figure is 5 or 0, is exactly divisible by 5; and any number whose last figure is 0 is exactly divisible by 10.

---

54. If the divisor is equal to the dividend, what will the quotient be? Generally, how will the quotient compare with 1?

55. What is a prime number? Give an example.

56. By what numbers only will the product of prime factors be divisible?

57. If an even number be multiplied by a whole number, will the product be odd or even?

58. Is an odd number divisible by an even number?

59. What numbers are exactly divisible by 2? What numbers by 4?

60. If the last figure of a number be 5 or 0, by what number may it be divided?

OF DENOMINATE NUMBERS.

61. Simple numbers express a collection of units of the same kind, without expressing the particular value of the unit. For example, 40 and 55 are simple numbers, and the unit is 1, but they do not express whether the unit is 1 apple, 1 pound, or 1 horse.

A DENOMINATE number expresses the *kind* of unit which is considered. For example, 6 dollars is a denominate number, the *unit* 1 dollar being denominated, or named.

62. When two numbers have the same unit, they are said to be of the same denomination; and when two numbers have different units, they are said to be of different denominations. For example, 10 dollars and 12 dollars are of the same denomination; but 8 dollars and 20 cents express numbers of different denominations.

63. When all the units of a number are of the same kind, it is called a simple denominate number. But several numbers of different denominations are often connected together, forming a single expression, as 3 dollars 15 cents. Such are called *compound denominate numbers.*

64. In Federal money we pass from one denomination to another according to the scale of tens, as in simple numbers; but in other denominate numbers, such, for example, as yards, feet, and inches, the scale is different, and we must consider how many units of each denomination make one unit of the next higher.

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61. What do simple numbers express? What is a denominate number? What is the unit of 6 dollars?

62. When two numbers have the same unit, what do you say of them? When they have different units? Are 6 dollars and 4 dollars of the same denomination? Are 4 dollars and 4 cents? What is the unit of each?

63. What is a simple denominate number? What is a compound denominate number? Give an example.

64. In Federal money, what is the scale in passing from one denomination to another? How does this compare with the scale in simple numbers? How is it in other denominate numbers? What is always to be considered?

OF FEDERAL MONEY.

[Before proceeding further, let the pupil study carefully from Art. 107 to and including Art. 109, page 149.]

65. Federal money is the currency of the United States. Its denominations, or names, are Eagles, Dollars, Dimes, Cents, and Mills.

The coins of the United States are of gold, silver, and copper, and are of the following denominations.

1. Gold—Eagle, half-eagle, quarter-eagle.

2. Silver—Dollar, half-dollar, quarter-dollar, dime, half-dime.

3. Copper—Cent, half-cent.

If a given quantity of gold or silver be divided into 24 equal parts, each part is called a *carat.* If any number of carats be mixed with such a number of carats of a less valuable metal, that there be 24 carats in the mixture, then the compound is said to be as many carats fine as it contains carats of the more precious metal, and to contain as much alloy as it contains carats of the baser. For example, if 20 carats of gold be mixed with 4 of silver, the mixture is called gold of 20 carats fine, and 4 parts alloy.

66. The standard of the gold coin in the United States, is 22 carats of gold, 1 of silver, and 1 of copper. The standard for silver coins is 1489 parts of pure silver, to 179 of pure copper. The copper coins are of pure copper.

The eagle contains 270 grains of standard gold; the dollar 416 grains of standard silver; and the cent 11 pennyweights of copper.

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65. What is the currency of the United States? What are its denominations? What are the coins of the United States? Which gold? Which silver? Which copper? What do you understand by gold 20 carats fine?

66. What is the standard of the gold coin? What of the silver? What of the copper? What is the value, in gold, of the eagle? *What is the* value, in silver, of the dollar? What in copper of the *cent?*

TABLE.

10 Mills marked (*m*) make	1 Cent, marked	*ct.*
10 Cents - - - - - -	1 Dime, - -	*d.*
10 Dimes - - - - - -	1 Dollar, - -	$.
10 Dollars - - - - - -	1 Eagle, - -	*E.*

67. The above table is read, ten mills make one cent, ten cents one dime, ten dimes one dollar, ten dollars one eagle.' It is thus seen, that ten units of each denomination make one unit of the denomination next higher, the same as in simple numbers. Therefore,

The denominations of Federal money here expressed may be added, subtracted, multiplied, and divided, by the same rules that have already been given for simple numbers.

NUMERATION TABLE.

Tens of dollars or eagles.	Dollars.	Tens of cents or dimes.	Cents.	Mills.	
			5	7,	is read, 5 cents and 7 mills, or 57 mills.
		1	6	4,	- - 16 cents and 4 mills, or 164 mills.
6	2,	1	2	0,	- - 62 dollars 12 cents and no mills.
2	7,	6	2	3,	- - 27 dollars, 62 cents and 3 mills.
4	0,	0	4	1,	- - 40 dollars, 4 cents and 1 mill.

---

67. Repeat the table. How many units of either denomination make one of the next higher? How do the units of simple numbers increase from the right to the left? How may Federal money be added, subtracted, multiplied, and divided?

68. In numerating Federal money, what is the figure on the right called? The second? The third? The fourth? How do you separate the dollars from the cents? How is Federal money generally *read?*

68. As dimes are tens of cents, the second line may either be read 16 cents and 4 mills, or 1 dime 6 cents and 4 mills. And as the eagles are tens of dollars, the third line may be read 62 dollars and 12 cents, or 6 eagles 2 dollars and 12 cents.

The comma, or separatrix, is generally used to separate the cents from the dollars. It is not usual to place the comma between the cents and mills. Thus, $67,25 6 is read 67 dollars 25 cents and 6 mills.

Federal money is generally read in dollars cents and mills.

REDUCTION OF FEDERAL MONEY.

69. Reduction of Federal money consists in changing its denominations without altering its value. It is divided into two parts.

1st. To reduce from a higher denomination to a lower, as from dollars to cents.

2d. To reduce from a lower denomination to a higher, as from mills to dollars.

From the table it appears,

1st. *That cents may be changed into mills by annexing a cipher.*

Thus, 8 cents are equal to 80 mills.

2d. *That dollars may be changed into cents by annexing two ciphers, and into mills by annexing three.*

For example, 12 dollars are equal to 1200 cents, or to 12000 mills. The reason of these rules is evident, since 10 mills make a cent, 100 cents a dollar, and 1000 mills a dollar.

---

69. What is reduction? How many kinds of reduction are there? Name them. How may cents be changed into mills? How may dollars be changed into cents? How into mills? To how many cents are 12 dollars equal? To how many mills are they equal? How many cents in 4 dollars? How many in 6 dollars? How many mills in 9 dollars? How many mills in 5 dollars? How many cents in 3 dollars? In 8 dollars? In 7 dollars?

EXAMPLES.

1. Reduce 25 eagles 8 dollars 65 dimes and 35 cents, to the denomination of cents.

	OPERATION
Eagles the highest denomination - -	25
Dollars make one eagle - - - - -	10
Product in dollars - - - - - -	250
Add the number in the denomination of dollars	8
	258
The number of dimes in a dollar - -	10
Product in dimes - - - - - -	2580
Add the number in the denomination of dimes	65
	2645
Number of cents in a dime - - -	10
Product in cents - - - - - -	26450
Cents to be added - - - - - -	35
Total number of cents - - - -	26485

2. In 3 dollars 60 cents and 5 mills: how many mills?
3 dollars=300 cents,
60 cents to be added,
360=3600 mills, to which add the 5 mills.
Ans. 3605.

3. In 37 dollars 37 cents 8 mills: how many mills? *Ans.* 37378.

4. In 375 dollars 99 cents 9 mills: how many mills? *Ans.* 375999.

5. How many mills in 67 cents? *Ans.* ——

6. How many mills in $54? *Ans.* 54000.

7. How many cents in $125? *Ans.* 12500.

8. In $400, how many cents? How many mills?

9. In $375, how many cents? How many mills?

10. How many mills in $4? In $6? In $10,14 cents?

11. How many mills in $40,36 cents 8 mills?

12. How many mills in 71,45 cents 3 mills?

70. As we change dollars into cents by adding two ciphers, and cents into mills by adding one, it follows that, to change mills into dollars cents and mills, we have the following

RULE.

Cut off the right-hand figure for mills, and the figures to the left will be cents. Then cut off the two next figures for cents, and the remaining figures to the left will be dollars.

The reason of the rule is this: by cutting off the first right-hand figure, we in fact divide by 10, and thus reduce the mills to cents. Then by cutting off the next two figures, we divide by 100; and thus reduce the cents to dollars.

EXAMPLES.

1. How many dollars cents and mills, are there in 67897 mills? *Ans.* $67,89 7.
2. Set down 104 dollars 69 cents and 8 mills. *Ans.* $104,69 8.
3. Set down 4096 dollars 4 cents and 2 mills. *Ans.* ——
4. Set down 100 dollars 1 cent and 1 mill. *Ans.* $100,01 1.
5. Write down 4 dollars and 6 mills. *Ans.* 4,00 6.
6. Write down 109 dollars and 1 mill. *Ans.* ——
7. Write down 65 cents and 2 mills. *Ans.* $0,65 2.
8. Write down 2 mills. *Ans.* $0,00 2.
9. Reduce 1607 mills, to dollars and cents. *Ans.* ——
10. Reduce 170464 mills, to dollars. *Ans.* $170,46 4.
11. Reduce 8674416 mills, to dollars.
12. Reduce 94780900 mills, to dollars.
13. Reduce 74164210 mills, to dollars.

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70. How do you change mills into cents? How do you change cents into dollars? How do you separate the mills from the cents? How the cents from the dollars?

71. The parts of a dollar are sometimes expressed fractionally, as in the following

TABLE.

$1	=100 cents,	$\frac{1}{8}$ of a dollar	$=12\frac{1}{2}$ cents,
$\frac{1}{2}$ of a dollar	= 50 cents,	$\frac{1}{10}$ of a dollar	= 10 cents,
$\frac{1}{3}$ of a dollar	$=33\frac{1}{3}$ cents,	$\frac{1}{16}$ of a dollar	$= 6\frac{1}{4}$ cents,
$\frac{1}{4}$ of a dollar	= 25 cents,	$\frac{1}{20}$ of a dollar	= 5 cents,
$\frac{1}{5}$ of a dollar	= 20 cents,	$\frac{1}{2}$ of a cent	= 5 mills.

ADDITION OF FEDERAL MONEY.

1. Charles gives $9\frac{1}{2}$ cents for a top, and $3\frac{1}{2}$ cents for 6 quills: how much do they cost him? *Ans.* 13 *cents.*

2. John gives $1,$37\frac{1}{2}$ for a pair of shoes, 25 cents for a penknife, and $12\frac{1}{2}$ cents for a pencil: how much does he pay for all?

We first recollect that half a cent is equal to 5 mills. We then place the mills under each other, the cents under cents, and the dollars under dollars. We then add as in simple numbers.

OPERATION.

$1,	37 5
	25
	12 5
$1,	75

3. James gives 50 cents for a dozen oranges, $12\frac{1}{2}$ cents for a dozen apples, and 30 cents for a pound of raisins: how much for all?

OPERATION.

$0,	50
	12 5
	30
$0,	92 5

72. Hence, for the addition of Federal money, we have the following

71 How many cents in a dollar? In half a dollar? In a third a dollar? In a fourth of a dollar? In the fifth of a dollar? In the eighth of a dollar? In the tenth of a dollar? In the sixteenth of a dollar? In the twentieth of a dollar? How many mills in half a cent?

72. How do you set down Federal money for addition? How do you add up the *columns?* How do you place the separating point?

RULE.

I. *Set down the numbers to be added so that units of the same value shall fall under each other.*

II. *Then add up the several columns as in simple numbers, and place the separating point in the sum directly under those in the columns.*

EXAMPLES.

1. Add $67, 21 4, $10, 04 9, $6, 04 1, $0, 27 1, together.

(1.)	(2.)	(3.)
$ *cts. m.*	$ *cts. m.*	$ *cts. m.*
67, 21 4	59, 31 6	81, 05 8
10, 04 9	87, 42 5	67, 41 2
6, 04 1	48, 87 2	95, 37 6
0, 27 1	56, 70 8	87, 06 4
		$330, 90 5

(4.)	(5.)	(6.)
$375, 02 1	$27, 09 8	$7, 00 9
2, 09 6	325, 59 2	0, 01 1
0, 47 9	25, 60 3	0, 00 1
3, 01 2	9, 99 9	46, 67 9
$380, 60 8		

APPLICATIONS.

1. A grocer purchased a box of candles for 6 dollars 89 cents; a box of cheese for 25 dollars 4 cents and 3 mills; a keg of raisins for 1 dollar 12½ cents, (or 12 cents and 5 mills;) and a cask of wine for 40 dollars 37 cents 8 mills: what did the whole cost him?

Ans. $73, 43 6.

2. A farmer purchased a cow for which he paid 30 dollars and 4 mills; a horse for which he paid 104 dollars 60 cents and 1 mill; a wagon for which he paid 85 dollars and 9 mills: how much did the whole cost?

Ans. ——

3. A man is indebted to A, $630,49; to B, $25, to C, 87½ cents; to D, 4 mills: how much does he owe?
Ans. $656,36 9.

4. Bought 1 gallon of molasses at 28 cents per gallon; a half pound of tea for 78 cents; a piece of flannel for 12 dollars 6 cents and 3 mills; a plough for 8 dollars 1 cent and 1 mill; and a pair of shoes for 1 dollar and 20 cents: what did the whole cost? *Ans.* ——

5. Bought 6 pounds of coffee for 1 dollar 12½ cents; a wash-tub for 75 cents 6 mills; a tray for 26 cents 9 mills; a broom for 27 cents; a box of soap for 2 dollars 65 cents 7 mills; a cheese for 2 dollars 87½ cents: what is the whole amount? *Ans.* $7,95 2.

6. What is the entire cost of the following articles: viz. 2 gallons of molasses 57 cents; half a pound of tea 37½ cents; 2 yards of broadcloth $3,37½ cents; 8 yards of flannel $9,87 5; two skeins of silk 12½ cents, and 4 sticks of twist 8½ cents?

SUBTRACTION OF FEDERAL MONEY.

1. John gives 9½ cents for a pencil, and 8 cents for a top: how much more does he give for the pencil than top? *Ans.* $0,01 5.

2. A man buys a cow for $26,37, and a calf for $4,50: how much more does he pay for the cow than calf?

We set down the numbers as in addition, and then subtract them as in simple numbers.

OPERATION.

$$
\begin{array}{r}
\$26,\ 37 \\
4,\ 50 \\
\hline
\$21,\ 87
\end{array}
$$

73. Hence, for subtraction of Federal money, we have the following

RULE.

Write the lesser number under the greater so that units of the same order shall fall directly under each other; then subtract as in simple numbers, and place the separating point in the remainder directly under that above.

---

73. How do you set down the numbers for subtraction? How do you subtract them? Where do you place the separating point in the remainder?

EXAMPLES.

	(1.)		(2.)
From	$204, 67 9	From	$8976, 40 0
take	98, 71 4	take	610, 09 8
Remainder	$105, 96 5	Remainder	$8366, 30 2

(3.)	(4.)	(5.)
$620, 00 0	$327, 00 1	$2349,
19, 02 1	2, 09 0	29, 33
$600, 97 9	$324, 91 1	$2319, 67

6. What is the difference between $6 and 1 mill? Between $9,75 and 8 mills? Between 75 cents and 6 mills? Between $87,35 4 and 9 mills?

7. From $107,00 3 take $0,47 9. *Ans.* $106,52 4.

APPLICATIONS.

1. A man's income is $3000 a year; he spends $187,50: how much does he lay up? *Ans.* $2812,50.

2. A man purchased a yoke of oxen for $78, and a cow for $26,00 3: how much more did he pay for the oxen than for the cow? *Ans.* ——

3. A man buys a horse for $97,50, and gives a hundred dollar bill: how much money ought he to receive back? *Ans.* $2,50.

4. How much must be added to $60,03 9 to make the sum $1005,40? *Ans.* ——

5. A man sold his house for $3005, this sum being $98,03 9 more than he gave for it: what did it cost him? *Ans.* $2906,96 1.

6. A man bought a pair of oxen for $100, and sold them again for $75,37½: did he make or lose by the bargain, and how much? *Ans.* He lost $24,62 5.

7. A man starts on a journey with $100; he spends $87,57: how much has he left? *Ans.* $12,43.

8. How much must you add to $40,17 3 to make $100? *Ans.* ——

9. A man purchased a pair of horses for $450, but finding one of them injured, the seller agreed to deduct $106,32 5: what had he to pay? *Ans.* $343,67 5.

MULTIPLICATION OF FEDERAL MONEY.

1. John gives 3 cents apiece for 6 oranges: how much do they cost him? *Ans.* 18 *cents.*

2. John buys 6 pairs of stockings, for which he pays 25 cents a pair: how much do they cost him?

3. A farmer sells 8 sheep for $1,25 each: how much does he receive for them?

We multiply the cost of one sheep by the number of sheep, and the product is the entire cost.

OPERATION.
$1,25
8
$10,00

74. Hence, for the multiplication of Federal money by a simple number, we have the following

RULE.

Multiply as in simple numbers, and the product will be the answer in the lowest denomination mentioned in the multiplicand; then reduce the product to dollars and cents.

EXAMPLES.

1. Multiply 375 dollars 28 cents and 2 mills, by 8.

OPERATION.	(2.)
$385, 28 2	$475, 87
8	9
Product $3082, 25 6	Product $4282, 83

(3.)	(4.)	(5.)
$3, 00 4	$89, 07 9	$81, 99 2
12	7	6
$	$623, 55 3	$

(6.)	(7.)	(8.)
$87, 57 3	$497, 04 3	$157, 92 3
11	10	12

74. How do you multiply Federal money? What will be the denomination of the product? How will you then reduce it to dollars and cents?

APPLICATIONS.

1. What will 55 yards of cloth come to at 37 cents per yard? *Ans.* $20,35.

2. What will 300 bushels of wheat come to at $1,25 per bushel? *Ans.* $375.

3. What will 85 pounds of tea come to at 1 dollar 37½ cents per pound?

In this example we first consider that ½ of a cent is equal to 5 mills. Then as $1,37 5 contains more figures than 85, we multiply by the 85, knowing that the product will be the same whichever number be made the multiplier. The product 116875 is in mills, which is reduced to dollars and cents as before.

OPERATION.
1375
85
6875
11000
116875
Ans. $116,87 5

4. What will a firkin of butter containing 90 pounds come to at 25½ cents per pound? *Ans.* $22,95.

5. What is the cost of a cask of wine containing 29 gallons, at 2 dollars and 75 cents per gallon? *Ans.* $79,75.

6. A bale of cloths contains 95 pieces, costing 40 dollars 37½ cents each: what is the cost of the whole bale? *Ans.* ——

7. What is the value of 300 hats at 3 dollars and 25 cents apiece? *Ans.* $975.

8. What is the value of 9704 oranges at 3½ cents each? *Ans.* $339,64.

9. What will be the cost of 356 sheep at 3¼ dollars a head? *Ans.* $1157.

10. What will be the cost of 47 barrels of apples at 1¾ dollars per barrel? *Ans.* ——

11. What is the value of 6000 bricks at 4,37½ per thousand? *Ans.* $26,25.

12. What is the cost of a box of oranges containing 450, at 2½ cents apiece? *Ans.* ——

13. What is the cost of 307 yards of linen at 68½ cents per yard? *Ans.* ——

DIVISION OF FEDERAL MONEY.

75. To divide a sum expressed in dollars, cents, and mills, by a simple number.

RULE.

I. *If the number to be divided contains dollars cents and mills, divide as in simple numbers, and separate the quotient into dollars cents and mills.*

II. *But if the number to be divided contains only dollars, or dollars and cents, bring it to mills by annexing one or more ciphers: then divide as in simple numbers, and separate the quotient as before.*

The answer is always sufficiently exact when it is true within 1 mill, and therefore the remainder in mills may always be neglected. But in common business the quotient figure in mills is neglected. When, however, such quotient figure is greater than 5, one may be added to the cents. The sign + is added in the examples, to show that the division may be continued.

EXAMPLES.

1. Divide \$4, 62 4 by 4; also, \$87, 25 6 by 5.

OPERATION.	OPERATION.
4)\$4, 62 4	5)\$87, 25 6
\$1, 15 6	\$17, 45 $1\frac{1}{5}$

2. Divide \$37 by 8.

In this example we first reduce the \$37 to mills by annexing three ciphers. The quotient will then be mills, and can be reduced to dollars and cents, as before.

OPERATION.
8)\$37, 00 0
\$ 4, 62 5

3. Divide \$56, 17 by 16. *Ans.* \$3, $51\frac{1}{16}$.

75. How do you divide in Federal money? When the number to be divided contains only dollars, how do you divide? When is the answer sufficiently exact? In common business are the mills considered? When they exceed five, what do you do? How do you denote that the *division* may be continued?

4. Divide $495, 70 4 by 129. *Ans.* $3, 84+.
5. Divide $12 into 200 equal parts. *Ans.* $0, 06.
6. Divide $400 into 600 equal parts. *Ans.* ——
7. Divide $857 into 51 equal parts. *Ans.* $16, 80+.
8. Divide $6578, 95 in 157 equal parts. *Ans.* $41, 90 4+.
9. Divide $248,54 by 125. *Ans.* —— +.
10. Divide $100 by 33. *Ans.* $3, 03 0+.
11. Divide $120463,2 by 1728. *Ans.* $69, 7125.

APPLICATIONS.

1. A man bought a piece of cloth containing 72 yards, for which he paid $252: what did he pay per yard? *Ans.* $3,50.

2. If $600 be divided equally among 26 persons, what will be each one's share? *Ans.* 23,07+.

3. Divide $18000 in 40 equal parts: what is the value of each part? *Ans.* ——

4. Divide $3769,25 into 50 equal parts: what is one part? *Ans.* $75,38 5.

5. A farmer purchased a farm containing 725 acres, for which he paid $18306,25: what did it cost him per acre? *Ans.* $25,25.

6. A merchant buys 15 bales of goods at auction, for which he pays $1000: what do they cost him per bale? *Ans.* $66,66 6+.

7. A drover pays $1250 for 500 sheep: what shall he sell them for apiece, that he may neither make nor lose by the bargain? *Ans.* ——

8. The dairy of a farmer produces $600, and he has 25 cows: how much does he make by each cow? *Ans.* $24.

9. A farmer receives $840 for the wool of 1400 sheep: how much does each sheep produce him? *Ans.* $0,60.

10. A merchant buys a piece of goods containing 105 yards, for which he pays $262,50: he wishes to sell it *so as to* make $52,50: how much must he ask per *yard?* *Ans.* ——

APPLICATIONS IN THE FOUR PRECEDING RULES.

1. A farmer sold a yoke of oxen for $80,75; 6 cows for $29 each; 30 sheep at $2,50 a head; and 3 colts, one for $25, the other two for $30 apiece: what did he receive for the whole lot? *Ans.* $414,75.

2. A merchant buys 6 bales of goods, each containing 20 pieces of broadcloth, and each piece of broadcloth contained 29 yards; the whole cost him $15660: how many yards of cloth did he purchase, and how much did it cost him per yard? *Ans.* { 3480 *yards.* $4,50 *per yard.* }

3. A man dies leaving an estate of $33000 to be equally divided among his 4 children, after his wife shall have taken her third. What was the wife's portion, and what the part of each child? *Ans.* { }

4. A person sells 3 cows at $25 each; and a yoke of oxen for $65: he agrees to take in payment 60 sheep: how much do his sheep cost him per head?

Ans. $2, 33 3+.

5. A person settling with his butcher, finds that he is charged with 126 pounds of beef at 9 cents per pound; 85 pounds of veal at 6 cents per pound; 6 pairs of fowls at 37 cents a pair; and three hams at $1,50 each: how much does he owe him? *Ans.* $23,16.

6. A farmer agrees to furnish a merchant 40 bushels of rye at 62 cents per bushel, and to take his pay in coffee at 16 cents a pound: how much coffee will he receive? *Ans.* ——

7. A farmer bargains with his tailor for a new coat every six months, a new vest every three months, and three pairs of pantaloons a year: the coats to cost $29,50 each, the vests $3 apiece, and the pantaloons $12 a pair: at the end of two years how much does he owe him?

Ans. $214.

8. A bookseller sells 14 dozen of books at $4,56 per dozen. He takes in part payment 6 reams of letter paper at $3,50 per ream. How much is still due him?

Ans. $42,84.

9. A farmer raises 300 bushels of wheat, for which he receives $1,37½ per bushel; 500 bushels of potatoes at 29 cents a bushel; 1000 bushels of oats at 34 cents a bushel; and 75 tons of hay for which he receives 16 dollars per ton: how much does the whole come to?

Ans. $2097,50.

10. A farmer has 6 ten-acre lots, in each of which he pastures 6 cows: each cow produces 112 pounds of butter, for which he receives 18¾ cents per pound: the expenses of each cow are 5 dollars and a half: how much does he make by his dairy? *Ans.* $547,92.

11. A drover goes to New York with 40 horses, of which he sells 20 for 125 dollars apiece, and the other 20 for 119 dollars apiece. He sells 17 cows at 24 dollars a head, 12 fat oxen at $130 a pair, and 30 sheep at $3½ a head: how much does he receive for them all?

Ans. ——

12. A person settling with his grocer finds that he has purchased 12 pounds of tea at 85½ cents a pound; 85 pounds of brown sugar at 9¼ cents a pound; 64 pounds of coffee at 7¼ cents a pound; and 38 pounds loaf sugar at 12½ cents a pound: how much does he owe?

13. A man lets out 2000 sheep, with the condition that he is to have three-fourths of what they produce after deducting the expenses of shearing: they yield 4 pounds of wool a head, which is sold at 47½ cents per pound: the expense of shearing is one-tenth of the whole: what does the owner of the sheep receive? *Ans.* ——

14. A man lets out his farm on shares. He is to have half the grain, one-third the price of the hay, and one quarter the increase of the live-stock. At the end of the time, there have been raised, 500 bushels of wheat worth $1,87½ a bushel, 300 bushels of oats worth 37½ cents a bushel, 250 bushels of corn worth 80 cents a bushel, 65 tons of hay worth $18 a ton; and the increase of the live-stock has been, 5 two years old worth $8 apiece, 8 calves worth $5 apiece, 10 sheep worth $2 apiece, a colt worth $36, and a pair of steers worth $28,50: how much was the owner of the land to receive? *Ans.* $1056,12½.

BILLS OF PARCELS.

New York, May 1st, 1847.

Mr. James Spendthrift

Bought of Benj. Saveall.

16 pounds of tea at 85 cents per pound - - -
27 pounds of coffee at 15½ cents per pound - -
15 yards of linen at 66 cents per yard - - -

$27, 68 5

Rec'd payment, *Benj. Saveall.*

Albany, June 2d, 1847.

Mr. Jacob Johns *Bought of Gideon Gould.*

36 pounds of sugar at 9½ cents per pound - -
3 hogsheads of molasses, 63 galls. each, at 27 cents a gallon - - - - - - - - - -
5 casks of rice, 285 pounds each, at 5 cents per pound - - - - - - - - - - - - -
2 chests of tea, 86 pounds each, at 96 cents per pound - - - - - - - - - - - - -

Total cost $290,82

Rec'd payment, For Gideon Gould,
Charles Clark.

Hartford, November 21st, 1847.

Gideon Jones *Bought of Jacob Thrifty.*

69 chests of tea at $55,65 per chest - - -
126 bags of coffee, 100 pounds each, at 12½ cents per pound - - - - - - - -
167 boxes of raisins at $2,75 per box - - -
800 bags of almonds at $18,50 per bag - - -
9004 barrels of shad at $7,50 per barrel - - -
60 barrels of oil, 32 gallons each, at $1,08 per gallon - - - - - - - - - - -

Amount $90277,70

Received *the above* in full, Jacob Thrifty.

ENGLISH MONEY.

76. The denominations of English money, are guineas, pounds, shillings, pence, and farthings.

TABLE.

4 farthings marked *far.*	make	1 penny, marked	*d.*
12 pence	- - -	1 shilling, -	*s.*
20 shillings	- - -	1 pound, -	£.
21 shillings	- - -	1 guinea.	

£	*s.*	*d.*	*far.*
1 =	20 =	240 =	960
	1 =	12 =	48
		1 =	4

Note 1. This table is read, 4 farthings make 1 penny, 12 pence make 1 shilling, 20 shillings 1 pound.

Note 2. Farthings are generally expressed in fractions of a penny. Thus, for 1 farthing, write $\frac{1}{4}$*d.*, for 2 farthings $\frac{1}{2}$*d.*, and for 3 farthings, $\frac{3}{4}$*d.*

REDUCTION OF DENOMINATE NUMBERS.

77. Reduction is changing the unit or denomination of a number, without altering the value of the number.

For example, 42 dollars and 35 cents are expressed in different denominations.

But 42 dollars are equal to 4200 cents.
Add - - - - - - 35 cents,
and the sum 4235 cents,

is equal to 42 dollars and 35 cents. Here we have brought the numbers to the same denomination without altering their value.

---

76. What are the denominations of English money? Repeat the table. How many farthings in 1 shilling? In 1 pound? How are farthings generally written?

77. What is reduction? How many pounds and shillings in 24 shillings? How many feet in a yard? How many inches in a *foot?* How many feet in 3 yards? How many inches in 3 yards? *How many feet* in 72 inches? How many yards?

Again, if we have 24 shillings, we can reduce them to pounds and shillings; for, since 20 shillings make 1 pound, 24 shillings are equal to £1 4*s*. Here we have again changed the denomination without altering the value.

We may take, as another example, 3 yards and reduce it to inches. Now, since 3 feet make a yard, and 12 inches a foot, we have

$3 \times 3 = 9$ feet; and $9 \times 12 = 108$ inches.

If, on the contrary, it were required to bring inches into yards, we should first divide by 12, to bring them into feet, and then by 3 to bring the feet into yards. Thus,

108 inches $\div 12 = 9$ feet; and 9 feet $\div 3 = 3$ yards.

78. From the above illustrations we see, that reduction of denominate numbers generally, like that of Federal money, is divided into two parts.

1st. To change the unit of a number from a higher denomination to a lower.

2d. To change the unit of a number from a lower denomination to a higher.

CASE I.

79. To reduce denominate numbers from a higher denomination to a lower.

RULE.

I. *Consider how many units of the next lower denomination make one unit of the higher.*

II. *Multiply the higher denomination by that number, and add to the product the number belonging to the lower: we shall then have the equivalent number in the next lower denomination.*

III. *Proceed in a similar way through all the denominations to the last; the last sum will be the required number.*

---

78. Into how many parts may reduction of denominate numbers be divided? Name them. Does the term reduction imply a change in value?

79. How do you reduce numbers from a higher to a lower denomination? *Repeat the rule.*

EXAMPLES.

1. Reduce 9 yards and 6 feet to inches.

We first bring the yards to feet, and then add the 6 feet, after which we reduce the whole to inches.

OPERATION.

```
  9
  3
 --
 27
  6 feet to be added
 --
 33
 12
---
396 inches.
```

2. Reduce £27 6s. 8d. to the denomination of pence.

We first bring the pounds to shillings and then add the 6s.; we then bring the shillings to pence and add in the 8d., giving for the answer, 6560 pence.

OPERATION.

```
£27 6s. 8d.
 20
---
540
  6s.
----
546s.
 12
----
6552
   8d.
-----
6560d.
```

In reducing, we often add the next lower denomination mentally, without setting it down. Thus, when we multiply by 20, we add the 6s. without writing it down, making in the product 6 in the units' place: and when we multiply by 12 we say, 12 times 6 are 72 and 8d. to be added make 80.

OPERATION.

```
£27 6s. 8d.
 20
----
546s.
 12
----
6560
```

3. In £1465 14s. 5d., how many farthings? *Ans.* 1407092.

4. In £45 12s. 10d., how many pence? *Ans.* 10954.

5. In 87 guineas, how many farthings? *Ans.* 87696.

6. In £145 16s. 11d., how many pence? *Ans.* 35003.

CASE II.

80. To reduce denominate numbers from a lower denomination to a higher.

RULE.

I. *Consider how many units of the given denomination make one unit of the next higher, and take this number for a divisor: divide the given number by it and set down the remainder, if there be any.*

II. *Divide the quotient thus obtained by the number of units in the next higher denomination, and set down the remainder.*

III. *Proceed in the same way through all the denominations to the highest: the last quotient with the several remainders annexed, will give the answer sought, and if there be no remainders, the last quotient will be the answer.*

EXAMPLES.

1. Reduce 3138 farthings to the denomination of pounds.

OPERATION.

```
4)3138
12)784 . 2 far. rem.
2|0)6|5 . . 4d. rem.
      3 . . 5s. rem.
Ans. £3 5s. 4d. 2far.
```

In this example we first divide by 4, the number of farthings in a penny; the quotient is 784 pence, and 2 farthings over. The 784 pence are then divided by 12, the number of pence in a shilling. The quotient is 65 shillings, and four pence over. The 65 shillings are then divided by 20, the number of shillings in a pound; the quotient is £3 and a remainder of 5 shillings. Hence, £3 5*s*. 4*d*. 2*far*. is the value of 3138 farthings.

2. Reduce 3658 inches to yards.

Ans. 101 *yards*, 1 *foot*, 10 *inches*.

3. In 80 guineas, how many pounds? *Ans*. £84.

---

80. In reducing from a lower denomination to a higher what do you first do? What next? and what next? Is this rule applicable to all denominate numbers?

4. In 1549 farthings, how many pounds, shillings and pence? *Ans.* £1 12*s.* 3¼*d.*

5. Reduce 1046 pence to pounds. *Ans.* £4 7*s.* 2*d.*

6. Reduce 4704 pence to guineas. *Ans.* 18 *guineas* 14*s.*

7. In 6169 pence, how many £? *Ans.* £25 14*s.* 1*d.*

PROOF OF REDUCTION.

81. After a number has been reduced from a higher denomination to a lower, by the first rule, let it be reduced back by the second; and after a number has been reduced from a lower denomination to a higher, by the second rule, let it be reduced back by the first rule. If the results agree, the work is supposed right.

EXAMPLES.

1. Reduce £15 7*s.* 6*d.* to the denomination of pence.

OPERATION.	PROOF.
15	12)3690
20	2|0)30|7 . . . 6*d.* rem.
307	15 . . . 7*s.* rem.
12	
3690	*Ans.* £15 7*s.* 6*d.*

2. In £31 8*s.* 9*d.* 3*far.*, how many farthings? Also the proof.

3. In £87 14*s.* 8½*d.*, how many farthings? Also the proof.

4. In £407 19*s.* 11¾*d.*, how many farthings? Also the proof.

AVOIRDUPOIS WEIGHT.

82. The standard avoirdupois pound of the United States, as determined by Mr. Hassler, is the weight of 27.7015 cubic inches of distilled water. By this weight are weighed all coarse articles, such as hay, grain, chandlers' wares, and all the metals, except gold and silver.

---

81. How do you prove reduction?

In this weight the words *gross* and *net* are used. Gross weight is the weight of the goods, with the boxes, casks, or bags in which they are contained. Net is the weight of the goods only; or what remains after deducting from the gross weight the weight of the boxes, casks, or bags.

According to the old method of weighing, which was adopted from the English system, 112 pounds make what was called one hundred weight. But at the present time the merchants in our principal cities buy and sell by the 100 pounds.

TABLE.

16 drams *dr.*	make	1 ounce,	marked	*oz.*
16 ounces	- -	1 pound,	- - -	*lb.*
25 pounds	- -	1 quarter,	- - -	*qr.*
4 quarters	- -	1 hundred weight,		*cwt.*
20 hundred weight,		1 ton,	- - - -	*T.*

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
1 =	20 =	80 =	2000 =	32000 =	512000
	1 =	4 =	100 =	1600 =	25600
		1 =	25 =	400 =	6400
			1 =	16 =	256
				1 =	16

NOTE 1. Read the table across the page downward, thus, 16 drams make 1 ounce, 16 ounces make 1 pound, 25 pounds make 1 quarter, 4 quarters 1 hundred, and 20 hundreds 1 ton.

NOTE 2. We can always see from the tables, by reading them from the left to the right, the value of any unit expressed in each of the lower denominations. Thus, 1 ounce is equal to 16 drams, 1 pound equal to 16 ounces, equal to 256 drams, &c.

---

82. What is the standard avoirdupois pound of the United States? For what is this weight used? What is the meaning of the terms *gross* and *net?* What is a hundred weight? How are goods now generally bought and sold? How is the table to be read? How can you determine, from the table, the value of any unit in units of the lower *denominations?*

EXAMPLES.

1. Reduce 5*T.* 8*cwt.* 3*qr.* 24*lb.* 13*oz.* 14*dr.* to drams

We first multiply by 20 and add in the 8 hundred: we next multiply by 4 and add in the 3*qr.*; next by 25 and add in the 24*lb.*; next by 16 and then add in the 13*oz.*; and finally by 16 and add in the 14*dr.*

OPERATION.

```
      5
     20
    ---
    108
      4
    ---
    435
     25
  -----
  10899
     16
 ------
 174397
     16
-------
2790366 drams.
```

2. Reduce 27*T.* 17*cwt.* 29*qr.* 21*lb.* to ounces.
Ans. 903136*oz.*

3. Reduce 94*T.* 19*cwt.* 1*qr.* to quarters.
Ans. 7597*qr.*

4. Reduce 3124446 drams to tons.

```
4×4=16 { 4)3124446 . . . 2
       {  4)781111 . . . 3×4+2=14dr.
4×4=16 {  4)195277 . . . 1
       {   4)48819 . . . 3×4+1=13oz.
5×5=25 {   5)12204 . . . 4
       {    5)2440 . . . 0+4=4lb.
             4)488 . . . 0qr.
          2|0)12|2 . . . 2cwt.
                 6
```

Ans. 6*T.* 2*cwt.* 0*qr.* 4*lb.* 13*oz.* 14*dr.*

5. Reduce 108910592 drams to tons.
Ans. 212*T.* 14*cwt.* 1*qr.* 7*lb.*

6. Reduce 2998128 ounces to tons.
Ans. 93*T.* 13*cwt.* 3*qr.* 8*lb.*

TROY WEIGHT.

83. Gold, silver, jewels, and liquors, are weighed by this weight. The standard Troy pound of the United States, as determined by Mr. Hassler, is the weight of 22.794377 cubic inches of distilled water. Hence, it is less than the pound avoirdupois. Its denominations are pounds, ounces, pennyweights, and grains.

TABLE.

24 grains, *gr.*	make	1 pennyweight, marked	*pwt.*
20 pennyweights	-	1 ounce - - - -	*oz.*
12 ounces	- - -	1 pound - - - -	*lb.*

lb.	*oz.*	*pwt.*	*gr.*
1=	12=	240=	5760
	1=	20=	480
		1=	24

EXAMPLES.

1. Reduce 16*lb.* 11*oz.* 15*pwt.* to pennyweights.

In this example, we first multiply by the number of ounces in a pound, and then add the ounces; we then multiply by 20, and add the pennyweights.

OPERATION.

```
 16 lb.
 12 oz.
----
 192
  11 oz. added.
----
 203
  20 pwt. in an oz.
----
4060
  15 pwt. added.
----
4075 pennyweights.
```

2. In 25*lb.* 9*oz.* 0*pwt.* 20*gr.*, how many grains?
Ans. 148340.

---

83. What things are weighed by Troy weight? What is the standard *pound?* What are its denominations? Repeat the table.

3. Reduce 6490*gr.* to pounds.

We first divide by the number of grains in a *pwt.*; then by the *pwts.* in an *oz.*; then by the ounces in a *lb.*

OPERATION.

```
24) 6490
    2|0)27|0   10gr. remainder.
       12)13 . . 10pwt. remainder.
           1 . .  1oz. remainder.
    Ans. 1lb. 1oz. 10pwt. 10gr.
```

4. In 678618 grains, how many pounds?

Ans. 117*lb.* 9*oz.* 15*pwt.* 18*gr.*

5. Reduce 8794*pwt.* to pounds.

Ans. 36*lb.* 6*oz.* 14*pwt.*

6. Reduce 756412*pwt.* to pounds. *Ans.* ——

7. In 897264*lb.*, how many grains? *Ans.* ——

APOTHECARIES' WEIGHT.

84. This weight is used by apothecaries and physicians in mixing their medicines. Its denominations are pounds, ounces, drams, scruples, and grains. The pound and ounce are the same as the pound and ounce in the Troy weight; the difference between the two weights consists in the different divisions and subdivisions of the ounce.

TABLE.

20 grains, *gr.*	make	1 scruple,	marked ℈.
3 scruples	- -	1 dram,	- - - ʒ.
8 drams	- - -	1 ounce,	- - ℥.
12 ounces	- - -	1 pound,	- - ℔.

℔	℥	ʒ	℈	gr.
1=	12=	96=	288=	5760
	1=	8=	24=	480
		1=	3=	60
			1=	20

84. What is the use of the Apothecaries' weight? What are its denominations? Of what value are the pound and the ounce? Repeat the table.

EXAMPLES.

1. Reduce 9℔ 8℥ 6ʒ 2℈ 12*gr.*, to grains.

We first multiply by the number of ounces in a *lb.*, and at the same time add in the ounces. We next multiply by the number of drams in an ounce, and add in the drams: we then multiply by the number of scruples in a dram, and add in the scruples; and lastly, we multiply by 20 and add in the grains.

OPERATION.

```
        9
       12
      116 ounces.
        8
      934 drams.
        3
     2804 scruples.
       20
Ans. 56092 grains.
```

2. Reduce 27℔ 9℥ 6ʒ 1℈ to scruples. *Ans.* 8011 *scruples.*

3. Reduce 94℔ 11℥ 1ʒ to drams. *Ans.* 9113 *drams.*

LONG MEASURE.

85. This measure is used to measure distances, lengths, breadths, heights, depths, &c. Its denominations are barleycorns, inches, feet, yards, fathoms, rods, furlongs, and miles.

TABLE.

3 barleycorns, *bar.*	make	1 inch, marked	*in.*
12 inches	- - -	1 foot, - -	*ft.*
3 feet	- - -	1 yard - -	*yd.*
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	-	1 rod, perch, or pole,	*rd.*
40 rods	- - -	1 furlong, - -	*fur.*
8 furlongs or 320 rods		1 mile, - -	*mi.*
3 miles	- - -	1 league, - -	*L.*
60 geographical or $69\frac{1}{2}$ statute miles		1 degree, - -	*deg.* or °
360 degrees	- - -	a great circle, or circumference of the earth.	

85. When is Long Measure used? What are its denominations? Repeat the table. What is a fathom? What is a hand?

mi.	fur.	rd.	yd.	ft.	in.
1=	8=	320=	1760=	5280=	63360
	1=	40=	220=	660=	7920
		1=	$5\frac{1}{2}$=	$16\frac{1}{2}$=	198
			1=	3=	36
				1=	12

NOTE. A fathom is a length of six feet, and is generally used to measure the depth of water.

A hand is 4 inches, and is used to measure the height of horses.

EXAMPLES.

1. In 675*ft.* 10*in.* 2*bar.*, how many barleycorns?

We first reduce the feet to inches and then add in the 10 inches: we next reduce the inches to barleycorns and add in the 2 barleycorns.

OPERATION.

```
      675
       12
     8110
        3
Ans. 24332 barleycorns.
```

2. In 59*mi.* 7*fur.* 38*rd.*, how many rods?
Ans. 19198*rd.*

3. In 194656*bar.*, how many feet?

We first divide by the number of barleycorns in an inch, and then by the number of inches in a foot.

OPERATION.

```
3)194656
12)64885 . . . 1bar.
    5407 . . . 1in.
```

Ans. 5407*ft.* 1*in.* 1*bar.*

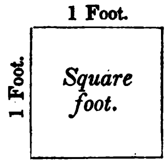
4. In 115188 rods, how many miles?
Ans. 359*mi.* 7*fur.* 28*rd.*

5. In 719*m.* 7*fur.* 16*rd.*, how many feet?
Ans. 3801204.

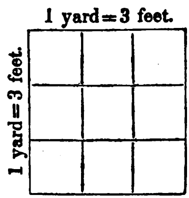
6. In 118°, how many statute and how many geographical miles?
Ans. 8201 *statute miles.* 7080 *geographical miles.*

LAND OR SQUARE MEASURE.

86. Land or square measure is used in measuring land, or any thing in which length and breadth are both considered.



A square is the space included between four equal lines, drawn perpendicular to each other. Each line is called a side of the square. If each side be one foot, the figure is called a *square foot.*



If the sides of the square be each one yard, the square is called a *square yard.* In the large square there are nine small squares, the sides of which are each one foot. Therefore, the square yard contains 9 square feet.

The number of small squares that is contained in any large square is always equal to the product of two of the sides of the large square. As in the figure, $3\times3=9$ square feet. The number of square inches contained in a square foot is equal to $12\times12=144$.

TABLE.

144 square inches, *sq. in.*	make	1 square foot,	*Sq. ft.*
9 square feet	- -	1 square yard,	*Sq. yd.*
$30\frac{1}{4}$ square yards	- -	1 square pole,	*P.*
40 square poles	- -	1 rood, -	*R.*
4 roods	- - -	1 acre, -	*A.*
640 acres	- - -	1 square mile,	*M.*

---

86. For what is Square Measure used? What is a square? If each side be one foot, what is it called? If each side be a yard, what is it called? How many square feet does the square yard contain? How is the number of small squares contained in a large square found? Repeat the table.

A.	*R.*	*P.*	*Sq. yd.*	*Sq. ft.*	*Sq. in.*
1=	4=	160=	4840=	43560=	6272640
	1=	40=	1210=	10890=	1568160
		1=	$30\frac{1}{4}$=	$272\frac{1}{4}$=	39204
			1=	9=	1296
				1=	144

87. The Surveyor's or Gunter's chain is generally used in surveying land. It is 4 poles or 66 feet in length, and is divided into 100 links.

TABLE.

$7\frac{92}{100}$ inches	make	1 link, marked	*l.*
4 rods or 66 *ft.*	- -	1 chain,	*c.*
80 chains	- - -	1 mile,	*mi.*
1 square chain	- -	16 square poles,	*P.*
10 square chains	- -	1 acre,	*A.*

Land is generally estimated in square miles, acres, roods, and square poles or perches.

EXAMPLES.

1. In 32*M.* 25*A.* 3*R.*, how many square poles?

We first bring the square miles to acres by multiplying by 640, and then add in the 25 acres. We next reduce to roods and add in the 3 roods: we then reduce to poles.

OPERATION.

```
         32
        640
      20505
          4
      82023 roods
         40
Ans. 3280920 P.
```

2. In 19*A.* 2*R.* 37*P.*, how many square poles?
Ans. 3157*P.*

3. In 175 square chains, how many square poles?
Ans. 2800*P.*

87. What chain is used in surveying land? How long is it? How is it divided? Repeat the table. How is land generally estimated?

4. In 37456 square inches, how many square feet?

$$12 \times 12 = 144 \left\{ \begin{array}{l} 12)37456 \\ \quad 12)3121 \;..\; 4 \\ \qquad 260 \;..\; 1 \end{array} \right.$$

$$1 \times 12 + 4 = 16.$$

Ans. 260*Sq. ft.* 16*Sq. in.*

5. In 14972 square rods, how many acres?

Ans. 93*A.* 2*R.* 12*P.*

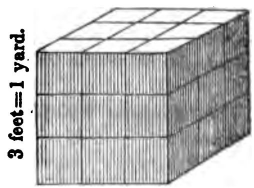
6. In 3674139*P.*, how many square miles?

Ans. 35*M.* 563*A.* 1*R.* 19*P.*

SOLID OR CUBIC MEASURE.

88. Solid or cubic measure is used in measuring stone, timber, earth, and such other things as have three dimensions, length, breadth, and thickness. Its denominations are tons, cords, yards, feet, and inches.

A cube is a body, or solid, having six equal faces, which are squares. If the sides of the cube be each one foot long, the solid is called a cubic or solid foot. But when the sides of the cube are one yard, as in the figure, the cube is called a cubic or solid yard. The



3 feet=1 yard.

base of the cube, which is the face on which it stands, contains $3 \times 3 = 9$ square feet. Therefore 9 cubes, of one foot each, can be placed on the base. If the solid were one foot high it would contain 9 cubic feet; if it were 2 feet thick it would contain two tiers of cubes, or 18 cubic feet; and if it were 3 feet high, it would contain three tiers, or 27 cubic feet. Hence, *the content of a solid is equal to the product of its length, breadth, and*

---

88. For what is solid or cubic measure used? What are its denominations? What is a cube? What is a cubic or solid foot? What is a cubic yard? How many cubic feet in a cubic yard? What is the content of a solid equal to? Repeat the table. What is a cord of wood? How many solid feet does it contain?

thickness. Therefore, 1 cubic foot contains $12\times12\times12=1728$ cubic inches.

TABLE.

1728 solid inches, *S. in.*	make	1 solid foot,	*S. ft.*
27 solid feet	- - -	1 solid yard,	*S. yd.*
40 feet of round, or 50 feet of hewn timber,		1 ton, - -	*Ton.*
128 solid feet $=8\times4\times4$, that is, a pile 8 feet in length, 4 feet in width, and 4 feet in height,	make	1 cord of wood -	*C.*

NOTE. A cord foot, is one foot in length of the pile which makes a cord. It contains sixteen solid feet.

EXAMPLES.

1. Reduce 14 tons of round timber to solid inches. *Ans.* 967680 *solid inches.*

2. In 55 cords of wood, how many solid feet? *Ans.* 7040.

3. In 25 cords of wood, how many cord feet? *Ans.* *cord ft.*

4. Reduce 3058560 cubic inches to tons of round timber.

We first divide by 1728, the number of solid inches in a solid foot, and next by 40, the number of solid feet in a ton.

OPERATION.

```
1728)3058560(1770
     1728
     -----
     13305
     12096
     ------
      12096      4|0)177|0
      12096          ---
      -----          44 . . . 10
      00000
```

Ans. 44 *tons* 10*ft.*

5. Reduce 28160 solid feet to cords. *Ans.* 220 *cords.*

6. Reduce 174964 cord feet to cords. *Ans.* 21870 *cords,* 4 *cord feet.*

7. In 7645900 solid inches, how many tons of hewn timber? *Ans.* 88 *tons* 24 *S. ft.* 1228 *S. in.*

CLOTH MEASURE.

89. Cloth measure is used for measuring all kinds of cloth. Its denominations are Ells French, Ells English, Ells Flemish, yards, quarters, nails, and inches.

TABLE.

$2\frac{1}{4}$ inches, *in.*	make	1 nail, marked	*na.*
4 nails	- - -	1 quarter of a yard,	*qr.*
4 quarters	- -	1 yard, - - -	*yd.*
3 quarters	- -	1 Ell Flemish, -	*E. Fl.*
5 quarters	- -	1 Ell English -	*E. E.*
6 quarters	- -	1 Ell French -	*E. Fr.*

EXAMPLES.

1. In 35*yd.* 3*qr.* 3*na.*, how many nails?

We first reduce the yards to quarters and add in the 3*qr.*; we next reduce the quarters to nails and add in the 3 nails.

OPERATION.

```
      35
       4
     ---
     143
       4
     ---
Ans. 575 na.
```

2. Reduce 49 Ells English to nails. *Ans.* 980*na.*

3. Reduce 51 Ells Flemish, 2*qr.* 3*na.* to nails. *Ans.* 623*na.*

4. In 3278 nails, how many yards?

We first divide by 4, which brings the number to quarters, and then again by 4, which brings it to yards.

OPERATION.

```
4)3278
 4)819 . . 2na.
   204 . . 3qr.
Ans. 204yd. 3qr. 2na.
```

5. Reduce 340 nails to Ells Flemish. *Ans.* 28*E. Fl.* 1*qr.*

6. In 67 quarters, how many yards? *Ans.* 16*yd.* 3*qr.*

---

89. For what is cloth measure used? What are its denominations? Repeat the table.

LIQUID MEASURE.

90. The standard gallon of the United States is the wine gallon of Great Britain, and contains 231 cubic inches. This is the standard for all liquids. The denominations of liquid measure are tuns, pipes, hogsheads, barrels, gallons, quarts, pints, and gills.

TABLE.

4 gills, *gi.*	make	1 pint,	marked *pt.*
2 pints	- - -	1 quart,	- - - *qt.*
4 quarts	- - -	1 gallon,	- - - *gal.*
31½ gallons	- - -	1 barrel,	- - - *bar.*
63 gallons	- - -	1 hogshead,	- - *hhd.*
2 hogsheads	- -	1 pipe,	- - - *pi.*
2 pipes or 4 hogsheads		1 tun,	- - - *tun.*

tun.	*pi.*	*hhd.*	*bar.*	*gal.*	*qt.*	*pt.*	*gi.*
1 =	2 =	4 =	8 =	252 =	1008 =	2016 =	8064
	1 =	2 =	4 =	126 =	504 =	1008 =	4032
		1 =	2 =	63 =	252 =	504 =	2016
			1 =	31½ =	126 =	252 =	1008
				1 =	4 =	8 =	32
					1 =	2 =	8
						1 =	4

EXAMPLES.

1. In 5 tuns 1 hogshead of wine, how many gallons?

OPERATION.

```
     5
     4
    --
    21
    63
    --
    63
   126
   ----
Ans. 1323 gal.
```

We first multiply by 4, the number of hogsheads in a tun, and add in the 1 hogshead, after which we reduce to gallons.

---

90. **What is measured by liquid measure? What are its denominations? Repeat the table. What is the content of the wine gallon?**

2. Reduce 12 pipes 1 hogshead and 1 quart of wine to pints. *Ans.* 12602*pt.*

3. In 1 tun of cider, how many gills? *Ans.* 8064.

4. In 10584 quarts of wine, how many tuns?

```
              4)10584
63=7×9 { 7)2646
         { 9)378
              4)42
       Ans. 10tuns 2hhd.
```

5. Reduce 201632 gills to tuns. *Ans.* 25*tuns* 1*gal.*

6. Reduce 16128 gills of cider to tuns. *Ans.* 2*tuns.*

ALE OR BEER MEASURE.

91. In the English ale or beer measure 36 gallons make a barrel, and 54 gallons a hogshead. Its denominations are hogsheads, barrels, gallons, quarts, and pints.

TABLE.

2 pints, *pt.*	make	1 quart,	marked	*qt.*
4 quarts	-	1 gallon,	-	*gal.*
36 gallons	-	1 barrel,	-	*bar.*
54 gallons	-	1 hogshead,	-	*hhd.*

hhd.	*bar.*	*gal.*	*qt.*	*pt.*
1 =	$1\frac{1}{2}$ =	54 =	216 =	432
	1 =	36 =	144 =	288
		1 =	4 =	8
			1 =	2

EXAMPLES.

1. Reduce 47*bar.* 16*gal.* 4*qt.* to pints. *Ans.* 13672*pt.*

2. In 27*hhd.* of beer, how many pints? *Ans.* 11664.

3. In 55832 pints of beer, how many hogsheads? *Ans.* ——

4. In 64972 quarts of beer, how many barrels? *Ans.* 451*bar.* 7*gal.*

---

91. What are the denominations of ale or beer measure? Repeat the table.

DRY MEASURE.

92. Dry measure is used in measuring all dry articles, such as grain, fruits, roots, salt, coal, &c. Its denominations are chaldrons, bushels, pecks, quarts, and pints.

TABLE.

2 pints, *pt.*	make	1 quart,	marked *qt.*
8 quarts	- - -	1 peck,	- - - *pk.*
4 pecks	- - -	1 bushel,	- - *bu.*
36 bushels	- -	1 chaldron,	- - *ch.*

ch.	*bu.*	*pk.*	*qt.*	*pt.*
1=	36=	144=	1152=	2304
	1=	4=	32=	64
		1=	8=	16
			1=	2

NOTE. A gallon dry measure contains 268.8 cubic inches. A Winchester bushel is $18\frac{1}{2}$ inches in diameter, 8 inches deep, and contains 2150.4 cubic inches. This is the standard bushel adopted in the United States.

EXAMPLES.

1. In 372 bushels, how many pints? *Ans.* 23808.
2. In 5 chaldrons 31 bushels, now many pecks? *Ans.* ——
3. In 17408 pints, how many bushels? *Ans.* 272.
4. In 4220 pints, how many chaldrons? *Ans.* 1*ch.* 29*bu.* 3*pk.* 6*qt.*

TIME.

93. The denominations of time are years, months, weeks, days, hours, minutes, and seconds.

---

92. For what is dry measure used? What are its denominations? Repeat the table. What is the content of a gallon? How large is a Winchester bushel? What is the standard in the United States?

60 seconds, *sec.*	make	1 minute,	marked	*m.*
60 minutes	- - -	1 hour,	- - -	*hr.*
24 hours	- - -	1 day,	- - -	*da.*
7 days	- - -	1 week,	- - -	*wk.*
4 weeks	- - -	1 month,	- - -	*mo.*
13*mo.* 1*da.* and 6*hrs.*, or 365*da.* 6*hrs.*		1 common or Julian year,	-	*yr.*

yr.	*wk.*	*da.*	*hr.*	*m.*	*sec.*
1 =	52 =	$365\frac{1}{4}$ =	8766 =	525960 =	31557600
	1 =	7 =	168 =	10080 =	604800
		1 =	24 =	1440 =	86400
			1 =	60 =	3600
				1 =	60

The whole days only are reckoned. The odd six hours, by accumulating for 4 years, make one day, so that every fourth year contains 366 days. This is called the Bissextile, or Leap year.

Although the year is reckoned at 365*da.* 6*hr.*, it is in fact but 365*da.* 5*hr.* 48*m.* 48*sec.*, and the difference by accumulating for 100 years makes about 1 day, so that the centennial years are not leap years. The leap years are exactly divisible by 4. They are 1840, 1844, &c.

The year is also divided into 12 calendar months, which contain an unequal number of days.

		Names.	*No. of Days.*
1	month	January,	31
2	- - -	February,	28
3	- - -	March,	31
4	- - -	April,	30
5	- - -	May,	31
6	- - -	June,	30
7	- - -	July,	31
8	- - -	August,	31
9	- - -	September,	30
10	- - -	October,	31
11	- - -	November,	30
12	- - -	December,	31
		Total	365

The additional day, when it occurs, is added to the month of February, so that this month has 29 days in the Leap year.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

EXAMPLES.

1. How many seconds in a year of 365*da*. 6*hr*.?

We first reduce the days to hours and add in the 6 hours. We then multiply by 60, which brings the whole to minutes, after which we again multiply by 60, which reduces the number to seconds.

OPERATION.

```
       365
        24
      1466
      730
      8766   hours.
        60
    525960   minutes.
        60
Ans. 31557600 seconds.
```

2. In 12 years of 365*da*. 23*hr*. 57*m*. 39*sec*. each, how many seconds? *Ans*. 379467108*sec*.

3. In 49 weeks, how many minutes? *Ans*. 493920.

4. In 126230400 seconds, how many years of 365 days?

We first divide by 60, which brings the number into minutes. We then divide again by 60, which brings it into hours, then by 24, which brings it into days; and lastly, by 365, which gives the quotient in years.

OPERATION.

```
           6|0)12623040|0
              6|0)210384|0
           {     4)35064
4×6=24     {      6)8766
              365)1461(4
                  1460
                     1
```

Ans. 4 *years* and 1 *day*.

93. What are the denominations of time? How long is a year? How many days in a common year? How many days in a Leap year? How many calendar months in a year? Name them, and the number of days in each. How many days has February in the leap year? How do you remember which of the months have 30 days, and which 31?

5. In 756952018 seconds, how many years of 365 days each? *Ans*. 24*yr*. 1*da*. 26*m*. 58*sec*.

6. In 5927040 minutes, how many weeks? *Ans*. 588.

CIRCULAR MEASURE OR MOTION.

94. Circular measure is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies. Every circle is supposed to be divided into 360 equal parts, called degrees. Each degree is divided into 60 minutes, and each minute into 60 seconds.

TABLE.

60 seconds ″	make	1 minute,	marked ′.
60 minutes	- -	1 degree,	- - °.
30 degrees	- -	1 sign,	- - - s.
12 signs or 360°		1 circle,	- - - c.

c.	s.	°.	′.	″.
1 =	12 =	360 =	21600 =	1296000
	1 =	30 =	1800 =	108000
		1 =	60 =	3600
			1 =	60

EXAMPLES.

1. Reduce 5s. 29° 25′ to minutes. *Ans*. 10765′.
2. In 2 circles, how many seconds? *Ans*. ——
3. In 32295 minutes, how many circles? *Ans*. 1c. 5s. 28° 15′.
4. In 27894 seconds, how many degrees? *Ans*. 7° 44′ 54″.

TABLE OF PARTICULARS.

12 things	make	1 dozen.
12 dozen	- - - - - -	1 gross.
12 gross, or 144 dozen	-	1 great gross.

---

94. For what is circular measure used? How is every circle supposed to be divided? Repeat the table.

How many things make a dozen? How many dozen a gross? How many gross a great gross? How many things make a score? How many pounds a quintal of fish? How many sheets a quire of paper? How many *quires* a ream?

ALSO,

20 things	make	1 score.
112 pounds	- - - - - -	1 quintal of fish.
24 sheets of paper	- - -	1 quire.
20 quires	- - - - - -	1 ream.

BOOKS.

A sheet folded in two leaves is called a folio.		
" folded in four leaves	- -	a quarto, or 4to.
" folded in eight leaves	- -	an octavo, or 8vo.
" folded in twelve leaves	-	a duodecimo, or 12mo.
" folded in eighteen leaves	-	an 18mo.

EXAMPLES—FROM A HIGHER TO A LOWER DENOMINATION.

1. How many hours in 344*wk.* 6*da.* 17*hr.* *A.* 57953.
2. In 6 signs, how many minutes? *Ans.* ——
3. In 15 tons of hewn timber, how many solid inches? *Ans.* 1296000.
4. How many times will a wheel, 16 feet and 6 inches in circumference, turn round in the distance of 84 miles? *Ans.* 26880.
5. What will 28 rods and 129 feet of land cost, at 12 dollars a square foot? *Ans.* $93024.
6. In 59*lb.* 13*pwt.* 5*gr.*, how many grains? *Ans.* 340157.
7. In £85 8*s.*, how many guineas? *Ans.* 81 *guineas* 7*s.*
8. How many cords are there in a pile of wood that is 36 feet long, 6 feet high, and 4 feet wide? *Ans.* 6 *cords*, and 6 *cord feet.*
9. A man has a journey to perform of 288 miles; supposing him to travel 12 hours each day for 6 days in succession, at what rate must he travel per hour to accomplish it in that time? *Ans.* ——

When a sheet is folded in two leaves, what is it called? When folded in four leaves? When folded in eight leaves? When folded in twelve? When folded in eighteen?

10. How many yards of carpeting which is one yard in width, will be required to carpet a room 18 feet wide and 20 feet long? *Ans.* 40.

11. Reduce 346 Ells Flemish to Ells English. *Ans.* $207\frac{3}{5}$*E. E.*

12. Suppose the number of inhabitants in the United States to be 12 millions, how long would it take to count them, counting at the rate of 50 a minute? *Ans.* 166 *days* 16 *hours.*

13. A merchant wishes to bottle a cask of wine containing 63 gallons, into bottles containing one pint each: how many bottles are necessary? *Ans.* 504.

14. There is a cube, or square piece of wood, 2 feet or 24 inches each way; how many small cubes of one inch each way can be sawed from it, allowing no waste in sawing? *Ans.* 13824.

15. A merchant wishes to ship 285 bushels of flax-seed, in casks containing 7 bushels 2 pecks each: what number of casks are required? *Ans.* 38.

16. In two leagues, how many inches? *Ans.* ——

17. How many times will a wheel, 10 feet and 6 inches in circumference, turn round in going from New Haven to Hartford, the distance being 34 miles? *Ans.* $17097\frac{18}{126}$ *times.*

18. How many times will a ship, 104 feet and 8 inches long, sail her length in going from New York to China, it being about 12000 miles? *Ans.* $605350\frac{400}{1256}$ *times.*

19. In 29 pieces of Holland, each containing 36 Ells Flemish, how many nails? *Ans.* 12528.

20. In 3*hhd.* 13*gal.* 2*qt.*, how many halfpints?

21. In 12*T.* 15*cwt.* 1*qr.* 19*lb.* 12*dr.*, how many drams? *Ans.* 6539276.

22. How many seconds old is a man, who has lived 32 years and 40 days? *Ans.* 1013299200.

23. In 24 cords of wood, how many solid inches? *Ans.* 5308416.

24. How many barleycorns will reach round the globe, the circumference being about 25000 miles?

25. In 583*A.* 3*R.* 10*P.*, how many square rods?
Ans. 93410.

26. In 190 yards, how many nails? Ans. ——

27. How many seconds from the Declaration of Independence, July 4th, 1776, to July 4th, 1838? Ans. 1956571200.

28. How much will 3 loads of hay come to at 3 cents a pound, each load weighing 18*cwt.* 3*qr.* 24*lb.*?

29. In 24*hhd.* 18*gals.* 2*qts.* of molasses, how many pints? Ans. 12244.

30. If you should buy a piece of cloth containing 341*yd.* 3*qr.* 1*na.*, how many nails in the piece?

31. In 197111024 square miles, how many square inches? Ans. 791300155893350400.

32. Reduce 18 tons of round timber to cubic inches.

33. Change 24 pipes into gills. Ans. 96768 *gills.*

34. Reduce 95 barrels of beer to pints. Ans. ——

35. Change 84 chaldrons of coal to pecks. Ans. 12096.

36. Suppose your age to be 15*yr.* 2*wk.* 5*da.* 13*hr.* 38*m.* 48*sec.*, how many seconds old are you? Ans. ——

37. If a ship has sailed 9s. 13° 25′, how many seconds has she made? Ans. 1020300″.

38. How many square feet in 35*A.* 2*R.* 24*P.*?

39. How many inches, from Hartford to the White Mountains in New Hampshire, the distance being about 241 miles? Ans. 15269760.

40. In 302 Ells English, how many yards?

41. In 24*hhd.* of sugar, each 11*cwt.* 25*lb.*, how many pounds? Ans. 27000.

42. How many grains in 30 pieces of metal, each weighing 9*oz.* 5*pwt.*? Ans. 133200.

43. What will 12*cwt.* 4*qr.* 12*lb.* of sugar cost at 12 cents a pound? Ans. $157,44.

44. What will 2*hhd.* 16*gal.* 3*qt.* 1*pt.* of molasses cost at 6 cents a pint? Ans. $68,58.

45. In 14 bales of cloth, each 17 pieces, each piece 56 ells Flemish, how many yards, ells English, and ells French? Ans. ——

46. How many pounds, ounces, pennyweights, and grains of gold in 704121 grains? Ans. ——

EXAMPLES—FROM A LESS TO A GREATER DENOMINATION.

1. In 171360 pence, how many pounds? *Ans.* £714.
2. How much will 63*lb.* 7*oz.* 10*pwt.* 11*gr.* of gold cost at 20 cents per grain? *Ans.* $73298,20.
3. In 243648 farthings, how many dollars at 6*s.* each?
4. How many pounds of gold can you buy for $55104, at 20 cents per grain? *Ans.* 47*lb.* 10*oz.*
5. How many tons of sugar can you buy for $146470, at 2 cents per dram?
Ans. 14*T.* 6*cwt.* 0*qr.* 7*lb.* 6*oz.* 12*dr.*
6. Reduce 1720320 drams to tons. *Ans.* ——
7. What is the weight of 470 bags of sugar, each bag weighing 26 pounds? *Ans.* 122*cwt.* 20*lb.*
8. In 55799 grains of laudanum, how many pounds?
9. At 6 cents a pint, how many barrels of beer can you buy for $820,80? *Ans.* 47*bar.* 18*gal.*
10. In 97397 grains, how many pounds Troy?
11. How many degrees in 9511603200 barleycorns? *Ans.* 720.
12. How long will it take to count 2000000 at the rate of 50 a minute? *Ans.* 27*da.* 18*hr.* 40*m.*
13. In 1296000″, how many signs? *Ans.* 12.
14. In 63360 inches, how many miles? *Ans.* ——
15. In 31557600 seconds, how many years? *Ans.* 1.
16. In 2016 pints, how many tuns? *Ans.* 1.
17. In 4014489600 square inches, how many miles? *Ans.* 1.
18. In 11520 grains, how many pounds? *Ans.* 2.
19. In 123456720 minutes, how many years? *Ans.* 234*yr.* 265*da.* 8*hr.*
20. In 811480″, how many signs? *Ans.* 7s. 15° 24′ 40″.
21. In 2654208 solid inches, how many cords? *Ans.* 12.
22. If there is 15713280 inches in the distance from New York to Boston, how many miles? *Ans.* 248.
23. If you have lived 399794890 seconds, how many years is that equal to?
Ans. 12*yr.* 244*da.* 6hr. 8m. 10sec

24. In 4320 sheets of paper, how many reams?

25. In 31556928 seconds, how many years of 36 days? *Ans.* 1*yr.* 5*hr.* 48*m.* 48*sec*

26. The surface of the earth contains 791300159907840000 square inches: how many squar miles? *Ans.* 197111025

27. How many yards in 760 nails? *Ans.* 47*yd.* 2*qr*

ADDITION OF DENOMINATE NUMBERS.

1. John buys a knife for 1*s.* 8*d.*, and a bunch of quill for 1*s.* 2*d.*: what do they cost him? *Ans.* 2*s.* 10*d*

2. James gives 4*s.* 9*d.* for a pair of shoes, and 2*s.* 4*d* for a pair of stockings: how much do they cost him? *Ans.* 7*s.* 1*d*

3. How many hours in 8*hr.*+6*hr.*+7*hr.*+9*hr.*? *Ans.* 30*hr*

4. In 8*yd.*+7*yd.*+5*yd.*+6*yd.*, how many yards? *Ans.* 26*yd*

5. How many pounds shillings and pence in £4 8*s.* 9*d.*, £27 14*s.* 11*d.*, and £156 17*s.* 10*d.*?

OPERATION.

£	*s.*	*d.*
4	8	9
27	14	11
156	17	1(
£189	1*s.*	

We write the denominations under each other, and draw a line beneath them. We then add up the column of pence, and find the sum to be 30. But 30 pence are equal to 2 shillings and 6 pence: we therefore write down the 6 and carry 2 to the shillings. We then find the sum of shillings to be 41; that is, 2 pounds and 1 shilling o Carrying the 2 to the column of pounds, we find the to be 189.

95. Addition of denominate numbers, like that of ple numbers, teaches how to express the value of s numbers by a single one, which is called their sur

95. What is addition of denominate numbers? How do down the numbers for addition? Where do you begin to add do you do with the first sum? What do you write down do you carry to the next column? What do you do at *How do you prove addition?*

RULE.

I. *Set down the numbers to be added so that all the denominations of the same kind shall stand in the same column.*

II. *Begin with the column of the lowest denomination, and add it up as in whole numbers.*

III. *Then consider how many units of this denomination make one unit of the next higher, and divide the sum by this number. Write down the remainder under the units of its kind, and carry the quotient to the next column, as in addition of simple numbers.*

IV. *Proceed in the same way for all the columns to the last, and set down the entire sum of the last column.*

The proof is the same as in the addition of simple numbers.

EXAMPLE ILLUSTRATING PRINCIPLES.

What is the sum of £16 18*s*. 9*d*., £14 13*s*. 8*d*., and £15 17*s*. 6*d*.?

OPERATION.

	£	s.	d.
	16	18	9
	14	13	8
	15	17	6
Sum	47	9	11
	30	11	2
Proof	47	9	11

NOTE.—In simple numbers 10 units of any one of the columns, make one unit of the next left-hand column, (ART. 8.) We therefore carry one for every 10. But in denominate numbers the higher denominations are formed differently. For example, 12 pence make 1 shilling, the unit of the next higher denomination; and 20 shillings make 1 pound. In passing from pence to shillings, we must therefore carry 1 for every 12, and 1 for every 20 in passing from shillings to pounds. And, in general, we must carry 1 for so many units of the lower denomination as make one unit of the next higher.

Ex. 1. In simple numbers, how many units of one order make one unit of the next higher order? How do you carry in simple numbers? How do you carry in passing from pence to shillings? In passing from shillings to pounds? Generally, how do you carry?

(1.)

£	s.	d.
173	13	5
87	17	7¾
75	18	7½
25	17	8¼
10	10	10½
373	18	3

(2.)

£	s.	d.
705	17	3½
354	17	2¾
175	17	3¾
87	19	7½
52	12	7¾
1377	4	1¼

(3.)

£	s.	d.
104	18	9½
404	17	8¾
467	11	10¼
597	14	4¼
22	18	5

TROY WEIGHT.

OPERATION.

lb.	oz.	pwt.	gr.
11	8	18	19
114	9	6	16
223	10	17	12
350	5	2	23

Adding up the grains, we find their sum to be 47; that is, 1*pwt.* and 23*gr.*: setting down 23, and carrying 1 to the pennyweights, we find their sum to be 42; that is, 2*oz.* and 2*pwt.* Carrying 2 to the ounces, we find their sum to be 29; that is, 2*lb.* and 5*oz.*: carrying 2 to the pounds and adding, we find their sum to be 350.

(1.)

	lb.	oz.	pwt.	gr.
Add	100	10	19	20
	432	6	0	5
	80	3	2	1
	7	0	0	9
	0	11	10	23
	0	0	8	9
Sum				

(2.)

lb.	oz.	pwt.	gr.
171	6	13	14
391	11	9	12
230	6	6	13
94	7	3	18
42	10	15	20
31	0	0	21

APOTHECARIES' WEIGHT.

(1.)

℔	℥	ʒ	℈	gr.
24	7	2	1	16
17	11	7	2	19
36	6	5	0	7
15	9	7	1	13
9	3	4	1	9

(2.)

℥	ʒ	℈	gr.
11	2	1	17
7	4	2	14
4	0	1	19
2	5	2	11
10	1	2	16

(3.)

ʒ	℈	gr.
3	2	15
0	1	13
2	2	11
7	0	17
5	2	14

AVOIRDUPOIS WEIGHT.

(1.)

cwt.	*qr.*	*lb.*	*oz.*	*dr.*
14	1	25	14	9
13	2	20	1	15
9	3	6	7	3
10	0	18	12	11
7	2	27	3	2
6	1	19	8	1
4	3	0	15	5
12	2	0	0	13

(2.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*
15	12	1	10	10
71	8	2	6	0
83	19	3	15	5
36	7	0	20	14
47	11	1	27	11
63	5	2	19	7
12	13	1	14	9
9	7	0	5	10

3. A merchant bought 4 barrels of potash of the following weights, viz.: 1st, 3*cwt.* 1*qr.* 25*lb.* 12*oz.* 3*dr.*; 2d, 4*cwt.* 1*qr.* 21*lb.* 4*oz.*; 3d, 4*cwt.*; 4th, 3*cwt.* 3*qr.* 27*lb.* 15*oz.* 15*dr.*: what was the entire weight of the four barrels? *Ans.* 16*cwt.* 0*qr.* 0*lb.* 0*oz.* 2*dr*

LONG MEASURE.

(1.)

L.	*mi.*	*fur.*	*rd.*
16	2	7	39
327	1	2	20
87	0	1	15
1	1	1	1

(2.)

yd.	*ft.*	*in.*	*bar.*
90	2	11	2
155	1	9	1
327	0	7	0
50	2	1	2

CLOTH MEASURE.

(1.)

E.Fl.	*qr.*	*na.*
126	4	4
65	3	1
72	1	3
157	2	3

(2.)

yd.	*qr.*	*na.*
4	3	2
5	4	1
6	1	0
25	2	2

(3.)

E.E.	*qr.*	*na.*
128	5	1
20	3	1
19	1	4
15	3	1

11*

LAND OR SQUARE MEASURE.

(1.)

Sq. yd.	*Sq. ft.*	*Sq. in.*
97	4	104
22	3	27
105	8	2
37	7	127

(2.)

M.	*A.*	*R.*	*P.*
1	700	3	37
6	375	2	25
7	450	1	31
11	30	0	25

3. There are 4 fields, the 1st contains 12*A*. 2*R*. 38*P*.; the 2d, 4*A*. 1*R*. 26*P*.; the 3d, 85*A*. 0*R*. 19*P*.; and the 4th, 57*A*. 1*R*. 2*P*.: how many acres in the four fields?
Ans. 159*A*. 2*R*. 5*P*.

SOLID OR CUBIC MEASURE.

(1)

S. yd.	*S. ft.*	*S. in.*
65	25	1129
37	26	132
50	1	1064
22	19	17

(2.)

C.	*S. ft.*
16	127
17	12
18	119
37	104

(3.)

C.	*Cord feet.*
87	9
26	7
16	6
19	5

LIQUID MEASURE.

(1.)

hhd.	*gal.*	*qt.*	*pt.*
127	65	3	2
12	60	2	3
450	29	0	1
21	0	2	3
14	39	1	2

(2.)

tun	*pi.*	*hhd.*	*gal.*	*qt.*
14	2	1	27	3
15	1	2	25	2
4	2	1	27	1
5	0	1	62	3
7	1	2	21	2

DRY MEASURE.

(1.)

ch.	*bu.*	*pk.*	*qt.*	*pt.*
27	25	3	7	1
59	21	2	6	3
2	1	2	7	1
5	9	1	8	2

(2.)

ch.	*bu.*	*pk.*	*qt.*	*pt.*
141	36	3	7	2
21	32	2	4	1
85	9	1	0	3
10	4	4	1	3

TIME.

(1.)

yr.	*mo.*	*wk.*	*da.*	*hr.*
4	11	3	6	20
3	10	2	5	21
5	8	1	4	19
101	9	3	7	23
55	8	4	6	17

(2.)

wk.	*da.*	*hr.*	*m.*	*sec.*
8	8	14	55	57
10	7	23	57	49
20	6	14	42	01
6	5	23	19	59
2	2	20	45	48

APPLICATIONS IN ADDITION.

1. Add 46*lb.* 9*oz.* 15*pwt.* 16*gr.*, 87*lb.* 10*oz.* 6*pwt.* 14*gr.*, 100*lb.* 10*oz.* 10*pwt.* 10*gr.*, and 56*lb.* 3*pwt.* 6*gr.* together.
Ans. 291*lb.* 6*oz.* 15*pwt.* 22*gr.*

2. What is the weight of forty-six pounds, eight ounces, thirteen pennyweights, fourteen grains; ninety-seven pounds, three ounces; and one hundred pounds, five ounces, ten pennyweights, and thirteen grains?

3. Add the following together: 29*T.* 16*cwt.* 1*qr.* 14*lb.* 12*oz.* 9*dr.*, 18*cwt.* 3*qr.* 1*lb.*, 50*T.* 3*qr.* 4*oz.*, and 2*T.* 1*qr.* 14*dr.* *Ans.* 82*T.* 16*cwt.* 0*qr.* 16*lb.* 1*oz.* 7*dr.*

4. What is the weight of 39*T.* 10*cwt.* 1*qr.* 27*lb.* 15*oz.* 12*dr.*, 17*cwt.* 6*lb.*, 12*cwt.* 3*qr.*, and 2*qr.* 8*lb.* 9*dr.*?

5. Add the following together: 19℔ 10℥ 4ʒ 2℈ 16*gr.*, 9℥ 7ʒ 17*gr.*, and 3℔ 6℥ 5ʒ 1℈ 18*gr.*
Ans. 24℔ 3℥ 1ʒ 2℈ 11*gr.*

6. Add together 19*yd.* 2*qr.* 3*na.*, 14*yd.* 2*qr.* 1*na.*, 32*yd.* 1*na.*, 2*qr.* 2*na.*, and 57*yd.* 3*qr.* 2*na.*

7. What is the sum of the following: 64*deg.* 38*mi.* 4*fur.* 26*rd.* 15*ft.* 10*in.* 2*bar.*, 49*mi.* 7*fur.* 38*rd.* 12*ft.* 9*in.* 1*bar.*, 6*fur.* 20*rd.*, and 9*mi.* 3*fur.* 29*rd.* 9*ft.* 8*in.*?
Ans. 65*deg.* 28*mi.* 6*fur.* 35*rd.* 5*ft.* 4*in.*

8. What is the sum of the following: 314*A.* 2*R.* 39*P.* 200*Sq. ft.* 136*Sq. in.*; 16*A.* 1*R.* 20*P.* 10*Sq. ft.*; 3*R.* 36*P.*; and 4*A.* 1*R.* 16*P.*?

9. What is the solid content of 64*ton* 33*ft.* 800*in.*, 9*ton* 1200*in.*, 25*ft.* 700*in.*, and 95*ton* 31*ft.* 1500*in.*?
Ans. 170ton 11ft. 744in.

10. What is the area of the four following pieces of land; the first containing 20*A*. 3*R*. 15*P*. 250*Sq.ft.* 116*Sq. in.*; the second, 19*A*. 1*R*. 39*P*.; the third, 2*R*. 10*P*. 60*Sq.ft.*; and the fourth, 5*A*. 6*P*. 50*Sq. in.*?

Ans. 45*A*. 3*R*. 31*P*. 38$\frac{3}{4}$ *Sq.ft.* 22*Sq. in.*

11. Add together, 49*ton* 19*ft.* 1666*in.*, 19*ton* 10*ft.* 1001*in.*, 16*ton* 36*ft.* 109*in.*, and 4*ton* 17*ft.* 1727*in.*

Ans. 90*ton* 4*ft.* 1047*in.*

12. Add together, 67*tuns* 2*hhd.* 60*gal.* 3*qt.* 1*pt.* 3*gi.*, 19*tuns* 3*hhd.* 10*gal.* 2*qt.* 1*gi.*, 47*tuns* 1*hhd.* 20*gal.* 1*qt.* 1*pt.* 1*gi.*, 90*tuns* 2*hhd.* 10*gal.* 3*qt.* 2*gi.*

Ans. 225*tuns* 1*hhd.* 39*gal.* 2*qt.* 1*pt.* 3*gi.*

13. Add together, 1*tun* 1*pi.* 116*gal.* 3*qt.*, 1*pi.* 48*gal.*, 5*tuns* 1*pi.* 86*gal.* 3*qt.*, 102*gal.*, and 4*tuns*.

Ans. 12*tuns* 1*pi.* 101*gal.* 2*qt.*

14. Add together, 49*tuns* 3*hhd.* 4*gal.* 3*qt.* 1*pt.* 2*gi.*, 19*tuns* 2*hhd.* 37*gal.* 1*qt.*, 1*hhd.* 51*gal.*, and 74*tuns* 3*hhd.* 19*gal.* 2*qt.* 1*pt.* 2*gi.*

Ans. 144*tuns* 2*hhd.* 49*gal.* 3*qt.* 1*pt.*

15. Add together, 96*bu.* 3*pk.* 2*qt.* 1*pt.*, 46*bu.* 3*pk.* 1*qt.* 1*pt.*, 2*pk.* 1*qt.* 1*pt.*, and 23*bu.* 3*pk.* 4*qt.* 1*pt.*

Ans. 168*bu.* 2*qt.*

16. What is the sum of the following: 49*yr.* 320*da.* 14*hr.* 49*m.* 37*sec.*, 360*da.* 19*hr.* 8*m.* 45*sec.*, 76*yr.* 200*da.*, 16*yr.* 150*da.* 20*hr.* 54*m.* 45*sec.*?

17. What length of time is there in 24*yr.* 67*da.* 19*hr.* 43*m.* 34*sec.*, 300*da.* 10*hr.*, 290*da.* 50*m.*, and 86*yr.* 320*da.* 51*m.*? *Ans.* 112*yr.* 247$\frac{1}{2}$*da.* 7*hr.* 24*m.* 34*sec.*

18. Add together the following: 9s. 20° 34′ 37″, 17° 36′ 44″, 7s. 28° 39′ 14″, 8s. 24° 38′ 55″.

19. What is the sum of 5s. 20° 30′ 40″, 7s. 54′, 8s. 9° 45″, and 29° 16′ 54″? *Ans.* 21s. 29° 42′ 19″.

20. What quantity of paper is there in 76 reams 19 quires 23 sheets; 16 reams 8 quires 13 sheets; 4 reams; and 90 reams 11 quires 8 sheets?

Ans. 187 reams 19 *quires* 20 *sheets*.

SUBTRACTION OF DENOMINATE NUMBERS.

1. John has 3*s*. 6*d*. and gives 1*s*. 4*d*. for a knife: how much has he left? *Ans*. 2*s*. 2*d*.

2. James has 4*s*. 8*d*. and gives 2*s*. 3*d*. for a bunch of quills: how much has he left? *Ans*. 2*s*. 5*d*.

3. What is the difference between £27 16*s*. 8*d*. and £19 17*s*. 9*d*.?

OPERATION.

£	*s*.	*d*.
	20	12
27	16	8
19	17	9
7	18	11

In this example we cannot take 9*d*. from 8*d*.; we therefore add 12*d*. to the 8*d*., making 20*d*., and then say, 9 from 20, 11 remains. Set down the 11, and carry 1 to 17, making 18: then say, 18 from 36 leaves 18: set it down and carry 1 to 19, making 20: 20 from 27 leaves 7.

The reason of this process is evident; for, adding equivalent numbers to the minuend and subtrahend does not alter the remainder.

96. Hence, for the subtraction of denominate numbers, we have the following

RULE.

I. *Set down the less number under the greater, placing the same denominations directly under each other.*

II. *Begin with the lowest denomination, and if the number expressing that denomination be less than the number directly over it, make the subtraction as in simple numbers; but if it be greater, subtract it from the upper number increased by so many units as make one unit of the next higher denomination, and then add one to the next higher denomination in the subtrahend, as in subtraction of simple numbers.*

III. *Do the same for all the denominations, and set down the several remainders, and they will form the true remainder.*

---

96. How do you write down the numbers for subtraction? Where do you begin to subtract? When the number to be subtracted is less than the one above it, what do you do? When it is greater, what do you do?

PROOF.

97. Add the remainder to the subtrahend—their sum should be equal to the minuend.

EXAMPLES.

(1.)

	A.	*R.*	*P.*
From	18	3	28
take	15	2	30
Remainder	3	0	38
Proof	18	3	28

(2.)

	T.	*cwt.*	*qr.*	*lb.*
From	4	12	3	20
take	2	18	2	26
Remainder	1	14	0	19
Proof	4	12	3	20

(3.)

	lb.	*oz.*	*pwt.*	*gr.*
From	273	0	0	0
take	98	10	18	21
Remainder				

(4.)

	lb.	*oz.*	*pwt.*	*gr.*
From	18	9	10	8
take	9	10	15	20
Remainder				

(5.)

	T.	*cwt.*	*qr.*	*lb.*	*oz.*
From	7	14	1	3	6
take	2	6	3	4	11
Remainder					

(6.)

	cwt.	*qr.*	*lb.*	*oz.*	*dr.*
From	14	2	12	10	8
take	6	3	16	15	3
Remainder					

APPLICATIONS.

1. From 38*mo.* 2*wk.* 3*da.* 7*hr.* 10*m.*, take 10*mo.* 3*wk.* 2*da.* 10*hr.* 50*m.* *Ans.* 27*mo.* 3*wk.* 20*hr.* 20*m.*
2. From 176*yr.* 8*mo.* 3*wk.* 4*da.*, take 91*yr.* 9*mo.* 2*wk.* 6*da.* *Ans.* 84*yr.* 11*mo.* 5*da.*
3. From 6*tuns*, take 3*hhd.* 15*gal.* 3*qt.* *Ans.* ——
4. From £3, take 3*s.* *Ans.* £2 17*s*
5. From 2*lb.*, take 20*gr.* Troy. *Ans.* 1*lb.* 11*oz.* 19*pwt.* 4*gr.*
6. From 8℔, take 1℔ 1℥ 23 2℈. *Ans.* ——
7. From 9*T.*, take 1*T.* 1*cwt.* 2*qr.* 20*lb.* 15*oz.* 14*dr.* *Ans.* 7*T.* 18*cwt.* 1*qr.* 4*lb.* 0*oz.* 2*dr.*
8. From 3 miles, take 3*fur.* 19*rd.* *Ans.* 2*mi.* 4*fur.* 21*rd.*

---

97. How do you prove subtraction?

9. The revolution commenced April 19, 1775, and a general peace took place January 20, 1783: how long did the war continue? *Ans.* 7*yr.* 9*mo.* 1*da.*

10. America was discovered by Columbus, October 11, 1492: what was the length of time to July 25, 1838?

11. I purchased 167*lb.* 8*oz.* 16*pwt.* 10*gr.* of silver, and sold 98*lb.* 10*oz.* 12*pwt.* 19*gr.*: how much had I left?
Ans. 68*lb.* 10*oz.* 3*pwt.* 15*gr.*

12. I bought 19*T.* 11*cwt.* 1*qr.* 27*lb.* 12*oz.* 12*dr.* of old iron, and disposed of 17*T.* 13*cwt.* 2*qr.* 19*lb.* 14*oz.* 10*dr.*: what had I left? *Ans.* ——

13. I purchased 101℔ 11℥ 7ʒ 2℈ 19*gr.* of medicine, and sold 17℔ 2℥ 3ʒ 1℈ 5*gr.*: how much remained unsold? *Ans.* 84℔ 9℥ 4ʒ 1℈ 14*gr.*

14. Take 34℔ 9℥ 4ʒ 2℈ from 93℔ 10℥ 5ʒ 1℈ 19*gr.*

15. From 46*yd.* 1*qr.* 3*na.*, take 42*yd.* 3*qr.* 1*na.* 2*in.*
Ans. 3*yd.* 2*qr.* 1*na.* $\frac{1}{4}$*in.*

16. It is about 25000 miles round the globe; if a man shall have travelled 43 miles 17 rods 9 inches, what distance will remain? *Ans.* ——

17. Bought 7 cords of wood, and 2 cords 78 feet having been stolen, how much remains? *Ans.* 4*C.* 50*ft.*

18. I had 15 yards of cloth: having sold 3*yd.* 2*qr.* 1*na.*, what remains? *Ans.* ——

19. Bought a hogshead of wine, and by an accident 8*gal.* 3*qt.* 1*pt.* leaked out: what remains?
Ans. 54*gal.* 1*pt.*

20. I have 73*A.* of land; if I should sell 5*A.* 3*R.* 1*P.*, how much should I have left? *Ans.* ——

21. A owes B £100: what will remain due after he has paid him 3*s.* 6½*d.*? *Ans.* £99 16*s.* 5½*d.*

22. A farmer raised 136 bushels of wheat; if he sells 49*bu.* 2*pk.* 7*qt.* 1*pt.*, how much will he have left?

23. From 174*hhd.* 10*gal.* 1*qt.* 1*pt.* of beer, take 86*hhd.* 17*gal.* 2*qt.* 1*pt.* *Ans.* 87*hhd.* 46*gal.* 3*qt.*

24. A farmer had 576*bu.* 1*pk.* 2*qt.* of wheat; he sold 139*bu.* 2*pk.* 3*qt.* 1*pt.*: how much remained unsold?

25. A merchant bought 17*cwt.* 2*qr.* 14*lb.* of sugar, of which he sold at one time 3*cwt.* 2*qr.* 20*lb.*; at another 6*cwt.* 1*qr.* 5*lb.*: how much remained unsold?

26. There are two men, the oldest is 81*yr*. 6*mo*. 3*wk*. 1*da*. 21*hr*. 16*sec*.; the youngest 29*yr*. 10*mo*. 2*wk*. 4*da*. 16*hr*. 38*m*. 45*sec*.: what is the difference of their ages?

27. What is the difference of time between 31*yr*. 10*mo*. 2*wk*. 4*da*. 7*hr*. 24*m*. 49*sec*., and 10*yr*. 10*mo*. 2*wk*. 2*da*. 7*hr*. 59*m*. 14*sec*.? *Ans*. 21*yr*. 1*da*. 23*hr*. 25*m*. 35*sec*.

28. A merchant had six debtors, who together owed him £2917 10*s*. 6*d*.; five of them owed him £1675 13*s*. 9*d*.: what did the sixth owe? *Ans*. ——

29. Bought 12*cwt*. 3*qr*. 27*lb*. of pork, and sold at one time 4*cwt*. 26*lb*.; at another 3*cwt*. 3*qr*.; at another 2*cwt*. 1*lb*.: what remained on hand? *Ans*. 3*cwt*.

30. Bought a hogshead of molasses, and sold at one time 10*gal*. 3*qt*. 1*pt*. 2*gi*.; at another 12*gal*. 3*qt*. 1*pt*. 3*gi*.; at another 15*gal*. 3*qt*. 1*pt*. 2*gi*.: how much remains unsold? *Ans*. ——

31. Bought a piece of cloth containing 145*yd*. 3*qr*., and sold 95*yd*. 2*qr*. 3*na*.: how much remained?

Ans. 50*yd*. 1*na*.

32. A merchant has £375 10*s*.: if he takes £122 11*s*. 3*d*. to pay for goods, how much will he have left?

33. A merchant bought 375*T*. 15*cwt*. 3*qr*. 19*lb*. 7*oz*. 12*dr*. of sugar, and sold 205*T*. 17*cwt*. 1*qr*. 27*lb*. 9*oz*. 15*dr*.: how much remained on hand?

Ans. 169*T*. 18*cwt*. 1*qr*. 16*lb*. 13*oz*. 13*dr*.

APPLICATIONS IN ADDITION AND SUBTRACTION.

1. Sold a merchant one quarter of beef for £2 7*s*. 9*d*.; one cheese for 9*s*. 7*d*.; 20 bushels of corn for £4 10*s*. 11*d*.; and 40 bushels of wheat for £19 12*s*. 8½*d*.: how much did the whole come to? *Ans*. £27 0*s*. 11½*d*.

2. Bought of a silversmith a teapot, weighing 3*lb*. 4*oz*. 9*pwt*. 21*gr*.; one dozen of silver spoons, weighing 2*lb*. 1*oz*. 1*pwt*.; 2 dishes weighing 16*lb*. 10*oz*. 15*pwt*. 16*gr*.: how much did the whole weigh?

Ans. 22*lb*. 4*oz*. 6*pwt*. 13*gr*.

3. Bought one hogshead of sugar, weighing 9*cwt.* 2*qr.* 27*lb.* 14*oz.*; one barrel weighing 3*cwt.* 27*lb.*, and a second barrel weighing 2*cwt.* 3*qr.* 26*lb.* 4*oz.*: how much did the whole weigh? *Ans.* ——

4. A merchant buys two hogsheads of sugar, one weighing 8*cwt.* 3*qr.* 21*lb.*, the other, 9*cwt.* 2*qr.* 6*lb.*; he sells two barrels, one weighing 3*cwt.* 1*qr.* 12*lb.* 14*oz.*, the other, 2*cwt.* 3*qr.* 15*lb.* 6*oz.*: how much remains on hand? *Ans.* 12*cwt.* 23*lb.* 12*oz.*

5. A man sets out upon a journey and has 200 miles to travel; the first day he travels 9 leagues 2 miles 7 furlongs 30 rods; the second day 12 leagues 1 mile 1 furlong; the third day 14 leagues; the fourth day 15 leagues 2 miles 5 furlongs 35 rods: how far had he then to travel? *Ans.* 14*L.* 1*mi.* 1*fur.* 15*rd.*

6. A farmer has two meadows, one containing 9*A.* 3*R.* 37*P.*, the other contains 10*A.* 2*R.* 25*P.*; also three pastures, the first containing 12*A.* 1*R.* 1*P.*, the second containing 13*A.* 3*R.*, and the third 6*A.* 1*R.* 39*P.*: by how many acres does the pasture exceed the meadow land? *Ans.* ——

7. Supposing the Declaration of Independence to have been published at precisely 12 o'clock on the 4th of July, 1776, how much time elapsed to the 1st of January, 1833, at 25 minutes past 3. P. M.?

Ans. 56*yr.* 181*da.* 3*hr.* 25*m.*

8. A farmer has three granaries, one for wheat, one for rye, and one for corn: he fills them all. His wheat granary contains 657*bu.* 3*pk.* 6*qt.*; the corn granary 257*bu.* 1*pk.* 1*qt.*; the rye granary 459*bu.* 2*pk.* 7*qt.*: how much grain had he in all, and how much more wheat than rye? *Ans.* { In all 1374*bu.* 3*pk.* 6*qt.*
Wheat more than rye 198*bu.* 7*qt.* }

9. A father was born on the 8th of December, 1759, his first son on the 4th of June, 1795: what was the difference of their ages? *Ans.* 35*yr.* 5*mo.* 27*da.*

10. A merchant has a bill to pay of £600. He has £250 19*s.* 8*d.* in cash, a good note against A for £75 10*s.* 6*d.*, and a note against B for £37 11*s.* 9*d*: how much money does he want to make the payment?

MULTIPLICATION OF DENOMINATE NUMBERS.

1. Charles pays 6*d.* for a pencil: how much will buy two pencils? How much will buy 3 pencils? 4 pencils? 5 pencils? 6 pencils?

2. James puts 1 quart and 1 pint into a measure: how much could he put in a measure of twice the size? In a measure of three times the size? 4 times the size? 5 times the size? 6 times the size?

3. What is the product of 2*s.* 4*d.* multiplied by 2? by 3? by 4? by 5? by 6? by 7? by 8? by 9?

4. What is the product of 1*yd.* 1*qr.* multiplied by 2? by 3? by 4? by 5? by 6? by 7? by 8? by 9?

EXAMPLE ILLUSTRATING PRINCIPLES.

Let it be required to multiply £3 9*s.* 10*d.* by 4?

In this example we say, 4 times 10*d.* are 40*d.*, equal to 3*s.* and 4*d.* Set down the 4*d.* in the lower line. Then 4 times 9*s.* are 36*s.*, and 3*s.* to carry make 39*s.*, equal to £1 and 19*s.* over: set down the 19*s.* Then 4 times £3 are £12, and £1 to carry make £13.

OPERATION.

£	s.	d.
3	9	10
		4
£12	36s.	40d.
£13	19s.	4d.

We may conclude from the example that, to multiply a denominate number by a simple one, is to take the denominate number as many times as there are units in the multiplier.

CASE I.

98. When the simple number does not exceed 12.

RULE.

I. *Write down the denominate number and set the multiplier under the lowest denomination.*

---

98. What is required when you multiply a denominate number by a simple one? When the simple number does not exceed 12, *how do* you write it down? How do you begin to multiply? How *do you carry*?

II. *Multiply the lowest denomination by the multiplier, and divide the product by so many units as make one of the next higher denomination, and set down the remainder as in addition.*

III. *Multiply the next higher denomination by the multiplier, and add the quotient to be carried from the last product; then reduce the sum to units of the next higher denomination, and proceed in the same way for all the denominations, setting down the entire product when you come to the last.*

EXAMPLES.

(1.)

£	*s.*	*d.*
17	15	9
		6
106	14	6

(2.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*
	9	3	27	12
				7
3	10	0	19	4

3. Multiply 9*s.* 6*d.* by 3. *Ans.* £1 8*s.* 6*d.*

4. What will 12 gallons of brandy cost at 9*s.* 6*d.* per gallon? *Ans.* £5 14*s.*

5. What will 9*cwt.* of butter cost at £1 11*s.* 5*d.* per *cwt.*? *Ans.* £14 2*s.* 9*d.*

APPLICATIONS.

1. What is the cost of 4 yards of cloth at £1 3*s.* 6*d.* per yard?

The amount per yard multiplied by the number of yards will evidently give the entire cost.

OPERATION.

£1	3*s.*	6*d.*
		4
£4	14*s.*	0*d.* *Ans.*

2. What will be the cost of 9 hats, at 9*s.* 9*d.* each? *Ans.* £4 7*s.* 9*d.*

3. A farmer has 11 bags of corn, each containing 2*bu.* 1*pk.* 3*qt.*: how much corn in all the bags? *Ans.* 25*bu.* 3*pk.* 1*qt.*

4. What is the cost of 12 bushels of wheat at 9*s.* 6*d.* per bushel? *Ans.* ——

5. How much sugar in 12 barrels, each containing 3*cwt.* 2*qr.* 27*lb.*? *Ans.* 2*T.* 5*cwt.* 0*qr.* 24*lb.*

6. In 7 loads of wood, each containing 1 cord and 2 cord feet, how many cords? *Ans.* 8 *cords* 6 *cord feet.*

CASE II.

99. When the simple number is greater than 12, and a composite number.

RULE.

Multiply the denominate number by one of the component parts, or factors, and then multiply the product by each of the other factors in succession: the last product is the one required.

EXAMPLES.

1. Multiply £6 2*s.* 9*d.* by 48=6×8. *A.* £294 12*s.*

2. What will 24 barrels of flour cost, at £2 11*s.* 8*d.* per barrel? *Ans.* £62.

3. What is the cost of 42*cwt.* of tallow, at £1 14*s.* 6*d.* per *cwt.*? *Ans.* ——

4. What is the cost of 120 dozen of candles, at 5*s.* 9*d.* per dozen? *Ans.* £34 10*s.*

5. How much water will be contained in 96 hogsheads, each containing 62*gal.* 1*qt.* 1*pt.* 1*gi.*? *Ans.* 5991 *gallons.*

CASE III.

100. When the simple number exceeds 12, and is not a composite number.

RULE.

Multiply the simple number by each of the denominations separately, and reduce each product to the highest denomination named. Then add the several products together, and their sum will be the answer sought.

---

99. How do you multiply when the simple number is greater than 12, and a composite number?

100. How do you multiply when the simple number exceeds 12, and is not a composite number?

EXAMPLES.

1. Multiply £5 3*s*. 8*d*. by 13.

13	13	13
8*d*.	3*s*.	£5
104*d*.=8*s*. 8*d*.	39*s*.=£1 19*s*.	£65

	£65		
	1	19*s*.	
		8*s*.	8*d*.
Ans.	£67	7*s*.	8*d*.

2. Multiply £6 8*s*. 9*d*. by 139.

139×9*d*.=1251*d*.=£ 5 4*s*. 3*d*.
139×8*s*. =1112*s*. =£ 55 12*s*.
139×£6 =£834 =£834 00
Ans. £894 16*s*. 3*d*.

3. Multiply £0 2*s*. 4*d*. by 195. *Ans*. £22 15*s*.

4. What is the cost of 46 bushels of wheat at 4*s*. 7¼*d*. per bushel? *Ans*. ——

5. What is the cost of 117*cwt*. of raisins at £1 2*s*. 3*d*. per *cwt*.? *Ans*. £130 3*s*. 3*d*.

APPLICATIONS.

1. If one share in a certain stock be valued at £13 8*s*. 9¼*d*., what is the value of 96 shares?
Ans. £1290 4*s*. 0*d*.

2. If one spoon weigh 3*oz*. 5*pwt*. 15*gr*., what is the weight of 120 spoons? *Ans*. ——

3. If a man travel 24*mi*. 7*fur*. 4*rd*. in one day, how far will he go in one month of 30 days?
Ans. 746*mi*. 5*fur*. 0*rd*.

4. If the earth revolve 0° 15′ of space per minute of time, how far does it revolve per hour? *Ans*. ——

5. If a man be 2*da*. 5*hr*. 17*m*. 19*sec*. in walking one degree, how long would it take him to walk round the earth, allowing 365¼ days to a year?
Ans. 2*yr*. 68*da*. 18*hr*. 54*m*.

6. If a man drink 3*gal*. 1*qt*. 1*pt*. of wine in a week, how much will he drink in 52 weeks? Ans. ——

7. A bond was given 21st of May, 1825, and was taken up the 12th of March, 1831: what will be the product, if the time which elapsed from the date of the bond till the day it was taken up be multiplied by 3?
Ans. 17*yr.* 5*mo.* 3*da.*

8. Bought 90*hhd.* of sugar, each weighing 12*cwt.* 2*qr.* 11*lb.*: what was the weight of the whole?

9. What is the cost of 18 sheep at 5*s.* 9½*d.* apiece?
Ans. £5 4*s.* 3*d.*

10. If one hat cost 11*s.* 6*d.*, what will 22 hats cost?

11. What is the weight of 1 dozen silver spoons, each weighing 3*oz.* 6*pwt.*? *Ans.* 3*lb.* 3*oz.* 12*pwt.*

12. What is the weight of 7 tierces of rice, each weighing 5*cwt.* 2*qr.* 16*lb.*? *Ans.* ——

13. Bought 4 packages of medicine, each containing 3℔ 4℥ 6ʒ 1℈ 16*gr.*: what is the weight of all?
Ans. 13℔ 7℥ 2ʒ 1℈ 4*gr.*

14. How far will a man travel in 5 days at the rate of 24*mi.* 4*fur.* 4*rd.* per day? *Ans.* ——

15. How much molasses is contained in 25*hhd.*, each hogshead having 61*gal.* 1*qt.* 1*pt.*? *A.* 1534*gal.* 1*qt.* 1*pt.*

16. How many yards of cloth in 36 pieces, each piece containing 25*yd.* 3*qr.*? *Ans.* ——

17. How much land is there in 9 fields, each field containing 12*A.* 2*R.* 25*P.*? *Ans.* 113*A.* 3*R.* 25*P.*

18. How many yards in 9 pieces, each 29*yd.* 2*qr.* 3*na.*? *Ans.* ——

19. If a vessel sails 5*L.* 2*mi.* 6*fur.* 36*rd.* in one week, how far will it sail in 8 weeks?
Ans. 47*L.* 1*mi.* 7*fur.* 8*rd.*

20. A farmer has 18 lots, and each lot contains 41*A.* 2*R.* 11*P.*: how many acres does he own? *Ans.* ——

21. There are three men whose mutual ages are 14 times 20*yr.* 5*mo.* 3*wk.* 6*da.*: what is the sum of their ages? *Ans.* 286*yr.* 11*mo.* 2*wk.*

22. Bought 90*hhd.* of sugar, each weighing 12*cwt.* 2*qr.* 14*lb.*: what is the weight of the whole?

23. If a vessel sail 49*mi.* 6*fur.* 8*rd.* in one day, how far will she sail in one month of 30 days?
Ans. 1493*mi.* 2*fur.*

24. Suppose each of 50 farmers to raise 125*bu.* 3*pk.* 6*qt.* of grain: how much do they all raise? *Ans.* ——

25 If one spoon weigh 3*oz.* 5*pwt.* 15*gr.*, what is the weight of 240 spoons? *Ans.* 65*lb.* 7*oz.* 10*pwt.*

26. If one man receive 3*yd.* 1*qr.* 1*na.* of cloth, how many yards will 11 men receive? *Ans.* ——

27. If a steamship in crossing the Atlantic goes 211*mi.* 4*fur.* 32*rd.* a day, how far will she go in 15 days? *Ans.* 3174 *miles.*

BILLS OF PARCELS.

A Hosier's Bill.—No. 1.

Philadelphia, Jan. 4, 1847.

Mr. Thomas Williams,

Bought of Richard Simpson.

		s.	d.	
8 Pairs of worsted stockings -	at	4	6	*per pair.*
5 Pairs of thread ditto - - -	at	3	2	
5 Pairs of black silk ditto - -	at	14	0	
6 Pairs of black worsted ditto	at	4	2	
4 Pairs of cotton ditto - - -	at	7	6	
2 Yards of fine flannel - -	at	1	8	*per yard.*

Total cost £9 0*s.* 2*d.*

A Mercer's Bill.—No. 2.

Mobile, July 13, 1847.

Mr. William George,

Bought of Peter Thompson.

		s.	d.	
15 Yards of satin - - - - -	at	9	6	*per yard.*
18 Yards of flowered silk -	at	17	4	
12 Yards of rich brocade - -	at	19	8	
16 Yards of sarcenet - - -	at	3	2	
13 Yards of Genoa velvet -	at	27	6	
23 Yards of lutestring - -	at	6	3	

Total cost £62 2*s.* 5*d.*

No. 3.

Cincinnati, May 1, 1847.

Mr. William Sampson,

Bought of John Strong.

			s.	d.
92	Ivory combs - - - - -	at	3	5¼
94	Pounds of colored thread -	at	6	9¼
102	Yards of durant - - -	at	1	8
104	Silk vests - - - - - -	at	6	7
106	Leghorns - - - - - -	at	11	9½
114	Pairs of nankin - - -	at	8	3¾
116	Pounds of white thread -	at	9	11¼
123	Pairs half hose - - -	at	3	6
148	Yards of muslin - - -	at	1	8

Total cost £291 14*s*. 10*d*.

Received payment, John Strong.

No. 4.

Chicago, April 16, 1847.

Williams & Lowry,

Bought of Alfred Robinson.

			s.	d.
90	Yards of broadcloth	at	8	4 *per yard.*
100	" " " -	at	10	6
112	Yards of satinet -	at	3	7½
126	" " " -	at	12	11¾
144	" " " -	at	19	11
162	" " " -	at	9	3
70	Yards of bombazine	at	19	7½
198	Yards of Italian silk	at	16	0½
132	" " " -	at	8	11
66	" " " -	at	16	11½

Ans. £752 14*s*. 1½*d*.

Received payment, Alfred Robinson,
per *John Nichols*.

DIVISION OF DENOMINATE NUMBERS.

1. Charles has 3*s.* and wishes to divide it equally between himself and two brothers: how much must he give to each? If he divides 2*s.* 6*d.*, how much? If he divides 2*s.*, how much? If he divides 1*s.* 6*d.*, how much? If he divides 1*s.*, how much?

2. John has a bushel of nuts and wishes to divide them equally among himself and three brothers: how much will each have?

3. If James divides 3*pk.* of corn among 4 persons, how much will each have? If he divides 2*pk.* 4*qt.*? If he divides 1*pk.*? If he divides 2*qt.*? If he divides 1*qt.*?

4. William has 8 eagles, 5 dollars, 6 dimes, 9 cents, and 5 mills, and wishes to divide them equally among five of his schoolmates: how many of each kind will each boy receive?

EXAMPLE ILLUSTRATING PRINCIPLES.

Divide £25 15*s.* 10*d.* equally among 8 persons.

In this example we find that 8 is contained in £25, 3 times and £1 over. Now this £1 has yet to be divided by 8, as well as the 15*s.* and 10*d.* Then multiplying the £1 by 20 and adding in the 15*s.* gives 35*s.*, which contains 8, 4 times and 3*s.* over. Multiplying the 3*s.* by 12 and adding in the 10*d.* gives 46*d.*, which contains 8, 5 times and 6*d.* over. The 6*d.* being reduced, gives 24 farthings, which contains 8, 3 times. Therefore, each of the denominations has been divided by 8.

OPERATION.

```
8)£25 15s. 10d.(£3
  24
  ---
  £ 1
    20
  8)35s.(4s.
    32
    ---
     3s.
    12
  8)46(5d.
    40
    ---
     6d.
     4
  8)24far.(3far.
```

Ans. £3 4*s.* 5¾*d.*

101. Therefore, a denominate number may be divided into any number of equal parts, by dividing each of its denominations by the divisor.

RULE.

I. *Set down the number to be divided in the order of its denominations from the highest to the lowest, and write the divisor on the left.*

II. *Find how often the divisor is contained in the figures of the highest denomination, and reduce the remainder, if there be any, to the next lower denomination, and add the figures of the dividend expressing that denomination, and then divide the sum by the divisor.*

III. *Proceed in the same way for all the denominations to the last, and if there be a remainder place the divisor under it, as in division of simple numbers. Each of the quotients will be of the same denomination as its dividend, and the several quotients connected together will be the entire quotient sought.*

EXAMPLES ILLUSTRATING PRINCIPLES.

102. When the divisor does not exceed 12, the division may be made after the manner of short division in simple numbers.

Ex. 1. Divide £25 15*s.* 4*d.* by 8.

OPERATION.

8)£25 15*s.* 4*d.*

£3 4*s.* 5*d.*

We first say 8 into 25, 3 times and £1 or 20*s.* over. Then after adding the 15*s.* we say, 8 into 35, 4 times and 3*s.* over. Then reducing the 3*s.* to pence and adding in the 4*d.*, we say 8 into 40, 5 times.

2. Divide £16 8*s.* 9*d.* by 5. *Ans.* £3 5*s.* 9*d.*

3. Divide £27 19*s.* 6*d.* by 9. *Ans.* £3 2*s.* 2*d.*

---

101. How may a denominate number be divided? How do you set down the number to be divided? How do you then divide? When there is a remainder, what do you do with it? Of what denomination will each of the quotients be?

102. When the divisor does not exceed 12, how may the division be performed?

4. Divide 36*bu.* 3*pk.* 7*qt.* by 7.

In this example we find that 7 is contained in 36 bushels 5 times and 1 bushel over. Reducing this to pecks and adding 3 pecks, gives 7 pecks, which contains 7, 1 time and no remainder. Multiplying 0 by 8 quarts and adding, gives 7 quarts to be divided by 7.

OPERATION.

```
7)36bu. 3pk. 7qt
  35
  --
   1
   4
  --
7)7pk.(1pk.
  7
  -
  0
  8
  -
7)7(1qt.
```

Ans. 5*bu.* 1*p*

5. Divide £821 17*s.* 9$\frac{3}{4}$*d.* by 4.
Ans. £205 9*s.* 5*d.* 1

6. Divide £55 14*s.* $\frac{3}{4}$*d.* by 7.
Ans. £7 19*s.* 1*d.* 3

7. Divide 16*cwt.* 3*qr.* 27*lb.* 6*oz.* by 7.
Ans. 2*cwt.* 1*qr.* 18*lb.*

8. Divide 49*yd.* 3*qr.* 3*na.* by 9. *Ans.*

9. Divide 131*A.* 1*R.* by 12. *Ans.* 10*A.* 3*R.*

103. When the divisor is a composite numbe exceeds 12, the work may be shortened by dividi the factors in succession, as in division of simple bers.

Ex. 1. Divide £28 2*s.* 4*d.* by the composite n 21. Here the factors are 3 and 7.

OPERATION.

```
7)£28  2s.  4d.
  £4   0s.  4d.
```

OPERATION.

```
3)£4  0s.  4d
  £1  6s.  9⅓
```

Hence, the answer sought is £1 6*s.* 9$\frac{1}{3}$*d.*

2. Divide £57 3*s.* 4*d.* by 35=5×7. *A.* £1 12

3. Divide £85 4*s.* by 72. *Ans.*

4. Divide £31 2*s.* 10$\frac{1}{2}$*d.* by 99. *Ans.* 6*s*

---

103. When the divisor is a composite number, how may t sion be *performed?*

PROOF OF MULTIPLICATION.

104. Divide the product by the multiplier, and if the quotient is equal to the multiplicand, the work may be considered right.

PROOF OF DIVISION.

105. Multiply the quotient by the divisor, and if the product is equal to the dividend, the work may be considered right.

GENERAL EXAMPLES.

1. Divide £1138 12*s.* 4*d.* by 53. *Ans.* £21 9*s.* 8*d.*
2. Divide 1417*cwt.* 7*lb.* by 79. *A.* 17*cwt.* 3*qr.* $18\frac{60}{79}$*lb.*
3. Divide £23 15*s.* $7\frac{1}{4}$*d.* by 37. *Ans.* —
4. Divide £199 3*s.* 10*d.* by 53. *Ans.* £3 15*s.* 2*d.*

APPLICATIONS.

1. Bought 65 yards of cloth for which I paid £72 14*s.* $4\frac{1}{2}$*d.*: what did it cost per yard? *Ans.* £1 2*s.* $4\frac{1}{2}$*d.*

2. Bought 64 gallons of brandy for £30 8*s.*: what did it cost per gallon? *Ans.* 9*s.* 6*d.*

3. Bought 144 reams of paper for £96: what did it cost me per ream? *Ans.* —

4. Sixty-three barrels of sugar contain 7*T.* 16*cwt.* 3*qr.* 12*lb.*: how much is there in each barrel?

Ans. 2*cwt.* 1*qr.* 24*lb.*

5. A farmer has a granary containing 232 bushels 3 ks 7 quarts of wheat, and he wishes to put it in 105 : how much will each bag contain? *Ans.* —

One hundred and seventy-six men consumed in a k 13*cwt.* 2*qr.* 15*lb.* 6*oz.* of bread: how much did each consume? *Ans.* 7*lb.* 12*oz.* 2*dr.*

7. If 62 yards of velvet cost £2 18*s.* 8*d.*, what will e yard cost? *Ans.* 11*d.* $1\frac{26}{62}$*far.*

8. If 92 yards of broadcloth cost £71 14*s.* 0*d.*, what the value of 1 yard? *Ans.* —

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104. How do you prove multiplication?
105. How do you prove division?

9. If 90 hogsheads of sugar weigh 56*T.* 14*cwt.* 3*qr.* 15*lb.*, what is the weight of 1 hogshead?

Ans. 12*cwt.* 2*qr.* 11*lb.*

10. When 192 shares of a certain stock are valued at £1290 4*s.* 0*d.*, what would be the cost of 1 share?

11. If the earth revolve 15° on its axis in 1 hour, how far does it revolve in 1 minute? *Ans.* 15′.

12. If 106 tons of iron cost £1001 9*s.* 7*d.*, what is the value of 1 ton? *Ans.* ——

13. If 57 gallons of wine cost £23 11*s.* 5¼*d.*, what is the cost of one gallon? *Ans.* 8*s.* 3¼*d.*

14. If 59 casks contain 44*hhd.* 53*gal.* 2*qt.* 1*pt.* of wine, what are the contents of one cask? *Ans.* ——

15. When 175*gal.* 2*qt.* of beer are drank in 52 weeks, how much is consumed in one week?

Ans. 3*gal.* 1*qt.* 1*pt.*

16. A rich man divided 168*bu.* 1*pk.* 6*qt.* of corn among 35 poor men: how much did each receive? *Ans.* ——

17. Suppose a man had 98*lb.* 2*oz.* 19*pwt.* 5*gr.* of silver: how much would he give 1 man if he divided it equally among 7 men? *Ans.* 14*lb.* 8*pwt.* 11*gr.*

18. Divide 9*hhd.* 28*gal.* 2*qt.* by 12. *Ans.* ——

19. What will be the share of 1 man, if 810*T.* 11*cwt.* 20*lb.* 13*oz.* 4*dr.* be divided equally among 346 men?

Ans. 2*T.* 6*cwt.* 3*qr.* 10*lb.* 5*oz.* 2*dr.*

20. What will be the quotient of 65*bu.* 1*pk.* 3*qt.* divided by 12? *Ans.* ——

21. Sold 8*lb.* of indigo for £19 13*s.* 8*d.*: how much was it a pound? *Ans.* £2 9*s.* 2½*d.*

22. I gave £8 6*s.* 2*d.* 2*far.* for 10 dozen of combs: how much did I pay for 1 dozen? *Ans.* ——

23. If I pay £12 14*s.* 5*d.* 3*far.* for 35 bushels of wheat, how much is it per bushel? *Ans.* 7*s.* 3*d.* 1*far.*

24. If a merchant paid £23 12*s.* 6*d.* for 84 yards of cloth, how much did he pay a yard? *Ans.* ——

25. Suppose a man has 246*mi.* 6*fur.* 36*rd.* to travel in 12 days: how far will that be in a day?

Ans. 20*mi.* 4*fur.* 23*rd.*

26. Suppose the distance from New York to Bristol, England, to be 3176 miles, and a steamship to complete the passage in 15 days: how far will she sail in one day at this rate? *Ans.* ——

27. If a steamboat should go 224 miles a day, how long would it take her to go to China, it being about 12000 miles? *Ans.* 53*da.* 13*hr.* 42*m.* $51\frac{96}{224}$*sec.*

28. How long would it take a balloon to go from the earth to the moon, allowing the distance to be about 240000 miles: the balloon ascending 34 miles per hour?

29. If a vessel sail 25° 42′ 40″ in 10 days, how far will she sail in one day? *Ans.* 2° 34′ 16″.

30. If you pay £56 8*s.* for 96 yards of cloth, how much do you pay a yard? *Ans.* ——

31. If one man can lift 201*lb.* 12*oz.*, how much can a boy lift, if a man lift 8 times as much as the boy? *Ans.* 25*lb.* 3*oz.* 8*dr.*

32. Divide a leap-year into 102 equal parts.

33. Divide a common year into 102 equal parts.

34. If 15 loads of hay contain 35*T.* 5*cwt.*, what is the weight of each load? *Ans.* ——

35. Divide 371*bu.* 1*pk.* of wheat equally among 270 men: what will each receive? *Ans.* 1*bu.* 1*pk.* 4*qt*

APPLICATIONS IN THE FOUR RULES.

New Orleans, July 1, 1847.

Mr. James Sears,

Bought of Albert Titus

3*lb.* of green tea at 7*s.* 6*d.* per pound, - -
27*yd.* of muslin at 1*s.* 6*d.* per yard, - - -
4*cwt.* of sugar at £2 2*s.* 8*d.* per cwt., - -
2*hhd.* of molasses at 2*s.* 6*d.* per gallon, -
6*lb.* of raisins at 1*s.* 7*d.* per pound, - -

Received payment, £27 18*s.* 2*d.*

Albert Titus.

2. A gentleman purchased of a silversmith, 2 dozen silver spoons, each weighing 3*oz.* 4*pwt.* 1*gr.*; 2 dozen of tea-spoons, each weighing 15*pwt.* 16*gr.*; 3 tankards, each

weighing 22*oz.* 14*pwt.* He sold him old silver to the amount of 6*lb.* 10*oz.* 3*pwt.*: how much remained to be paid for? *Ans.* 6*lb.* 9*oz.* 12*pwt.*

3. What will be the cost of 22 tons of hay at £2 1*s.* 10*d.* per ton? *Ans.* £46 0*s.* 4*d.*

4. If two hogsheads of wine cost £67 4*s.*, what does it cost per gallon? *Ans.* ——

5. If 4*cwt.* of sugar cost £14, what is it per pound? *Ans.* 8*d.* $1\frac{240}{400}$*far.*

6. A man paid £67 4*s.* for a pile of wood containing 64 cords; he sold 30 cords for £29 16*s.*: for how much must he sell the remainder per cord so as not to lose?

7. A printer uses one sheet of paper for every 16 pages of an octavo book: how much paper will be necessary to print 500 copies of a book containing 336 pages, allowing 2 quires of waste paper in each ream?*

Ans. 24 *reams* 5 *quires* 12 *sheets.*

8. Out of a pipe of wine, a merchant draws 12 bottles, each containing 1 pint 3 gills: he then fills six 5-gallon demijohns; then he draws off 3 dozen bottles, each containing 1 quart 2 gills: how much remained in the cask? *Ans.* 82*gal.* 1*pt.*

9. A man lends his neighbor £135 6*s.* 8*d.*, and takes in part payment 4 cows at £5 8*s.* apiece, also a horse worth £50: how much remained due? *Ans.* ——

10. A farmer has 6*T.* 8*cwt.* 2*qr.* 14*lb.* of hay to be removed in 6 equal loads: how much must be carried at each load? *Ans.* 1*T.* 1*cwt.* 1*qr.* 19*lb.*

11. A person at his death left landed estate to the amount of £2000, and personal property to the amount of £2803 17*s.* 4*d.* He directed that his widow should receive one eighth of the whole, and that the residue should be equally divided among his four children: what was the widow's and each child's portion?

Ans. { Widow's portion, £600 9*s.* 8*d.*
Each child's portion, £1050 16*s.* 11*d.* }

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* In packing and selling paper, the two outside quires of every ream are regarded as waste, and each of the remaining quires contains 24 perfect sheets: hence, in this example, the waste paper is considered as belonging *only to the* entire reams.

DIVISIONS OF ARITHMETIC.

106. The science of arithmetic, which treats of numbers, may be divided into three parts:

1st. That which treats of the properties of entire units, called the Arithmetic of Whole Numbers;

2d. That which treats of the parts of unity, called the Arithmetic of Fractions; and

3d. The application of the science of numbers to practical and useful purposes.

A portion of the first part has already been treated under the heads of Numeration, Addition, Subtraction, Multiplication, and Division.

The second part comes next in order, and naturally divides itself into two branches: viz.,

Vulgar or Common Fractions, in which the denominators are any number whatever; and Decimal Fractions, in which the unit is divided according to the scale of tens, hundreds, thousands, &c.

The third part embraces the applications of the principles of entire and fractional numbers to the ordinary transactions and business of life.

The uses and applications of figures are so numerous and so important, that the business of a single day cannot be conducted without them; and hence, no element of education is of greater value than a knowledge of the science of numbers.

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106. Of what does the science of arithmetic treat? Into how many parts may it be divided? Of what does the first part treat? Of what does the second part treat? What is the third part? Which part has been treated? Under how many heads? Into how many heads is the second part divided? What are they called? What *distinguishes* them? What does the third part embrace? Is a *knowledge* of the science of numbers important?

OF VULGAR OR COMMON FRACTIONS.

107. The unit 1 represents an entire thing; as 1 apple, 1 chair, 1 pound of tea.

If we suppose one thing, as one apple, or one pound of tea, to be divided into two equal parts, each part is called *one half.*

If the unit be divided into 3 equal parts, each part is called *one third.*

If the unit be divided into 4 equal parts, each part is called *one fourth.*

If the unit be divided into 12 equal parts, each part is called *one twelfth;* and when it is divided into any number of equal parts, we have a similar expression for each of the parts.

The equal parts of a thing are expressed thus:

$\frac{1}{2}$ is read	one half.	$\frac{1}{7}$ is read	one seventh
$\frac{1}{3}$ - -	one third.	$\frac{1}{8}$ - -	one eighth.
$\frac{1}{4}$ - -	one fourth.	$\frac{1}{10}$ - -	one tenth.
$\frac{1}{5}$ - -	one fifth.	$\frac{1}{15}$ - -	one fifteenth.
$\frac{1}{6}$ - -	one sixth.	$\frac{1}{50}$ - -	one fiftieth.

The expressions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., are called *fractions.*

108. Each fraction is expressed by two numbers; the number which is written above the line is called the *numerator;* and the one below it is called the *denominator,* because it gives a denomination or name to the fraction.

For example, in the fraction $\frac{1}{2}$, 1 is the numerator, and

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107. What does the unit 1 represent? If we divide it into two equal parts, what is each part called? If it be divided into three equal parts, what is each part? Into 4, 5, 6, &c., parts? What are such expressions called?

108. By how many numbers is each fraction expressed? What is the one above the line called? The one below the line? What does the denominator show? What does the numerator show? In the three-eighths, which is the numerator? Which the denominator? Into how many parts is the unit divided? How many parts are expressed? In the fraction nine-twentieths, into how many parts is the unit divided? How many parts are expressed?

2 the denominator. In the fraction $\frac{1}{3}$, 1 is the numerator, and 3 the denominator.

The denominator of every fraction shows into how many equal parts the unit, or single thing, is divided. For example, in the fraction $\frac{1}{2}$, the unit is divided into 2 equal parts; in the fraction $\frac{1}{3}$, it is divided into 3 equal parts; in the fraction $\frac{1}{4}$, it is divided into 4 equal parts, &c. In each of the fractions *one* of the equal parts is expressed. But suppose it were required to express 2 of the equal parts, as 2 halves, 2 thirds, 2 fourths, &c.

We should then write,

$\frac{2}{2}$	they are read	two halves.
$\frac{2}{3}$	- - -	two thirds.
$\frac{2}{4}$	- - -	two fourths.
$\frac{2}{5}$	- - -	two fifths, &c.

If it were required to express three of the equal parts, we should place 3 in the numerator; and generally,

The numerator shows how many of the equal parts are expressed in the fraction.

For example, three eighths are written,

$\frac{3}{8}$	and read	three eighths.
$\frac{4}{9}$	- -	four ninths.
$\frac{6}{13}$	- -	six thirteenths.
$\frac{9}{20}$	- -	nine twentieths.

109. When the numerator and denominator are equal, the numerator expresses all the equal parts into which the unit has been divided: therefore, the *value of the fraction is equal to* 1.

But if we suppose a second unit, of the same kind, to be divided into the same number of equal parts, those parts may also be expressed in the same fraction with the parts of the first unit.

109. When the numerator and denominator are equal, what is the value of the fraction? What is the value of the fraction three-halves? Of seven-fourths? Of sixteen-fifths? Of eighteen-sixths? Of twenty-five sevenths? Repeat the six principles. Write the fraction nineteen-fortieths:—also, 60 fourteenths—18 fiftieths—16 twentieths—17 thirtieths—41 one thousandths—69 ten thousandths—85 millionths?—106 fifths.

Thus, $\frac{3}{2}$	is read	three halves.
$\frac{7}{4}$	- -	seven fourths.
$\frac{16}{5}$	- -	sixteen fifths.
$\frac{18}{6}$	- -	eighteen sixths.
$\frac{25}{7}$	- -	twenty-five sevenths.

The denominator of the first fraction shows that a unit has been divided into 2 equal parts, and the numerator expresses that three such parts are taken. Now, two of the parts make up one unit, and the remaining part comes from the 2d unit: hence, the *value* of the fraction is $1\frac{1}{2}$; that is, one and one half.

The denominator of the second fraction shows that a unit has been divided into four equal parts, and the numerator expresses that 7 such parts are taken. Four of the 7 parts come from one unit, and the remaining 3 from a second unit: the *value* of the fraction is therefore equal to $1\frac{3}{4}$; that is, to one and three fourths. In the third fraction, the unit has been divided into 5 equal parts, and 16 such parts are taken. Now, since each unit has been divided into 5 equal parts, 15 of the 16 parts make 3 units, and the remaining part is 1 part of a fourth unit. Therefore, the *value* of the fraction is $3\frac{1}{5}$; that is, three and one fifth. The value of the fourth fraction is three, and of the fifth, three and four-sevenths. From what has been said, we conclude:

1st. *That a fraction is the expression of one or more parts of unity.*

2d. *That the denominator of a fraction shows into how many equal parts the unit or single thing has been divided, and the numerator expresses how many such parts are taken in the fraction.*

3d. *That the value of every fraction is equal to the quotient arising from dividing the numerator by the denominator.*

4th. *When the numerator is less than the denominator, the value of the fraction is less than 1.*

5th. *When the numerator is equal to the denominator, the value of the fraction is equal to 1.*

6th. *When the numerator is greater than the denominator, the value of the fraction is greater than* 1.

MENTAL EXERCISES IN COMMON FRACTIONS.

1. If a unit be divided into two equal parts, what is each part called? How do you express one of the parts?

2. If a unit be divided into three equal parts, what is each part called? How do you express one of the parts? How do you express two of them? How do you express three of them?

3. If a unit be divided into four equal parts, how do you express one of the parts? Two of the parts? Three of the parts? Four of the parts?

4. How many halves are there in one thing? How many fourths or quarters are there? How much greater is a half than a quarter?

5. If a unit be divided into five equal parts, what is each part called? How do you express three of the parts? Four of them? Five of them?

6. If a unit be divided into six equal parts, what is each part called? How do you express one-sixth? How do you express two of the parts? Three of them? How do you express six of them?

7. How many thirds are there in a unit? How many sixths are there? How much greater then is one-third than one-sixth?

8. If a unit be divided into seven equal parts, what is each part called? How do you express one part? Two parts? Four parts? Six parts? Seven parts?

9. If a unit be divided into eight equal parts, what is each part called? How do you express four of the parts? Five of them? Six of them? Seven of them? Eight of them?

10. How many fourths or quarters are there in a unit? How many eighths are there? How much greater, then, is a quarter than an eighth? How many eighths are equal to two quarters? How many to three quarters?

1. If a unit be divided into nine equal parts, what is

each part called? How do you express one part? Two parts? Four parts? Six parts? Nine parts?

12. If a unit be divided into ten equal parts, how do you express one of the parts? How do you express two of them? Ten of them?

13. How many fifths are there in one unit? How many tenths are there? How much greater is a fifth than a tenth? How many tenths are equal to two-fifths? How many tenths are equal to four-fifths? To five-fifths?

14. If a unit be divided into eleven equal parts, what is each part called? How do you express one of the parts? Two of them? Four of them? Six of them? Seven of them? Eleven of them?

15. If a unit be divided into twelve equal parts, what is each part called? How do you express it? How do you express five of the parts? Six of them? Twelve of them?

16. How many sixths are there in one unit? How many twelfths are there? How much greater is a sixth than a twelfth? How many twelfths are equal to two-sixths? How many to three-sixths? To four-sixths? To five-sixths? To six-sixths?

17. What is the half of one-half? What is the half of one-third? What is the half of one-fourth? What is the half of one-fifth? What is the half of one-sixth?

18. What is the sum of one-half and one-half? What is the sum of one-third and one-third? What is the sum of one-third and two-thirds?

19. What is the sum of one-fourth and two-fourths? Of one-fourth and three-fourths? Of two-fourths and two-fourths?

20. What is the sum of one-fifth and two-fifths? What is the sum of one-fifth and three-fifths? Of two-fifths and three-fifths?

21. What is the sum of one-sixth and four-sixths? What is their difference? What is the sum of three-sixths and two-sixths? What is their difference?

22. What is the sum of one-seventh and four-sevenths?

What is their difference? What is the sum of five-sevenths and two-sevenths? What is their difference?

23. What is the difference between five-eighths and three-eighths? What is their sum? What is the difference between six-eighths and one-eighth? What their sum?

24. What is the difference between seven-ninths and two-ninths? What is their sum? What is the difference between four-ninths and three-ninths? What is their sum?

25. What is the difference between one-tenth and six-tenths? What is their sum? What is the difference between six-tenths and four-tenths? What is their sum?

26. What is the difference between one-eleventh and four-elevenths? What is their sum? What is the difference between three-elevenths and eight-elevenths? What is their sum?

27. What is the sum of three-twelfths and six-twelfths? What is their difference? What is the sum of five-twelfths and seven-twelfths? What is their difference? What is the sum of eight-twelfths and four-twelfths? What is their difference?

28. How many halves are there in one? How many thirds? Fourths? Sevenths? Elevenths? Twelfths?

29. How many halves are there in two? How many thirds? Fourths? Fifths? Sixths? Sevenths? Twelfths?

30. How many thirds are there in three? How many fourths? Fifths? Sixths? Tenths? Twelfths?

31. How many fourths are there in four? How many fifths? How many sixths? How many sevenths? Eighths? Ninths? Twelfths?

32. How many sixths are there in five? How many sevenths? How many eighths? How many elevenths? How many twelfths?

33. How many sixths in nine? How many twelfths in ten? How many elevenths in six? In seven, how many? In eight?

34. How many twelfths in two? In four, how many? In five? How many ninths in five? In eight? In ten? eleven? In twelve?

35. How many whole units in two halves? In four halves? In five halves? In seven halves? In nine halves?

36. How many whole units are there in three-thirds? In six-thirds? In nine-thirds? In five-thirds? In eight-thirds? In eleven-thirds?

37. How many whole units are there in four-fourths? In six-fourths? In eight-fourths? In eleven-fourths? In sixteen? In twenty-nine? In thirty-six fourths?

38. How many whole units are there in five-fifths? In eight-fifths? In nine-fifths? In twelve-fifths? In twenty fifths? In twenty-six fifths? In twenty-eight fifths?

39. How many whole units are there in six-sixths? In twelve-sixths? In fifteen-sixths? In eighteen-sixths? In twenty-five sixths? In twenty-six sixths? In thirty-seven sixths?

40. How many whole units are there in seven-sevenths? In fourteen-sevenths? In nineteen-sevenths? In twenty-nine sevenths? In thirty-five sevenths? In forty-two sevenths?

41. How many whole units are there in eight-eighths? In twenty-four eighths? In sixteen-eighths? In thirty eighths? In thirty-four eighths? In forty eighths?

42. How many whole units are there in nine-ninths? In eighteen-ninths? In twenty-four ninths? In thirty-five ninths? In forty-five ninths?

43. How many whole units are there in ten-tenths? In twenty tenths? In thirty tenths? In forty-five tenths? In sixty tenths?

44. How many whole units are there in eleven-elevenths? In twenty-two elevenths? In thirty-three elevenths? In forty-four elevenths? In sixty-elevenths? In sixty-seven elevenths?

45. How many whole units are there in twelve-twelfths? In twenty-four twelfths? In twenty-six twelfths? In forty-eight twelfths? In fifty twelfths? In sixty twelfths?

46. What is the sum of one-half and one-fourth? What is the sum of one-half and three-fourths? Of three-fourths and six-fourths?

OF THE DIFFERENT KINDS OF VULGAR FRACTIONS.

110. There are six kinds of vulgar fractions—Proper, Improper, Simple, Compound, Mixed, and Complex.

A PROPER FRACTION is one in which the numerator is less than the denominator. The value of every proper fraction is less than 1. (See ART. 109.)

The following are proper fractions:

$$\frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{3}{4},\ \frac{3}{7},\ \frac{5}{8},\ \frac{9}{10},\ \frac{8}{9},\ \frac{5}{6}.$$

111. An IMPROPER FRACTION is one in which the numerator is equal to, or exceeds the denominator. Such fractions are called improper fractions because they are equal to, or exceed unity. When the numerator is equal to the denominator, the value of the fraction is 1; in every other case the value of an improper fraction is greater than 1.

The following are improper fractions:

$$\frac{3}{2},\ \frac{5}{3},\ \frac{6}{5},\ \frac{8}{7},\ \frac{9}{8},\ \frac{12}{6},\ \frac{14}{7},\ \frac{19}{2}.$$

112. A SIMPLE FRACTION is a single expression. A simple fraction may be either proper or improper.

The following are simple fractions:

$$\frac{1}{4},\ \frac{3}{2},\ \frac{5}{6},\ \frac{8}{7},\ \frac{9}{2},\ \frac{8}{3},\ \frac{6}{3},\ \frac{7}{5}.$$

113. A COMPOUND FRACTION is a fraction of a fraction, or several fractions connected together with the word *of* between them.

The following are compound fractions:

$$\frac{1}{2} \text{ of } \frac{1}{4},\ \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } \frac{1}{3},\ \frac{1}{6} \text{ of } 3,\ \frac{1}{7} \text{ of } \frac{1}{8} \text{ of } 4.$$

114. A MIXED FRACTION is made up of a whole number and a fraction. The whole numbers are sometimes called *integers*.

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110. How many kinds of Vulgar Fractions are there? What are they? What is a proper fraction? Is its value greater or less than 1?

111. What is an improper fraction? Why is it called improper? When is its value equal to 1?

112. What is a simple fraction? May it be either proper or improper?

113. What is a compound fraction?

114. What is a mixed fraction?

The following are mixed fractions:

$$3\frac{1}{2},\ 4\frac{1}{3},\ 6\frac{2}{8},\ 5\frac{3}{5},\ 6\frac{5}{8},\ 3\frac{1}{7}.$$

115. A COMPLEX FRACTION is one having a fractional numerator or denominator.

The following are complex fractions:

$$\frac{\frac{1}{7}}{5},\ \frac{2}{19\frac{1}{2}},\ \frac{\frac{2}{3}}{\frac{4}{5}},\ \frac{45\frac{1}{8}}{69\frac{1}{7}}.$$

116. The numerator and denominator of a fraction, taken together, are called the *terms* of the fraction. Hence, every fraction has two terms.

117. A whole number may be expressed fractionally by writing 1 below it for a denominator. Thus,

3 may be written	$\frac{3}{1}$	and is read,	3 ones.
5 - - -	$\frac{5}{1}$	- - -	5 ones.
6 - - -	$\frac{6}{1}$	- - -	6 ones.
8 - - -	$\frac{8}{1}$	- - -	8 ones.

But 3 ones are equal to 3, 5 ones to 5, 6 ones to 6, and 8 ones to 8. Hence, the value of a number is not changed by placing 1 under it for a denominator.

118. If an apple be divided into 6 equal parts,

$\frac{1}{6}$ will express	one of the parts,	
$\frac{2}{6}$ - -	two of the parts,	
$\frac{3}{6}$ - -	three of the parts,	
&c.,	&c.,	&c.,

and generally, the denominator shows into how many

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115. What is a complex fraction? Give an example of a proper fraction. Of an improper fraction. Of a simple fraction. Of a compound fraction. Of a mixed fraction. Is four-ninths a proper or improper fraction? What kind of a fraction is six-thirds? What is its value? What kind of a fraction is nine-eighths? What is its value? What kind of a fraction is one-half of a third? What kind of a fraction is two and one-sixth? Four and a seventh? Eight and a tenth?

116. What are the terms of a fraction? What are the terms of the fraction three-fourths? Of five-eighths? Of six-sevenths?

117. How may a whole number be expressed fractionally? Does this alter its value? Give an example.

14

equal parts the unit is divided, and the numerator how many of the parts are taken.

Hence, also, we may conclude that,

$\frac{1}{6}\times 2$; that is, $\frac{1}{6}$ taken 2 times $=\frac{2}{6}$,
$\frac{1}{6}\times 3$; that is, $\frac{1}{6}$ taken 3 times $=\frac{3}{6}$,
$\frac{1}{6}\times 4$; that is, $\frac{1}{6}$ taken 4 times $=\frac{4}{6}$;

and consequently we have,

PROPOSITION I. *If the numerator of a fraction be multiplied by any number, the denominator remaining the same, the value of the fraction will be multiplied as many times as there are units in the multiplier.* Hence,

To multiply a fraction by a whole number, we simply multiply the numerator by the number.

EXAMPLES.

1. Multiply $\frac{3}{8}$ by 8. *Ans.* $\frac{24}{8}$.
2. Multiply $\frac{7}{5}$ by 5. *Ans.* $\frac{35}{5}$.
3. Multiply $\frac{1}{7}$ by 9. *Ans.* ——
4. Multiply $\frac{8}{19}$ by 14. *Ans.* $\frac{112}{19}$.
5. Multiply $\frac{7}{6}$ by 20. *Ans.* $\frac{140}{6}$
6. Multiply $\frac{167}{81}$ by 25. *Ans.* $\frac{4175}{81}$.

119. If three apples be each divided into 6 equal parts, there will be 18 parts in all, and these parts will be expressed by the fraction $\frac{18}{6}$. If it were required to express but one-third of the parts, we should take in the numerator but one-third of 18; that is, the fraction $\frac{6}{6}$ would express one-third of $\frac{18}{6}$. If it were required to express one-sixth of the parts, we should take one-sixth of 18, and $\frac{3}{6}$ would be the required fraction.

In each case the fraction $\frac{18}{6}$ has been divided as many times as there were units in the divisor. Hence,

PROPOSITION II. *If the numerator of a fraction be divided by any number, the denominator remaining un-*

118. If an apple be divided in six equal parts, how do you express one of those parts? Two of them? Three of them? Four of them? Five of them? Repeat the proposition. How do you multiply a fraction by a whole number?

changed, the value of the fraction will be divided as many times as there are units in the divisor. Hence,

A fraction may be divided by a whole number, by dividing its numerator.

EXAMPLES.

1. Divide $\frac{28}{15}$ by 2, by 7, by 14. *Ans.* $\frac{14}{15}$, $\frac{4}{15}$, $\frac{2}{15}$.
2. Divide $\frac{112}{160}$ by 56, by 28, by 14, by 7. *Ans.* ——
3. Divide $\frac{400}{117}$ by 25, by 8, by 16, by 4. *Ans.* ——

120. Let us again suppose the apple to be divided into 6 equal parts. If now each part be divided into 2 equal parts, there will be 12 parts of the apple, and consequently each part will be but half as large as before.

Three parts in the first case will be expressed by $\frac{3}{6}$, and in the second by $\frac{3}{12}$. But since the value of each part in the second is only half the value of each part in the first fraction, it follows that,

$$\frac{3}{12} = \text{one-half of } \frac{3}{6}.$$

If we suppose the apple to be divided into 18 equal parts, three of the parts will be expressed by $\frac{3}{18}$, and since the parts are but one-third as large as in the first case, we have

$$\frac{3}{18} = \text{one-third of } \frac{3}{6}:$$

and since the same may be said of all fractions, we have

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119. If 3 apples be each divided into 6 equal parts, how many parts in all? If 4 apples be so divided, how many parts in all? If 5 apples be so divided, how many parts? How many parts in 6 apples? In 7? In 8? In 9? In 10? What expresses all the parts of the three apples? What expresses one-half of them? One-third of them? One-sixth of them? One-ninth of them? One-eighteenth of them? What expresses all the parts of four apples? One-half of them? One-third of them? One-fourth of them? One-sixth of them? One-eighth of them? One-twelfth of them? One twenty-fourth of them? Put similar questions for 5 apples, 6 apples, &c. Repeat the proposition. How may a fraction be divided by a whole number?

120. If a unit be divided in 6 equal parts and then into 12 equal parts, how does one of the last parts compare with one of the first? If the second division be into 18 parts, how do the parts compare? If into 24? What part of 24 is 6? If the second division be into 30 parts, how do they compare? If into 36 parts? Repeat the proposition. How may a fraction be divided by a whole number?

PROPOSITION III. *If the denominator of a fraction be multiplied by any number, the numerator remaining the same, the value of the fraction will be divided as many times as there are units in the multiplier.* Hence,

A fraction may be divided by any number, by multiplying the denominator by that number.

EXAMPLES.

1. What is $\frac{1}{2}$ of $\frac{1}{4}$? *Ans.* $\frac{1}{8}$.
2. What is $\frac{1}{5}$ of $\frac{3}{7}$? *Ans.* $\frac{3}{35}$.
3. Divide $\frac{3}{16}$ by 4. *Ans.* $\frac{3}{64}$.
4. Divide $\frac{17}{15}$ by 8. *Ans.* ——
5. Divide $\frac{16}{175}$ by 45. *Ans.* $\frac{16}{7875}$.

121. If we suppose the apple to be divided into 3 equal parts instead of 6, each part will be twice as large as before, and three of the parts will be expressed by $\frac{3}{3}$ instead of $\frac{3}{6}$. But this is the same as dividing the denominator 6 by 2; and since the same is true of all fractions, we have

PROPOSITION IV. *If the denominator of a fraction be divided by any number, the numerator remaining the same, the value of the fraction will be multiplied as many times as there are units in the divisor.* Hence,

A fraction may be multiplied by a whole number, by dividing the denominator by that number.

EXAMPLES.

1. Multiply $\frac{3}{4}$ by 2, by 4. *Ans.* $\frac{3}{2}$, $\frac{3}{1}$.
2. Multiply $\frac{16}{32}$ by 2, 4, 8, 16, 32.
Ans. $\frac{16}{16}$, $\frac{16}{8}$, $\frac{16}{4}$, $\frac{16}{2}$, $\frac{16}{1}$.
3. Multiply $\frac{8}{48}$ by 2, 4, 6, 8, 12, 16, 24, 48.
Ans. , , &c.

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121. If we divide 1 apple into three parts, and another into six, how much greater will the parts of the first be than those of the second? Are the parts larger as you decrease the denominator? If you divide the denominator by 2, how do you affect the parts? If you divide it by 3? By 4? By 5? By 6? By 7? By 8? Repeat the proposition. How may a fraction be multiplied by a whole number?

4. Multiply $\frac{19}{84}$ by 2, 4, 6, 12, 21, 42.
Ans. $\frac{19}{42}$, $\frac{19}{21}$, $\frac{19}{14}$, &c., &c.

5. Multiply $\frac{151}{200}$ by 5, 10, 20. *Ans.* $\frac{151}{40}$, $\frac{151}{20}$, $\frac{151}{10}$.

122. It appears from Prop. I., that if the numerator of a fraction be multiplied by any number, the value of the fraction will be *multiplied* as many times as there are units in the multiplier. It also appears from Prop. III., that if the denominator of a fraction be multiplied by any number, the value of the fraction will be *divided* as many times as there are units in the multiplier.

Therefore, when the numerator and denominator of a fraction are both multiplied by the same number, the increase from multiplying the numerator will be just equal to the decrease from multiplying the denominator: hence we have,

Proposition V. *If both terms of a fraction be multiplied by the same number, the value of the fraction will remain unchanged.*

EXAMPLES.

1. Multiply the numerator and denominator of $\frac{5}{7}$ by 7: this gives $\frac{5}{7}\times\frac{7}{7}=\frac{35}{49}$. *Ans.* $\frac{35}{49}$.

2. Multiply the numerator and denominator of $\frac{19}{12}$ by 3, by 4, by 5, by 6, by 9, by 12, by 15, by 20.

3. Multiply each term of $\frac{125}{102}$ by 7, by 8, by 12, by 14, by 15, by 17, by 45.

123. It appears from Prop. II., that if the numerator of a fraction be divided by any number, the value of the

---

122. If the numerator of a fraction be multiplied by a number, how many times is the fraction increased? If the denominator be multiplied by the same number, how many times is the fraction diminished? If then the numerator and denominator be both multiplied at the same time, is the value changed? Why not? Repeat the proposition.

123. If the numerator of a fraction be divided by a number, how many times will the value of the fraction be diminished? If the denominator be divided by the same number, how many times will the value of the fraction be increased? If they are both divided by the same number, will the value of the fraction be changed? Why not? Repeat the proposition.

fraction will be *divided* as many times as there are units in the divisor. It also appears from Prop. IV., that if the denominator of a fraction be divided by any number, the value of the fraction will be *multiplied* as many times as there are units in the divisor. Therefore, when the numerator and denominator of a fraction are divided by the same number, the *decrease* from dividing the numerator will be just equal to the *increase* from dividing the denominator: hence we have,

PROPOSITION VI. *If both terms of a fraction be divided by the same number, the value of the fraction will remain unchanged.*

EXAMPLES.

1. Divide both terms of the fraction $\frac{8}{16}$ by 4: this gives $\frac{4)8}{4)16}=\frac{2}{4}$. *Ans.* $\frac{2}{4}$.

2. Divide each term by 8: this gives $\frac{8)8}{8)16}=\frac{1}{2}$.

3. Divide each term of the fraction $\frac{32}{128}$ by 2, by 4, by 8, by 16, by 32.

4. Divide each term of the fraction $\frac{60}{180}$ by 2, by 3, by 4, by 5, by 6, by 10, by 12, by 15, by 20, by 30, by 60.

GREATEST COMMON DIVISOR.

124. Any number greater than unity that will divide two or more numbers without a remainder, is called their common divisor: and the greatest number that will so divide them, is called their GREATEST COMMON DIVISOR.

Before explaining the manner of finding this divisor, it is necessary to explain some principles on which the method depends.

One number is said to be a multiple of another when it contains that other an exact number of times. Thus, 24 is a multiple of 6, because 24 contains 6 an exact

124. What is a common divisor? What is the greatest common divisor of two or more numbers? When is one number said to be a multiple of another? What is the first principle? What is the second? What is the third? Give the rule for finding the greatest common divisor. How do you find the greatest common divisor of more than two numbers?

number of times. For a like reason 60 is a multiple of 12, since it contains 12 an exact number of times.

First Principle. Every number which exactly divides another number will also divide without a remainder any multiple of that number. For example, 24 is divisible by 8, giving a quotient 3. Now, if 24 be multiplied by 4, 5, 6, or any other number, the product so arising will also be divisible by 8.

Second Principle. If a number be separated into two parts, any divisor which will divide each of the parts separately, without a remainder, will exactly divide the given number. For, the sum of the two partial quotients must be equal to the entire quotient; and if they are both whole numbers, the entire quotient must be a whole number; for the sum of two whole numbers cannot be equal to a fraction.

For example, if 36 be separated into the parts 16 and 20, the number 4, which will divide both numbers 16 and 20, will also divide 36; and the sum of the quotients 4 and 5 will be equal to the entire quotient 9.

Third Principle. If a number be decomposed into two parts, then any divisor which will divide the given number and one of the parts, will also divide the other.

For, the entire quotient is equal to the sum of the two partial quotients; and if the entire quotient and one of the partial quotients be whole numbers, the other must also be a whole number; for no proper fraction added to a whole number can give a whole number.

Let it be required to find the greatest common divisor of the numbers 216 and 408.

It is evident that the greatest common divisor cannot be greater than the least number 216. Now, as 216 will divide itself, let us see if it will divide 408; for if it will, it is the greatest common divisor sought.

OPERATION.

```
216)408(1
    216
    ---
    192)216(1
        192
        ---
         24)192(8
            192
            ---
```

Making this division, we find a quotient 1 and a *remainder* 192; hence, 216 is not the greatest common divisor. Now we say, *that the greatest common divisor of the two given numbers is the common divisor of the less number* 216 *and the remainder* 192 *after the division.* For, by the second principle, any number which will exactly divide 216 and 192, will also exactly divide the number 408.

Let us now seek the common divisor between 216 and 192. Dividing the greater by the less, we have a remainder of 24; and from what has been said above, the greatest common divisor of 192 and 216 is the same as the greatest common divisor of 192 and 24, which we find to be 24. Therefore, 24 is the greatest common divisor of the given numbers 408 and 216: hence, to find the greatest common divisor,

Divide the greater number by the less, and then divide the divisor by the remainder, and continue to divide the last divisor by the last remainder until nothing remains. The last divisor will be the greatest common divisor sought.

NOTE. If it be required to find the greatest common divisor of more than two numbers, find first the greatest common divisor of two of them, then of that common divisor and one of the remaining numbers, and so on for all the numbers: the last common divisor will be the greatest common divisor of all the numbers.

EXAMPLES.

1. Find the greatest common divisor of 408 and 740. *Ans.* 4.

2. Find the greatest common divisor of 315 and 810.

3. Find the greatest common divisor of 4410 and 5670. *Ans.* 630.

4. Find the greatest common divisor of 3471 and 1869. *Ans.* 267.

5. Find the greatest common divisor of 1584 and 2772. *Ans.* ——

6. What is the greatest common divisor of 492, 744, and 1044? *Ans.* 12.

7. What is the greatest common divisor of 944, 1488, and 2088? *Ans.* 8.

8. What is the greatest common divisor of 216, 408, and 740? *Ans.* 4.

9. What is the greatest common divisor of 945, 1560, and 22683? *Ans.* 3.

10. What is the greatest common divisor of 204, 1190, 1445, and 2006? *Ans.* ——

LEAST COMMON MULTIPLE.

125. A number is said to be a *common multiple* of two or more numbers, when it can be divided by each of them separately, without a remainder. For example, 6 is a common multiple of 2 and 3, because it is exactly divisible by each of them. So likewise, 12 is a common multiple of 2, 3, 4, and 6, because it is divisible by each of them.

The *least common multiple* of two or more numbers, is the *least* number which they will separately divide without a remainder. For example, 12 is a common multiple of 2 and 3, but it is not the *least* common multiple, since 6 is also divisible by 2 and 3. Now 6 being the least number which is so divisible, it is the least common multiple of 2 and 3.

A factor of a number is any number that will divide it without a remainder; and a *prime factor* is any prime number which will so divide it.

126. To find the least common multiple of several numbers,

I. *Place the numbers on the same line, and divide by any prime number that will divide two or more of them without a remainder, and set down in a line below, the quotients and the undivided numbers.*

---

125. When is one number said to be a common multiple of two or more numbers? Of what numbers is 6 a common multiple? Of what numbers is 8 a common multiple? What is the least common multiple of two or more numbers? What is the difference between a common multiple and the least common multiple? What is a factor of any number? What is a prime factor?

II. *Then divide as before, until there is no number greater than 1 that will exactly divide any two of the numbers: then multiply together the numbers of the lower line, and the divisors, and the product will be the least common multiple. If, in comparing the numbers together, we find no common divisor, their product is the least common multiple.*

EXAMPLES.

1. Find the least common multiple of 3, 4, and 8.

OPERATION.

```
2)3 . . . 4 . . . 8
2)3 . . . 2 . . . 4
  3 . . . 1 . . . 2
```

Ans. $2\times1\times3\times2\times2=24$.

We first see, that 2 will divide 4 and 8, but as it will not divide 3, we bring down 3 into the 2d line: we again see that 2 is a common divisor of 2 and 4; and as there is no common divisor between any two of the numbers of the last line, it follows that $2\times1\times3$ multiplied by the two divisors, is the least common multiple.

2. Find the least common multiple of 3, 8, and 9.

OPERATION.

```
3)3 . . . 8 . . . 9
  1 . . . 8 . . . 3
```

$1\times8\times3\times3=72$.

We arrange the numbers in a line, and see that 3 will divide two of them. We then write down the quotients 1 and 3, and also the 8, which cannot be divided. Then as there is no common divisor between any two of the numbers, 1, 8, and 3, it follows that their product, multiplied by the divisor 3, will give the least common multiple sought.

3. Find the least common multiple of 6, 7, 8, and 10.
Ans. 840.

4. Find the least common multiple of 21 and 49.
Ans. 147.

5. Find the least common multiple of 2, 7, 5, 6, and 8.
Ans. 840.

---

126. Give the rule for finding the least common multiple. If the numbers have no common divisor, what is the least common multiple?

6. Find the least common multiple of 4, 14, 28, and 98. *Ans.* ——

7. Find the least common multiple of 13 and 6. *Ans.* 78.

8. Find the least common multiple of 12, 4, and 7. *Ans.* 84.

9. Find the least common multiple of 6, 9, 4, 14, and 16. *Ans.* 1008.

10. Find the least common multiple of 13, 12, and 4. *Ans.* 156.

11. What is the least common multiple of 11, 17, 19, 21, and 7? *Ans.* 74613.

REDUCTION OF VULGAR FRACTIONS.

127. Reduction of Vulgar Fractions is the method of changing their forms without altering their value.

A fraction is said to be in its lowest terms when there is no number greater than 1 that will divide the numerator and denominator without a remainder. The terms of the fraction are then said to have no common factor.

CASE I.

128. To reduce an improper fraction to its equivalent whole or mixed number.

Divide the numerator by the denominator, the quotient will be the whole number; and the remainder, if there be one, placed over the given denominator, will form the fractional part.

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127. What is reduction? When is a fraction said to be in its lowest terms? Is one-half in its lowest terms? Is two-fourths? Is three-fourths?

128. How do you reduce a fraction to its equivalent whole or mixed number? Does this reduction alter its value? Why not? What is four-halves equal to? Eight-fourths? Sixteen-eighths? Twenty fifths? Twenty-six sixths? Four-thirds? What is nine-fourths equal to? Five-fourths? Seventeen-sixths? Eighteen-sevenths?

EXAMPLES.

1. Reduce $\frac{84}{4}$ and $\frac{67}{9}$ to their equivalent whole or mixed numbers.

OPERATION.	OPERATION.
4)84	9)67
Ans. 21	*Ans.* $7\frac{4}{9}$

NOTE. It has been shown that the value of every fraction is equal to the quotient arising from dividing the numerator by the denominator: hence, the value of the fraction is not changed by the reduction.

2. Reduce $\frac{99}{8}$ to a whole or mixed number. *Ans.* $12\frac{3}{8}$.

3. In $\frac{19}{7}$ of yards of cloth, how many yards? *Ans.* ——

4. In $\frac{51}{9}$ of bushels, how many bushels? *Ans.* $5\frac{6}{9}$*bu.*

5. If I give $\frac{1}{3}$ of an apple to each one of 15 children, how many apples do I give? *Ans.* 5.

6. Reduce $\frac{327}{125}$, $\frac{3672}{153}$, $\frac{50287}{6941}$, $\frac{987625}{72301}$, to their whole or mixed numbers. *Ans.* $2\frac{77}{125}$, 24, $7\frac{1700}{6941}$, $13\frac{47712}{72301}$.

7. If I distribute 878 quarter-apples among a number of boys, how many whole apples do I use? *Ans.* ——

8. Reduce $\frac{62587}{3114}$, $\frac{4927}{109}$, $\frac{2641674}{278436}$, to their whole or mixed numbers. *Ans.* ——

9. Reduce $\frac{147254149}{4674}$, $\frac{145260}{108}$, $\frac{62015735}{7803}$, to their whole or mixed numbers.

Ans. $31504\frac{4453}{4674}$, 1345, $7947\frac{5294}{7803}$.

CASE II.

129. To reduce a mixed number to its equivalent improper fraction.

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129. How do you reduce a mixed number to its equivalent improper fraction? How many fourths are there in one? In two? In three? How many sixths in four and one-sixth? In eight and two-sixths? In seven and three-sixths? In nine and five-sixths? In ten and five-sixths? How many eighths in two and one-eighth? In three and three-eighths? In four and four-eighths? In five and six-eighths? In seven and seven-eighths? In eight and seven-*eighths?*

Multiply the whole number by the denominator of the fraction; to the product add the numerator, and place the sum over the given denominator.

EXAMPLES.

1. Reduce $4\frac{4}{5}$ to its equivalent improper fraction.

Here $4\times5=20$: then $20+4=24$; hence,

$\frac{24}{5}$ is the equivalent fraction. *Ans.* $\frac{24}{5}$.

This rule is the reverse of Case I. In the example $4\frac{4}{5}$ we have the integer number 4 and the fraction $\frac{4}{5}$. Now 1 whole thing is equal to 5 fifths, and 4 whole things are equal to 20 fifths; to which add the 4 fifths, and we obtain the 24 fifths.

2. Reduce $47\frac{5}{6}$ to its equivalent improper fraction. *Ans.* $\frac{287}{6}$.

3. Reduce $676\frac{37}{51}$, $874\frac{33}{9}$, $690\frac{47}{100}$, $367\frac{9}{104}$, to their equivalent improper fractions.

Ans. $\frac{34513}{51}$, $\frac{7899}{9}$, $\frac{69047}{100}$, $\frac{38177}{104}$.

4. Reduce $847\frac{36}{175}$, $874\frac{876}{104}$, $67426\frac{368}{879}$, to their equivalent improper fractions. *Ans.* ——

5. How many 200ths in $675\frac{187}{200}$? *Ans.* 135187.

6. How many 151ths in $187\frac{41}{151}$? *Ans.* 28278.

7. Reduce $625\frac{4}{13}$ to an improper fraction.

8. Reduce $156\frac{17}{407}$ to an improper fraction.

CASE III.

130. To reduce a fraction to its lowest terms.

I. *Divide the numerator and denominator by any number that will divide them both without a remainder, and then divide the quotients arising in the same way until there is no number greater than 1 that will divide both terms of the fraction without a remainder.*

---

130. When is a fraction in its lowest terms? (see Art. 127.) How do you reduce a fraction to its lowest terms by the first method? By the second? What are the lowest terms of two-fourths? Of six-eighths? Of nine-twelfths? Of sixteen thirty-sixths? Of ten-twentieths? Of fifteen twenty-fourths? Of sixteen-eighteenths? Of nine-eighteenths?

II. *Or, find the greatest common divisor of the numerator and denominator, and divide them by it. The value of the fraction will not be altered by the reduction.*

EXAMPLES.

1. Reduce $\frac{70}{175}$ to its lowest terms.

1ST METHOD.

$\frac{5)\ 70}{5)175}=\frac{7)14}{7)35}=\frac{2}{5}$, which are the lowest terms of the fraction, since no number greater than 1 will divide the numerator and denominator without a remainder.

2D METHOD, BY THE COMMON DIVISOR.

$$\begin{array}{r} 70)175(2 \\ 140 \\ \hline 35)70(2 \\ 70 \\ \hline \end{array}$$

Greatest common div. 35

$\frac{35)\ 70}{35)175}=\frac{2}{5}$ *Ans.*

2. Reduce $\frac{104}{312}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
3. Reduce $\frac{1049}{8392}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
4. Reduce $\frac{275}{440}$ to its lowest terms. *Ans.* ——
5. Reduce $\frac{351}{795}$ to its lowest terms. *Ans.* $\frac{117}{265}$.
6. Reduce $\frac{172}{1118}$ to its lowest terms. *Ans.* $\frac{2}{13}$.
7. Reduce $\frac{63}{81}$ to its lowest terms by the 2d method. *Ans.* $\frac{7}{9}$.
8. Reduce $\frac{315}{405}$ to its lowest terms by the 2d method. *Ans.* $\frac{7}{9}$.
9. Reduce $\frac{1157}{623}$ to its lowest terms by the 2d method. *Ans.* ——
10. Reduce $\frac{792}{1386}$ to its lowest terms by the 2d method. *Ans.* $\frac{4}{7}$.

CASE IV.

131. To reduce a whole number to an equivalent fraction having a given denominator.

Since the denominator of a fraction shows into how *many* equal parts unity has been divided, it is plain that if we multiply it by the number of units so divided, the product will be equal to the entire number of parts taken. Hence,

Multiply the whole number by the given denominator, and set the product over the said denominator.

EXAMPLES.

1. Reduce 6 to a fraction whose denominator shall be 4.

Since each unit is to be divided into 4 parts, it follows that the number of parts in 6 units will be expressed by $6\times4=24$; hence the required fraction is $\frac{24}{6}$.

2. Reduce 15 to a fraction whose denominator shall be 9. *Ans.* $\frac{135}{9}$.

3. Reduce 139 to a fraction whose denominator shall be 175. *Ans.* ——

4. Reduce 1837 to a fraction whose denominator shall be 181. *Ans.* ——

5. If the denominator be 837, what fractions will be formed from 327? From 889? From 575?

6. If the denominator be 216, what fractions will be formed from 876? From 306? From 5047?

CASE V.

132. To reduce a compound fraction to its equivalent simple one.

I. *Reduce all mixed numbers to their equivalent improper fractions by Case II.*

II. *Then multiply all the numerators together for a numerator, and all the denominators together for a denominator: their products will form the fraction sought.*

---

131. How do you reduce a whole number to an equivalent fraction having a given denominator? How many thirds in 1? In 2? In 3? In 4? If the denominator be 5, what fraction will you form of 5? Of 4? Of 9? Of 7? Of 8? With the denominator 6, what fraction will you form of 3? Of 4? Of 5? Of 6? Of 7? Of 9?

132. What is a compound fraction? How do you reduce a compound fraction to a simple one? When you find like factors in the numerator and denominator, what do you do with them? Does this alter the value of the fraction? What is one-half of one-half? One-half of one-third? One-third of one-fourth? One-sixth of one-seventh? *Three-halves of* one-eighth? Six-thirds of two-ones?

EXAMPLES.

1. Let us take the fraction $\frac{3}{4}$ of $\frac{5}{7}$.

First, $\frac{3}{4}=3\times\frac{1}{4}$: hence the fractions may be written $\frac{3}{4}$ of $\frac{5}{7}=3\times\frac{1}{4}$ of $\frac{5}{7}$; that is, three times one-fourth of $\frac{5}{7}$. But $\frac{1}{4}$ of $\frac{5}{7}=\frac{5}{28}$: hence we have,

$$\tfrac{3}{4} \text{ of } \tfrac{5}{7}=3\times\tfrac{5}{28}=\tfrac{15}{28};$$

a result which is obtained by multiplying together the numerators and denominators of the given fractions.

When the compound fraction consists of more than two simple ones, two of them can be reduced to a simple fraction as above, and then this fraction may be reduced with the next, and so on.

2. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{7}$ to a simple fraction.

Here, $\frac{1}{2}\times\frac{1}{3}\times\frac{5}{7}=\frac{5}{42}$. *Ans.* $\frac{5}{42}$.

3. Reduce $\frac{5}{3}$ of $\frac{3}{6}$ of $\frac{6}{7}$ to a simple fraction.

Here, $\frac{5}{3}\times\frac{3}{6}\times\frac{6}{7}=\frac{90}{126}=\frac{10}{14}=\frac{5}{7}$,

by dividing the numerator and denominator first by 9 and then by 2, as shown in Case III.

4. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{15}$ to a simple fraction. *Ans.* $\frac{2}{5}$.

5. Reduce $2\frac{1}{4}$ of $6\frac{1}{2}$ of 7 to a simple fraction. *Ans.* $\frac{819}{8}=102\frac{3}{8}$.

6. Reduce 5 of $\frac{1}{2}$ of $\frac{1}{7}$ of 6 to a simple fraction.

7. Reduce $6\frac{1}{3}$ of $7\frac{1}{4}$ of $6\frac{31}{27}$ to a simple fraction. *Ans.* $\frac{106343}{324}$.

METHOD BY CANCELLING

133. The work may often be abridged by striking out or *cancelling* common factors in the numerator and denominator, which is merely dividing both terms of the fraction by the same number. *In every operation in fractions, let this be done whenever it is possible.*

EXAMPLES.

1. Reduce $\frac{5}{3}$ of $\frac{3}{6}$ of $\frac{6}{7}$ to a simple fraction.

Here, $\frac{5}{\not 3}\times\frac{\not 3}{\not 6}\times\frac{\not 6}{7}=\frac{5}{7}$.

by cancelling or striking out the 3's and 6's in the numerator and denominator.

By cancelling or striking out the 3's we only divide the numerator and denominator of the fraction by 3; and in cancelling the 6's we divide by 6. Hence, *the value of the fraction is not affected by striking out like factors, which is merely dividing the numerator and denominator by the same number.*

2. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{15}$ to its simplest terms.

Here, $$\frac{\overset{2}{\cancel{6}}}{\cancel{8}}\times\frac{\cancel{8}}{\cancel{9}}\times\frac{\cancel{9}}{\underset{5}{\cancel{15}}}=\frac{2}{5}\ \textit{Ans.}$$

Besides cancelling the like factors 8 and 8 and 9 and 9, we also cancel the factor 3 common to 6 and 15, and write the quotients 2 and 5 above and below the numbers.

3. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{5}{9}$ of $\frac{27}{101}$ of $\frac{5}{13}$ to its simplest terms.

4. Reduce $\frac{41}{110}$ of $\frac{3}{19}$ of $\frac{41}{108}$ of $\frac{3}{7}$ to its simplest terms.

5. Reduce $3\frac{5}{8}$ of $\frac{5}{9}$ of $\frac{27}{301}$ of 49 to its simplest terms.

CASE VI.

134. To reduce complex fractions to simple ones.

Reduce the numerator and denominator, when necessary, to simple fractions; then multiply both terms by the denominator, with its terms inverted, and the product will be the equivalent simple fraction.

For, if we multiply the numerator and denominator of a fraction by any number whatever, the value of the fraction will not be altered (Art. 122). Let us then multiply both terms by the denominator with its terms inverted.

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133. How may the work often be shortened? Ought it to be so abridged?

134. What is a complex fraction? How do you reduce a complex to a simple fraction? If you invert the terms of any fraction, and then multiply it by the new fraction so found, what will the product be equal to?

EXAMPLES.

1. Reduce the complex fraction $\dfrac{\frac{4}{7}}{\frac{2}{9}}$ to a simple fraction.

Now, $$\frac{\frac{4}{7}}{\frac{2}{9}}=\frac{\frac{4}{7}\times\frac{9}{2}}{\frac{2}{9}\times\frac{9}{2}}=\tfrac{36}{14}.$$

2. Reduce $\dfrac{47\frac{5}{8}}{95}$ to a simple fraction. *Ans.* $\frac{381}{760}$.

3. Reduce $\dfrac{34\frac{5}{7}}{84}$ to a simple fraction. *Ans.* $\frac{81}{196}$.

4. Reduce $\dfrac{44}{147\frac{5}{9}}$ to a simple fraction. *Ans.* $\frac{99}{332}$.

5. Reduce $\dfrac{247}{\frac{5}{7}}$ to a simple fraction. *Ans.* ——

6. Reduce $\dfrac{\frac{147}{504}}{1789}$ to a simple fraction. *Ans.* $\frac{7}{42936}$.

7. Reduce $\dfrac{394\frac{74}{99}}{894\frac{547}{719}}$ to a simple fraction.

Ans. $\frac{28098520}{636899967}$.

CASE VII.

135. To reduce fractions of different denominators to equivalent fractions having a common denominator.

I. *Reduce complex and compound fractions to simple ones, and all whole or mixed numbers to improper fractions.*

II. *Then multiply the numerator and denominator of each fraction by the product of the denominators of all the others.*

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135. What is the first step in reducing fractions to a common denominator? What is the second? Does the reduction alter the *values of the* several fractions? Why not? When the numbers are *small, how* may the work be performed?

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{7}{3}$, and $\frac{4}{5}$ to a common denominator.

$1\times3\times5=15$, the new numerator of the 1st.
$7\times2\times5=70$ - - - - - 2d.
$4\times3\times2=24$ - - - - - 3d.
and $2\times3\times5=30$, the common denominator.

Therefore, $\frac{15}{30}$, $\frac{70}{30}$, and $\frac{24}{30}$, are the equivalent fractions.

It is plain, that this reduction does not alter the values of the several fractions, since the numerator and denominator of each are multiplied by the same number (see Prop. V).

2. When the numbers are small the work may be performed mentally. Thus,

$$\tfrac{1}{2},\ \tfrac{1}{4},\ \tfrac{2}{5}=\tfrac{20}{40},\ \tfrac{10}{40},\ \tfrac{16}{40}.$$

Here we find the first numerator by multiplying 1 by 4 and 5; the second, by multiplying 1 by 2 and 5; the third, by multiplying 2 by 4 and 2; and the common denominator by multiplying 2, 4, and 5 together.

3. Reduce $2\frac{1}{3}$, and $\frac{1}{2}$ of $\frac{1}{7}$ to a common denominator

$2\frac{1}{3}=\frac{7}{3}$; and $\frac{1}{2}$ of $\frac{1}{7}=\frac{1}{14}$.

$\frac{7}{3}$ and $\frac{1}{14}=\frac{98}{42}$ and $\frac{3}{42}$; the answers.

4. Reduce $5\frac{1}{2}$, $\frac{6}{7}$ of $\frac{1}{3}$, and 4, to a common denominator. *Ans.* $\frac{77}{14}$, $\frac{4}{14}$, $\frac{56}{14}$.

5. Reduce $\frac{7}{8}$, $\frac{135}{75}$, and 37, to a common denominator. *Ans.* $\frac{525}{600}$, $\frac{1080}{600}$, and $\frac{22200}{600}$.

6. Reduce 4, $\frac{31}{25}$, $\frac{62}{2}$, to a common denominator. *Ans.* $\frac{200}{50}$, $\frac{62}{50}$, and $\frac{1550}{50}$.

7. Reduce $7\frac{1}{2}$, $\frac{31}{18}$, $6\frac{1}{4}$, to a common denominator. *Ans.* $\frac{1080}{144}$, $\frac{248}{144}$, and $\frac{900}{144}$.

8. Reduce $4\frac{1}{9}$, $8\frac{1}{7}$, and $2\frac{1}{2}$ of 5, to a common denominator. *Ans.* ——

136. It is often convenient to reduce fractions to a common denominator by multiplying the numerator and denominator of each fraction by such a number as shall make the denominators the same in all.

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136. By what second method may fractions be reduced to a common denominator? *Will the value of either fraction be changed?*

EXAMPLES.

1. Let it be required to reduce $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator.

We see at once that if we multiply the numerator and denominator of the first fraction by 3, and the numerator and denominator of the second by 2, that they will have a common denominator.

The two fractions will be reduced to $\frac{3}{6}$ and $\frac{2}{6}$.

2. Reduce $\frac{1}{5}$ and $\frac{1}{3}$ to a common denominator.

If we multiply both terms of the first fraction by 3 and both terms of the second by 5, we have

$$\tfrac{1}{5}=\tfrac{3}{15}, \text{ and } \tfrac{1}{3}=\tfrac{5}{15}.$$

3. Reduce $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{3}{4}$ to a common denominator.
Ans. $\frac{2}{12}$, $\frac{1}{12}$, $\frac{9}{12}$.

4. Reduce $\frac{3}{7}$, $\frac{8}{28}$, $\frac{4}{14}$, to a common denominator.

5. Reduce $\frac{5}{8}$, $3\frac{5}{6}$, and $\frac{3}{4}$ to a common denominator.
Ans. $\frac{15}{24}$, $\frac{92}{24}$, $\frac{18}{24}$.

6. Reduce $6\frac{5}{12}$, $9\frac{1}{2}$, and $5\frac{7}{24}$ to a common denominator.
Ans. $\frac{154}{24}$, $\frac{228}{24}$, $\frac{127}{24}$.

7. Reduce $7\frac{5}{6}$, $\frac{4}{9}$, $\frac{1}{4}$, and $\frac{1}{9}$ to a common denominator.
Ans. $\frac{282}{36}$, $\frac{16}{36}$, $\frac{9}{36}$, $\frac{4}{36}$.

LEAST COMMON DENOMINATOR.

137. It is often necessary to reduce fractions to their *least common denominator.* For this purpose,

I. *Find the least common multiple of the denominators as in* ART. 126, *and it will be the least denominator sought.*

II. *Multiply the numerator of each fraction by the quotient which arises from dividing the common multiple by the denominator, and the products will be the numerators of the required fractions; under which write the least common denominator.*

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137. How do you reduce fractions to their least common denominator? Does this reduction affect the values of the fractions?

EXAMPLES.

1. Reduce $\frac{3}{7}$, $\frac{5}{8}$, and $\frac{2}{6}$ to their least common denominator.

OPERATION.

$$\begin{array}{l} 2)\underline{7\ ..\ 8\ ..\ 6} \\ \ \ \ \underline{7\ ..\ 4\ ..\ 3} \end{array} \quad \text{and } 3\times4\times7\times2=168$$

the least common denominator.

$$\frac{168}{7}\times3=24\times3=72 \text{ 1st numerator.}$$

$$\frac{168}{8}\times5=21\times5=105 \text{ 2d numerator.}$$

$$\frac{168}{6}\times2=28\times2=56 \text{ 3d numerator.}$$

Ans. $\frac{72}{168}$, $\frac{105}{168}$, and $\frac{56}{168}$.

2. Reduce $\frac{4}{5}$, $\frac{8}{9}$, and $\frac{3}{15}$ to their least common denominator. *Ans.* $\frac{36}{45}$, $\frac{40}{45}$, and $\frac{9}{45}$.

3. Reduce $14\frac{5}{4}$, $6\frac{3}{8}$, and $5\frac{1}{2}$ to their least common denominator. *Ans.* $\frac{122}{8}$, $\frac{51}{8}$, $\frac{44}{8}$.

4. Reduce $\frac{3}{15}$, $\frac{4}{24}$, and $\frac{8}{9}$ to their least common denominator. *Ans.* $\frac{72}{360}$, $\frac{60}{360}$, $\frac{320}{360}$.

5. Reduce $\frac{67}{120}$, $\frac{6}{40}$, $\frac{5}{2}$ to their least common denominator. *Ans.* $\frac{67}{120}$, $\frac{18}{120}$, $\frac{300}{120}$.

6. Reduce $\frac{41}{50}$, $3\frac{61}{20}$, $4\frac{1}{2}$ and 8 to a common denominator. *Ans.* $\frac{82}{100}$, $\frac{605}{100}$, $\frac{450}{100}$, $\frac{800}{100}$.

7. Reduce $3\frac{1}{8}$, $4\frac{4}{12}$, $8\frac{6}{18}$, $14\frac{7}{16}$ to their least common denominator. *Ans.* ——

8. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to fractions having the least common denominator. *Ans.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

9. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$, and $\frac{7}{10}$ to fractions having the least common denominator. *Ans.* $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{63}{90}$.

10. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$ to equivalent fractions having the least common denominator.

Ans. $\frac{16}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$, $\frac{33}{48}$, $\frac{34}{48}$.

EXAMPLES IN REDUCTION OF FRACTIONS.

1. Reduce $\frac{57}{456}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
2. Reduce $\frac{1429}{2858}$ to its lowest terms. *Ans.* $\frac{1}{2}$.
3. Reduce $\frac{3619}{6251}$ to its lowest terms. *Ans.* $\frac{11}{19}$
4. Reduce $\frac{468}{1184}$ to its lowest terms. *Ans.* $\frac{117}{296}$.
5. Reduce $\frac{1296}{3024}$ to its lowest terms. *Ans.* $\frac{3}{7}$.
6. Reduce $\frac{617}{4319}$ to its lowest terms. *Ans.* ——
7. Reduce $\frac{123}{456}$ to its lowest terms. *Ans.* $\frac{41}{152}$.
8. Reduce $45\frac{7}{8}$ to its equivalent improper fraction.
9. Reduce $16\frac{18}{100}$ to an improper fraction.
Ans. $\frac{1618}{100}$.
10. Reduce $149\frac{116}{137}$ to an improper fraction.
Ans. $\frac{20529}{137}$.
11. Reduce $\frac{12410}{19}$ to a whole or mixed number.
Ans. $653\frac{3}{19}$.
12. In $\frac{67856}{7}$ of pounds of sugar, how many pounds?
13. In $\frac{22085}{360}$ of *hhd.* of wine, how many *hhd.*?
Ans. $61\frac{125}{360}$*hhd.*
14. In $\frac{354}{18}$ bushels of wheat, how many bushels?
15. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{1}{8}$ of $\frac{7}{8}$ of $\frac{1}{3}$ to a simple fraction.
Ans. $\frac{7}{512}$.
16. In $\frac{6}{11}$ of $\frac{6}{7}$ of $\frac{1}{4}$ of 21 dollars, how many dollars?
Ans. $5\frac{5}{7}$.
17. Reduce $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{40}{1}$ to a simple fraction. *Ans.* $2\frac{5}{11}$.
18. I bought $\frac{8}{11}$ of $\frac{2}{3}$ of a ship: what part did I buy?
19. Sold $\frac{3}{5}$ of $\frac{19}{23}$ of 265 yards of cloth: how many did I sell? *Ans.* $131\frac{8}{23}$ *yards.*
20. In $\frac{7}{11}$ of $15\frac{7}{8}$ of $5\frac{7}{10}$ of 100 hogsheads of sugar, how many hogsheads? *Ans.* $5758\frac{13}{44}$.
21. Reduce $\frac{2}{9}$, 7, 8, and $5\frac{1}{7}$ to a common denominator.
Ans. $\frac{14}{63}$, $\frac{441}{63}$, $\frac{504}{63}$, $\frac{324}{63}$.
22. What is the least common denominator of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{6}$. $3\frac{4}{5}$? *Ans.* ——
23. What is the common denominator of $\frac{3}{4}$, $\frac{5}{6}$, and $12\frac{1}{3}$? *Ans.* $\frac{54}{72}$, $\frac{60}{72}$, $\frac{888}{72}$.

24. Reduce $\frac{2}{9}$, $\frac{3}{4}$, and $\frac{5}{8}$ of $\frac{11}{12}$ to a common denominator. *Ans.* ——

25. What is the common denominator of $\frac{1}{2}$ of $\frac{2}{3}$ of 2, $\frac{3}{4}$ of $\frac{4}{5}$ of $8\frac{1}{2}$, and $\frac{3}{5}$ of $9\frac{1}{3}$? *Ans.* $\frac{20}{30}$, $\frac{153}{30}$, $\frac{168}{30}$.

26. What is the common denominator of $\frac{1}{2}$ of 2 of $\frac{1}{3}$ of 3 of $\frac{1}{4}$ of 4, $\frac{1}{5}$ of 5 of $\frac{1}{6}$ of $6\frac{1}{2}$ of $\frac{1}{9}$ of 8, and $\frac{3}{5}$ of $2\frac{1}{2}$ of $3\frac{1}{2}$ of $\frac{1}{4}$? *Ans.* $\frac{432}{432}$, $\frac{416}{432}$, $\frac{567}{432}$.

REDUCTION OF DENOMINATE FRACTIONS.

138. We have seen (ART. **61**) that a denominate number is one in which the kind of unit is denominated or expressed. For a like reason, a denominate fraction is one which expresses the *kind of unit* that has been divided. Such unit is called the unit of the fraction. Thus, $\frac{2}{3}$ of a £ is a denominate fraction. It expresses that one £ is the unit which has been divided.

The fraction $\frac{3}{8}$ of a shilling is also a denominate fraction, in which the unit that has been divided is one shilling. These two fractions are of different denominations, the unit of the first being one pound, and that of the second, one shilling.

Fractions, therefore, are of the same denomination when they express parts of the same unit, and of different denominations when they express parts of different units.

REDUCTION *of denominate fractions consists in changing their denominations without altering their values.*

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138. What is a denominate number? What is a denominate fraction? What is the unit called? In two-thirds of a pound, what is the unit? In three-eighths of a shilling, what is the unit? In one-half of a foot, what is the unit? When are fractions of the same denomination? When of different denominations? Are one-third of a £ and one-fourth of a £ of the same or different denominations? One-fourth of a £ and one-sixth of a shilling? One-fifth of a shilling and one-half of a penny? What is reduction? How many shillings in a £? How many in £2? In 3? In 4? How many pence in 1*s.*? In 2? In 3? In 2*s.* 8*d.*? In 3*s.* 6*d.*? In 5*s.* 8*d.*? How many feet in 3 yards 2*ft.*? How many inches?

CASE I.

139. To reduce a denominate fraction from a lower to a higher denomination.

I. *Consider how many units of the given denomination make one unit of the next higher, and place* 1 *over that number forming a second fraction.*

II. *Then consider how many units of the second denomination make one unit of the denomination next higher, and place* 1 *over that number forming a third fraction; and so on, to the denomination to which you would reduce.*

III. *Connect all the fractions together, forming a compound fraction; then reduce it to a simple one by Case V.*

EXAMPLES.

1. Reduce $\frac{1}{3}$ of a penny to the fraction of a £.

OPERATION.

$\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$ = £$\frac{1}{720}$

The given fraction is $\frac{1}{3}$ of a penny. But one penny is equal to $\frac{1}{12}$ of a shilling: hence $\frac{1}{3}$ of a penny is equal to $\frac{1}{3}$ of $\frac{1}{12}$ of a shilling. But one shilling is equal to $\frac{1}{20}$ of a pound: hence $\frac{1}{3}$ of a penny is equal to $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a £ = £$\frac{1}{720}$. The reason of the rule is therefore evident.

2. Reduce $\frac{3}{8}$ of a barleycorn to the denomination of yards.

OPERATION.

$\frac{3}{8}$ of $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{3}$ = $\frac{3}{864}$ *yards.*

Since 3 barleycorns make an inch, we first place 1 over 3: then as 12 inches make a foot, we place 1 over 12, and as 3 feet make a yard, we next place 1 over 3.

3. Reduce $\frac{3}{4}$*oz.* avoirdupois to the denomination of tons. *Ans.* $\frac{3}{128000}$*T*

4. Reduce $\frac{5}{9}$ of a pint to the fraction of a hogshead. *Ans.* *hhd.*

5. Reduce $\frac{1}{3}$ of a farthing to the fraction of a £. *Ans.* £$\frac{1}{2880}$.

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139. How do you reduce a denominate fraction from a lower to a *higher* denomination? What is the first step? What the second? *What the third?*

6. Reduce $\frac{3}{8}$ of a gallon to the fraction of a hogshead.
Ans. $\frac{3}{504}$*hhd.*

7. Reduce $\frac{5}{8}$ of a shilling to the fraction of a £.
Ans. £$\frac{5}{160}$.

8. Reduce $\frac{187}{127}$ of a minute to the fraction of a day.
Ans. $\frac{187}{182880}$*da.*

9. Reduce $\frac{5}{8}$ of a pound to the fraction of a *cwt.*
Ans. *cwt.*

10. Reduce $\frac{1}{2}$ of an ounce to the fraction of a ton.
Ans. $\frac{1}{64000}$*T.*

CASE II.

140. To reduce a denominate fraction from a higher to a lower denomination.

I. *Consider how many units of the next lower denomination make one unit of the given denomination, and place* 1 *under that number forming a second fraction.*

II. *Then consider how many units of the denomination still lower make one unit of the second denomination, and place* 1 *under that number forming a third fraction, and so on, to the denomination to which you would reduce.*

III. *Connect all the fractions together, forming a compound fraction, and then reduce it to a simple one by Case V.*

EXAMPLES.

1. Reduce $\frac{1}{7}$ of a £ to the fraction of a penny.

OPERATION.
$\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}d.$

In this example $\frac{1}{7}$ of a pound is equal to $\frac{1}{7}$ of 20 shillings. But 1 shilling is equal to 12 pence: hence $\frac{1}{7}$ of a £$=\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}d.$ Hence, the reason of the rule is manifest.

2. Reduce $\frac{4}{7}$*cwt.* to the fraction of a pound.
Ans. $\frac{400}{7}$*lb.*

3. Reduce $\frac{2}{15}$ of a £ to the fraction of a penny.
Ans. $\frac{32}{1}$*d.*

140. What do you first do in reducing a denominate fraction to a lower denomination? What next? What next?

4. Reduce $\frac{1}{3}$ of a day to the fraction of a minute. *Ans.* 480*m.*

5. Reduce $\frac{3}{9}$ of an acre to the fraction of a pole. *Ans.* $\frac{480}{9}P.$

6. Reduce $\frac{6}{7}$ of a £ to the fraction of a farthing. *Ans.* $\frac{5760}{7}$*far.*

7. Reduce $\frac{3}{504}$ of a hogshead to the fraction of a gallon. *Ans.* $\frac{3}{8}$*gal.*

8. Reduce $\frac{4}{10}$ of a bushel to the fraction of a pint. *Ans.* $\frac{256}{10}$*pt.*

CASE III.

141. To find the value of a fraction in integers of a less denomination.

I. *Reduce the numerator to the next lower denomination, and then divide the result by the denominator.*

II. *If there be a remainder, reduce it to the denomination still less, and divide again by the denominator. Proceed in the same way to the lowest denomination. The several quotients, being connected together, will form the equivalent denominate number.*

EXAMPLES.

1. What is the value of $\frac{2}{3}$ of a £?

We first bring the pounds to shillings. This gives the fraction $\frac{40}{3}$ of shillings, which is equal to 13 shillings and 1 over. Reducing this to pence gives the fraction $\frac{12}{3}$ of pence, which is equal to 4 pence.

OPERATION.

```
  2
 20
3)40
 13s. . . . 1 Rem.
            12
          3)12
             4d.
Ans. 13s. 4d.
```

141. How much is one-half of a £? One-third of a shilling? One-half of a penny? How much is one-half of a *lb.* Avoirdupois? One-fourth of a ton? One-fourth of a *cwt.*? One-half of a quarter? One-fourth of a quarter? One-seventh of a quarter? One-fourteenth of a quarter? One twenty-eighth of a quarter? How do you find the value of a fraction in integers of a less denomination?

2. What is the value of $\frac{4}{5}$*lb*. Troy? *Ans*. 9*oz*. 12*pwt*.

3. What is the value of $\frac{5}{16}$ of a *cwt*.? *Ans*. 1*qr*. 6$\frac{1}{4}$*lb*.

4. What is the value of $\frac{5}{8}$ of an acre? *Ans*. 2*R*. 20*P*.

5. What is the value of $\frac{1}{6}$ of a £? *Ans*. ——

6. What is the value of $\frac{5}{6}$ of a hogshead?
Ans. 52*gal*. 2*qt*.

7. What is the value of $\frac{126}{504}$ of a hogshead?
Ans. *gal*. *qt*.

8. What is the value of $\frac{2}{9}$ of a guinea? *Ans*. 4*s*. 8*d*.

9. What is the value of $\frac{3}{5}$ of a *lb*. Troy?
Ans. *oz*. *pwt*.

10. What is the value of $\frac{7}{8}$ of a tun of wine?
Ans. 3*hhd*. 31*gal*. 2*qt*.

CASE IV.

142. To reduce a compound denominate number to a fraction of a given denomination.

Reduce the given number to the lowest denomination mentioned in it: then if the reduction is to be made to a denomination still less, reduce as in Case II.; but if to a higher denomination reduce as in Case I.

EXAMPLES.

1. Reduce 4*s*. 7*d*. to the fraction of a £.

OPERATION.

4*s*. 7*d*. = 55*d*.

Then, 55 of $\frac{1}{12}$ of $\frac{1}{20}$ = £$\frac{55}{240}$.

Ans. £$\frac{55}{240}$.

We first reduce the given number to the lowest denomination named in it, viz., pence. Then as the reduction is to be made to pounds, a higher denomination, we reduce by Case I.

2. What part of a pint is 2*pk*. 3*qt*.?

OPERATION.

2*pk*. 3*qt*. = 19*qt*.

19 of $\frac{2}{1}$ = 38 *pints*.

We first reduce to quarts, this being the lowest denomination named in it. We then reduce to the denomination of pints by Case II.

142. How do you reduce a compound denominate number to a fraction of a given denomination?

3. Reduce 2 feet 2 inches to the fraction of a yard.
Ans. $\frac{13}{18}$*yd*

4. Reduce 3 gallons 2 quarts to the fraction of a hogshead. *Ans.* $\frac{1}{18}$*hhd.*

5. Reduce 1*qr.* 7*lb.* to the fraction of a dram.
Ans.

6. What part of a hogshead is 3*qt.* 1*pt.* ? *Ans.* $\frac{1}{72}$.

7. What part of a mile is 6*ft.* 7*in.* ? *Ans.* $\frac{79}{63360}$.

8. What part of a mile is 1 inch ? *Ans.* $\frac{1}{63360}$.

9. What part of a month of 30 days, is 1 hour 1 minute 1 second ? *Ans.* ——

10. What part of 1 day is 3*hr.* 3*m.* ? *Ans.* $\frac{183}{1440}$.

11. What part is 3*hr.* 3*m.* of two days ? Of 3 ? Of 4 ? Of 10 ? Of 25 ?

EXAMPLES IN REDUCTION.

1. Reduce $\frac{8}{9}$ of a pound to the fraction of a *cwt.*
Ans. $\frac{2}{225}$*cwt.*

2. If you study Arithmetic $\frac{1}{224}$ part of an hour, what part is it of a week ? *Ans.* $\frac{1}{37632}$.

3. Bought $\frac{3}{4}$ of a pint of claret : what part of a hogshead ? *Ans.* $\frac{1}{672}$.

4. What part of a mile is $5\frac{1}{2}$ furlongs ? *Ans.* ——

5. Bought $\frac{3}{4}$*lb.* of cloves : what part of a ton ?
Ans. $\frac{3}{8000}$.

6. If a fly steps $\frac{1}{2}$ of a barleycorn, what part is it of a league ? *Ans.* $\frac{1}{1140480}$.

7. If a stone covers $\frac{3}{4}$ of a square inch of land, what part of an acre does it occupy ? *Ans.* $\frac{1}{8363520}$.

8. Bought $\frac{1}{2}$ of 3 pounds of raisins : what part of a *cwt.* ? *Ans.* ——

9. What part of a barrel is $\frac{1}{3}$ of $5\frac{1}{3}$ of $6\frac{1}{4}$ of a pint ?
Ans. $\frac{25}{567}$.

10. What part of a year is $\frac{1}{2}$ of $\frac{1}{6}$ of $2\frac{1}{2}$ of $3\frac{1}{3}$ of an hour ? Ans. $\frac{25}{315360}$.

11. What is $\frac{1}{2}$ of $2\frac{1}{3}$ of 400 bushels of wheat?
Ans. $466\frac{2}{3}$ *bushels.*

12. Reduce $\frac{7}{9}$ of a *cwt.* of sugar to the lower denominations. *Ans.* 3*qr.* 2*lb.* 12*oz.* $7\frac{1}{9}$*dr.*

13. I bought $\frac{5}{7}$ of a *hhd.* of wine: how many gallons did I buy? *Ans.* 45.

14. Reduce $\frac{197}{640}$ of a pound of laudanum to the lower denominations. *Ans.* 2℥ 3*gr.*

15. What is the value of $\frac{7}{13}$ of an acre? *Ans.* ——

16. A goldsmith received $\frac{8}{7}$ of a pound of gold: what is the value? *Ans.* 13*oz.* 14*pwt.* $6\frac{6}{7}$*gr.*

17. What is the value of $\frac{3}{8}$ of a chaldron of coal?

18. What is the value of $\frac{9}{10}$ of a yard?
Ans. 2*ft.* 8*in.* $1\frac{1}{5}$*bar.*

19. A man travelled $\frac{4}{7}$ of a mile: how many furlongs?
Ans. ——

20. Reduce $\frac{7}{13}$ of a day to the lower denominations.
Ans. 12*hr.* 55*m.* $23\frac{1}{3}$*sec.*

21. What is the value in grains of $\frac{1}{4800}$ pounds Troy?
Ans. $1\frac{1}{5}$*gr.*

22. What part of an inch is $\frac{1}{450}$ of an Ell English?

23. What part of a quart is $\frac{1}{576}$ of a tun? *Ans.* $1\frac{1}{5}$*qt.*

24. What is the value in gills of $\frac{1}{3}$ of $1\frac{1}{4}$ of $2\frac{2}{5}$ of a *hhd.*? *Ans.* 2016*gi.*

25. What part of a ton is 13*cwt.* 3*qr.* 20*lb.*?
Ans. $\frac{279}{400}$*T.*

26. What part of 4*cwt.* 1*qr.* 24*lb.* is 3*cwt.* 3*qr.* 17*lb.*?

27. What part of a pound Troy is 10*oz.* 13*pwt.* 8*gr.*?
Ans. $\frac{8}{9}$.

28. What part of a cord is 19*ft.* $1196\frac{4}{13}$*in.*?

29. What part of a mile is 13*fur.* 21*rd.* 18*ft.* 10*in.* $1\frac{11}{13}$*bar.*? *Ans.* $1\frac{4765}{6864}$*mi.*

30. What part of a year is 147*da.* 15*hr.*?
Ans. $\frac{1181}{2922}$.

31. What part of a hogshead is 27*gal.* 3*qt.* 1*pt.* of beer?
Ans. $\frac{223}{432}$.

ADDITION OF VULGAR FRACTIONS.

143. Addition of integer numbers teaches how to express all the units of several numbers by a single number.

Addition of fractions teaches how to express the value of several fractions by a single fraction.

It is plain, that we cannot add fractions so long as they have different units: for, $\frac{1}{2}$ of a £ and $\frac{1}{2}$ of a shilling make neither £1 nor 1 shilling.

Neither can we add parts of the same unit unless they are like parts; for $\frac{1}{3}$ of a £ and $\frac{1}{4}$ of a £ make neither $\frac{2}{3}$ of a £ nor $\frac{2}{4}$ of a £. But $\frac{1}{3}$ of a £ and $\frac{1}{3}$ of a £ may be added: they make $\frac{2}{3}$ of a £. So, $\frac{1}{4}$ of a £ and $\frac{2}{4}$ of a £ make $\frac{3}{4}$ of a £.

Hence, before fractions can be added, two things are necessary.

1st. *That the fractions be reduced to the same denomination.*

2d. *That they be reduced to a common denominator.*

CASE I.

144. When the fractions to be added are of the same denomination and have a common denominator.

Add the numerators together, and place their sum over the common denominator: then reduce the fraction to its lowest terms, or to its equivalent mixed number.

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143. What does addition of integer numbers teach? What does addition of fractions teach? What two things are necessary before fractions can be added? Can one-half of a £ be added to one-half of a shilling without reduction? Can one-half be added to one-fourth without reduction?

144. When the fractions are of the same denomination and have a common denominator, how do you find their sum? What is the sum of one-third and two-thirds? Of three-fourths, one-fourth, and four-fourths? Of three-fifths, six-fifths, and two-fifths? Of three-sixths, seven-sixths, and nine-sixths? Of one-eighth, three-eighths, and four-eighths?

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{3}{2}$, $\frac{6}{2}$, and $\frac{3}{2}$ together.

It is evident, since all the parts are halves, that the true sum will be expressed by the number of halves divided by 2: that is, by thirteen two's.

OPERATION.
$1+3+6+3=13$
Hence, $\frac{13}{2}$=sum.

2. Add $\frac{1}{7}$ of a £, $\frac{5}{7}$ of a £, and $\frac{9}{7}$ of a £ together.
Ans. $\frac{15}{7}$ of a £=£$2\frac{1}{7}$.

3. What is the sum of $\frac{3}{9}+\frac{4}{9}+\frac{6}{9}+\frac{13}{9}+\frac{16}{9}$?
Ans. $\frac{42}{9}=\frac{14}{3}$.

4. What is the sum of $\frac{3}{14}+\frac{8}{14}+\frac{9}{14}+\frac{5}{14}+\frac{3}{14}$.
Ans. 2.

CASE II.

145. When the fractions are of the same denomination but have different denominators.

Reduce compound and complex fractions to simple ones, mixed numbers to improper fractions, and all the fractions to a common denominator. Then add them as in Case I.

EXAMPLES.

1. Add $\frac{6}{2}$, $\frac{4}{3}$, and $\frac{2}{5}$ together.

By reducing to a common denominator, the new fractions are

$\frac{90}{30}+\frac{40}{30}+\frac{12}{30}=\frac{142}{30}$,

which, by reducing to the lowest terms becomes $4\frac{11}{15}$.

OPERATION.
$6\times3\times5=90$ 1st numerator.
$4\times2\times5=40$ 2d numerator.
$2\times3\times2=12$ 3d numerator.
$2\times3\times5=30$ the denominator.

2. Add $\frac{1}{7}$ of a £, $\frac{2}{9}$ of a £, and $\frac{5}{6}$ of a £ together.
Ans. £$\frac{453}{378}$=£$1\frac{75}{378}$=£$1\frac{25}{126}$.

3. Add together $\frac{1}{7}$, $\frac{1}{9}$, $4\frac{1}{3}$, and $6\frac{1}{5}$. *Ans.* $10\frac{248}{315}$.

4. Find the least common denominator (see ART. 137), and add the fractions $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$. *Ans.* ——

145. How do you add fractions which have different denominators? How do you reduce fractions of different denominators to equivalent fractions *having a common* denominator?

5. Find the least common denominator and add $\frac{6}{12}$, $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{6}{30}$. *Ans.* $1\frac{4}{5}$.

146. When there are mixed numbers, instead of reducing them to improper fractions we may add the whole numbers and the fractional parts separately, and then add their sums.

1. Add $19\frac{1}{7}$, $6\frac{2}{3}$, and $4\frac{4}{5}$ together.

OPERATION.

Whole numbers.	Fractional parts.
$19+6+4=29$.	$\frac{1}{7}+\frac{2}{3}+\frac{4}{5}=\frac{169}{105}=1\frac{64}{103}$.

Hence, $29+1\frac{64}{105}=30\frac{64}{105}$, the sum.

2. Add $3\frac{1}{4}$, $6\frac{5}{7}$, $8\frac{9}{15}$, and $65\frac{9}{8}$. *Ans.* $84\frac{193}{280}$.

CASE III.

147. When the fractions are of different denominations.

Reduce the fractions to the same denomination. Then reduce all the fractions to a common denominator, after which add them as in Case I.

EXAMPLES.

1. Add $\frac{2}{3}$ of a £ to $\frac{5}{6}$ of a shilling.

$\frac{2}{3}$ of a £ $=\frac{2}{3}$ of $\frac{20}{1}=\frac{40}{3}$ of a shilling.

Then, $\frac{40}{3}+\frac{5}{6}=\frac{240}{18}+\frac{15}{18}=\frac{255}{18}s.=\frac{85}{6}s.=14s.\ 2d.$

Or, the $\frac{5}{6}$ of a shilling might have been reduced to the fraction of a £ thus,

$\frac{5}{6}$ of $\frac{1}{20}=\frac{5}{120}$ of a £ $=\frac{1}{24}$ of a £.

Then, $\frac{2}{3}+\frac{1}{24}=\frac{48}{72}+\frac{3}{72}=\frac{51}{72}$ of a £: which being reduced, gives 14*s.* 2*d.* *Ans.* 14*s.* 2*d.*

2. Add $\frac{3}{8}$ of a yard to $\frac{5}{9}$ of an inch.
Ans. $\frac{253}{648}$*yd.* or $14\frac{1}{18}$*in.*

3. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. *Ans.* *da.* *hr.*

4. Add $\frac{4}{7}$ of a *cwt.*, $8\frac{5}{6}$*lb.*, and $3\frac{9}{10}$*oz.* together.
Ans. 2*qr.* 23*lb.* $1\frac{7}{30}$*oz.*

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146. How do you add when there are mixed numbers?
147. How do you add fractions of different denominations?

5. Add $1\frac{1}{4}$ miles, $\frac{7}{10}$ furlongs, and 30 rods together.
Ans. 1*m.* 3*fur.* 13*rd.*

148. The value of each of the fractions may be found separately, and their several values then added.

EXAMPLES.

1. Add $\frac{3}{5}$ of a year, $\frac{1}{3}$ of a week, and $\frac{1}{8}$ of a day together.

$\frac{3}{5}$ of a year $=\frac{3}{5}$ of $\frac{365}{1}$ days $=219$ days.
$\frac{1}{3}$ of a week $=\frac{1}{3}$ of 7 days $=$ 2 days 8 hours.
$\frac{1}{8}$ of a day $=$ - - - - - 3 hours.
Ans. 221*da.* 11*hr.*

2. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile together. *Ans.* *yd.* *ft.* *in.*

3. Add $\frac{3}{4}$ of a *cwt.*, $\frac{42}{2}$ of a *lb.*, 13*oz.*, and $\frac{1}{2}$ of a *cwt.* 6*lb.* together. *Ans.* 1*cwt.* 2*qr.* 2*lb.* 13*oz.*

EXAMPLES IN ADDITION.

1. I bought $22\frac{6}{7}$ bushels of wheat at one time, $19\frac{5}{12}$ at ancther, and $33\frac{5}{9}$ at another: how much did I buy in all?
Ans. $75\frac{209}{252}$*bu.*

2. What is the sum of $26\frac{2}{3}$, $18\frac{7}{8}$, $19\frac{3}{5}$, $13\frac{1}{4}$, and $\frac{117}{8}$?

3. A farmer owns three farms; the first contains $471\frac{11}{12}$ acres, the second $714\frac{19}{20}$, and the third $181\frac{3}{4}$: how many acres in all? *Ans.* $1368\frac{37}{60}$ *acres.*

4. Bought $\frac{1}{2}$ of $3\frac{1}{2}$ of 5*cwt.* of sugar at one time; at another, $\frac{1}{3}$ of $5\frac{1}{3}$ of 6*cwt.*; at another, $\frac{1}{5}$ of $\frac{6}{7}$ of 8*cwt.*: how much did I buy in all? *Ans.* $20\frac{331}{420}$*cwt.*

5. What is the value of $\frac{4}{7}$ of a ton, and $\frac{9}{10}$ of a *cwt.*?
Ans. 12*cwt.* 1*qr.* 7*lb.* $13\frac{5}{7}$*oz.*

6. How far is $\frac{3}{4}$ of a mile and $\frac{7}{10}$ of a furlong?

7. A man travelled $28\frac{3}{7}$ miles the first day, $33\frac{1}{14}$ the second day, and $29\frac{1}{21}$ miles the third day: how far did he travel in all? *Ans.* 90*mi.* 4*fur.* 15*rd.* 3*ft.* $11\frac{1}{7}$*in.*

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148. How may fractions of different denominations be added by the second method?

8. Add $5\frac{5}{8}$ days and $52\frac{5}{19}$ minutes together, and give the whole time. *Ans.* 5*da.* 20*hr.* 52*m.* $15\frac{15}{19}$*sec.*

9. What is the entire weight of $\frac{4}{7}$*cwt.* $8\frac{5}{8}$*lb.* and $3\frac{9}{10}$*oz.*? *Ans.* 2*qr.* 16*lb.* 3*oz.* $8\frac{32}{105}$*dr.*

10. Find the value of $\frac{1}{3}$ of a year, $\frac{1}{3}$ of a day, $\frac{1}{7}$ of $\frac{7}{3}$ of an hour, and $\frac{1}{3}$ of $\dfrac{2\frac{1}{2}}{2\frac{1}{3}}$ of a minute. *Ans.* ——

11. Bought 3 pieces of cloth; the first contained $\frac{1}{2}$ of 3 of $\frac{6}{7}$ of $\frac{2}{6}$ yards; the second $\frac{1}{4}$ of $\frac{4}{5}$ of 5; and the third $\frac{1}{6}$ of $\frac{3}{2}$ of $\frac{1}{3}$: what did they all contain?

Ans. 2*yd.* 2*qr.* $1\frac{1}{3}$*na.*

12. Add together $\frac{3}{7}$, $\frac{3}{5}$, 13, and $18\frac{3}{15}$. *Ans.* $31\frac{101}{105}$.

13. Add together $\frac{4}{16}$, $\frac{12}{7}$, 1, and $\frac{16}{9}$. *Ans.* $4\frac{187}{252}$.

14. Add together $38\frac{2}{7}$, $13\frac{1}{3}$, and $9\frac{3}{5}$. *Ans.* ——

15. Add together $6\frac{2}{4}$, $13\frac{3}{7}$, $17\frac{3}{15}$, and $132\frac{2}{8}$.

Ans. $169\frac{53}{140}$.

16. Add together $\frac{2}{3}$ of a week, $\frac{1}{2}$ of a day, and one hour. *Ans.* ——

17. Add together $1\frac{1}{7}$*cwt.* $17\frac{3}{5}$*lb.* and $7\frac{4}{5}$*oz.*

Ans. 1*cwt.* 1*qr.* 7*lb.* $7\frac{4}{105}$*oz.*

SUBTRACTION OF VULGAR FRACTIONS.

149. It has been shown that before fractions can be added together, they must be reduced to the same unit and to a common denominator. The same reductions must be made before subtraction.

SUBTRACTION *of Vulgar Fractions is the process of finding the difference between two fractions.*

CASE I.

150. When the fractions are of the same denomination and have a common denominator.

149. Can one-third of a shilling be subtracted from one-third of a £ without reduction? Can one-sixth of a shilling be subtracted from one-fifth of a shilling? What reductions are necessary before subtraction? What is subtraction?

150. How do you subtract fractions which have a common unit and the same denominator?

Subtract the less numerator from the greater and place the difference over the common denominator.

EXAMPLES.

1. What is the difference between $\frac{5}{8}$ and $\frac{3}{8}$?

Here we have $5-3=2$: hence, $\frac{2}{8}$=the difference.

2. What is the difference between $\frac{125}{365}$ and $\frac{67}{365}$. *Ans.* $\frac{58}{365}$.

3. From $\frac{335}{105}$ take $\frac{169}{105}$. *Ans.* $\frac{166}{105}$.

4. From $\frac{4978}{9765}$ take $\frac{1697}{9765}$. *Ans.* ——

5. From $\frac{18906}{327}$ take $\frac{909}{327}$. *Ans.* $\frac{17997}{327}$.

CASE II.

151. When the fractions are of the same denomination, but have different denominators.

Reduce mixed numbers to improper fractions, compound and complex fractions to simple ones, and all the fractions to a common denominator: then subtract them as in Case I.

EXAMPLES.

1. What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$?

Here, $\frac{5}{6}-\frac{1}{3}=\frac{5}{6}-\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$ answer.

2. What is the difference between $12\frac{1}{2}$ of $\frac{1}{6}$ and 2? *Ans.* $\frac{1}{12}$.

3. What is the difference between $2\frac{1}{2}$ of a £, and $\frac{3}{15}$ of a £? *Ans.* ——

4. From $\frac{1}{8}$ of 6, take $\frac{16}{17}$ of $\frac{1}{2}$. *Ans.* $\frac{19}{68}$.

5. From $\frac{1}{7}$ of $\frac{3}{8}$ of 7, take $\frac{3}{8}$ of $\frac{5}{4}$. *Ans.* ——

6. From $37\frac{11}{15}$, take $3\frac{5}{7}$ of $\frac{1}{3}$. *Ans.* $36\frac{52}{105}$.

CASE III.

152. When the fractions are of different denominations.

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151. How do you subtract fractions which have the same unit but different denominators? What is the difference between one-half and one-third?

152. How do you subtract fractions which are of different denominations?

Reduce the fractions to the same denomination: then reduce them to a common denominator, after which subtract as in Case I.

EXAMPLES.

1. What is the difference between $\frac{1}{2}$ of a £ and $\frac{1}{3}$ of a shilling?

$\frac{1}{3}$ of a shilling $=\frac{1}{3}$ of $\frac{1}{20}=\frac{1}{60}$ of a £.

Then, $\frac{1}{2}-\frac{1}{60}=\frac{30}{60}-\frac{1}{60}=\frac{29}{60}$ of a £ $=9s.\ 8d.$

2. What is the difference between $\frac{1}{2}$ of a day and $\frac{2}{3}$ of a second? *Ans.* 11*hr.* 59*m.* 59$\frac{1}{3}$*sec.*

3. What is the difference between $\frac{6}{9}$ of a rod and $\frac{3}{7}$ of an inch? *Ans.* 10*ft.* 11$\frac{4}{7}$*in.*

4. From 1$\frac{3}{4}$ of a *lb.*, Troy weight, take $\frac{1}{6}$ of an ounce.

5. What is the difference between $\frac{4}{15}$ of a hogshead, and $\frac{6}{19}$ of a quart? *Ans.* 16*gal.* 2*qt.* 1*pt.* 3$\frac{7}{95}$*gi.*

6. From $\frac{1}{2}$ of a £ take $\frac{3}{4}$ of a shilling. *Ans.* ——

7. From $\frac{3}{5}$*oz.* take $\frac{7}{8}$*pwt.* *Ans.* 11*pwt.* 3*gr.*

8. From 4$\frac{3}{7}$*cwt.* take 4$\frac{9}{10}$*lb.*
Ans. 4*cwt.* 1*qr.* 12*lb.* 15*oz.* 5$\frac{1}{35}$*dr.*

GENERAL EXAMPLES.

1. From $\frac{3}{8}$ of an ounce take $\frac{7}{8}$ of a *pwt.*
Ans. 6*pwt.* 15*gr.*

2. Take $\frac{4}{7}$ of a day and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of an hour from 3$\frac{2}{3}$ weeks. *Ans.* ——

3. A man engaged to work 41 days, but was absent by indisposition 6$\frac{9}{16}$ days: how many days did he work?
Ans. 34$\frac{7}{16}$ *days.*

4. What remains of a hogshead of vinegar if $\frac{1}{5}$ of it has leaked out? *Ans.* ——

5. A man has travelled 4*mi.* 1*fur.* 24*rd.*: how much farther must he go in order to make 6$\frac{1}{2}$ miles?
Ans. 2*mi.* 2*fur.* 16*rd.*

6. From 1 take $\frac{67}{100}$. *Ans.* ——

7. From 9 take 1$\frac{4}{5}$. *Ans.* 7$\frac{1}{5}$.

8. From $\frac{5}{7}$ of a degree take $\frac{3}{8}$ of a mile. *Ans.* ——

9. Take $\frac{1}{2}$ a square foot from $\frac{4}{11}$ of an acre.
Ans. 1*R.* 18*P.* 5*yd.* 4*ft.*

10. A man sold $\frac{15}{34}$ of a house to one man, $\frac{6}{34}$ to another, and $\frac{2}{17}$ to another: what part did he still own?

11. One man bought $\frac{1}{9}$ of $\frac{4\frac{1}{2}}{4\frac{1}{4}}$*cwt.* of iron, another $\frac{1}{9}$ of $\frac{5\frac{1}{2}}{5\frac{1}{5}}$*cwt.*: how much did one buy more than the other?
Ans. $3\frac{1201}{1989}$ *drams.*

12. From $\frac{1}{8}$ of 12, take $\frac{16}{17}$ of $\frac{1}{2}$. *Ans.* ——

13. From $\frac{3}{7}$ of $1\frac{1}{2}$ of 7, take $\frac{5}{4}$ of $\frac{3}{8}$. *Ans.* $4\frac{1}{32}$.

14. From $75\frac{7}{15}$, take $\frac{1}{3}$ of $3\frac{5}{7}$. *Ans.* ——

15. From $1\frac{1}{2}$ of a £ take $\frac{3}{4}$ of a shilling.
Ans. £1 9*s.* 3*d.*

16. From $1\frac{1}{5}$*oz.* take $\frac{7}{8}$*pwt.* *Ans.* ——

17. From $8\frac{6}{7}$*cwt.* take $4\frac{9}{10}$*lb.*
Ans. 8*cwt.* 3*qr.* 5*lb.* 13*oz.* $\cdot 0\frac{16}{35}$*dr.*

18. From $3\frac{1}{2}$*lb.*, Troy weight, take $\frac{1}{6}$*oz.* *Ans.* ——

19. What is the difference between $1\frac{1}{3}$ rods and $\frac{3}{7}$ of an inch? *Ans.* 21*ft.* $11\frac{4}{7}$*in.*

20. From $\frac{\frac{46}{49}}{\frac{37}{41}}$ take $\frac{\frac{21}{27}}{4\frac{2}{3}}$. *Ans.* ——

21. From $\frac{39}{57}$℔ take $\frac{25}{31}$℥. *Ans.* ——

MULTIPLICATION OF VULGAR FRACTIONS.

153. John gave $\frac{1}{3}$ of a cent for an apple. How much must he give for 2 apples? For 3 apples? For 4? For 5? For 6? For 7? For 8? For 9?

Charles gave $\frac{2}{5}$ of a cent for a peach. How much must he give for 2 peaches? For 3? For 4? For 5? For 6?

CASE I.

154. To multiply a fraction by a whole number.

Multiply the numerator, or divide the denominator by the whole number.

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154. **How do you multiply a fraction by a whole number?**

EXAMPLES ILLUSTRATING PRINCIPLES.

1. Multiply the fraction $\frac{5}{8}$ by 4.

When we propose to multiply a fraction by a whole number, it is required to take the fraction as many times as there are units in the multiplier, which may be done by multiplying the numerator (ART. 118), or by dividing the denominator (ART. 121).

OPERATION.

$$\frac{5}{8}\times 4=\frac{20}{8}=\frac{5}{2}=2\frac{1}{2};$$

or by dividing the denominator by 4, we have

$$\frac{5}{8}\times 4=\frac{5}{4)8}=\frac{5}{2}=2\frac{1}{2}$$

2. Multiply $\frac{37}{144}$ by 12. *Ans.* $3\frac{1}{12}$.
3. Multiply $\frac{47}{49}$ by 7. *Ans.* $6\frac{5}{7}$.
4. Multiply $\frac{175}{47}$ by 9. *Ans.* $\frac{1575}{47}$.
5. Multiply $\frac{127}{15}$ by 5. *Ans.* ——
6. Multiply $\frac{369}{145}$ by 49. *Ans.* $124\frac{101}{145}$.

155. When we multiply by a fraction it is required to take the multiplicand as many times as there are *units* in the fraction.

For example, to multiply 8 by $\frac{3}{4}$ is to take 8, $\frac{3}{4}$ times; that is, to take $\frac{3}{4}$ of 8, which is 6.

Hence, *when the multiplier is less than* 1 *we do not take the whole of the multiplicand, but only such a part of it as the fraction is of unity.* For example, if the multiplier be one-half of unity, the product will be half the multiplicand: if the multiplier be $\frac{1}{3}$ of unity, the product will be one-third of the multiplicand. Hence, *to multiply by a proper fraction does not imply increase, as in the multiplication of whole numbers. The product will always be such a part of the multiplicand as the multiplier is of unity.*

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155. What is required when we multiply by a fraction? What is the product of 8 multiplied by one-half? By one-fourth? By one-eighth? By three-halves? By six-halves? What is the product of 9 multiplied by one-half? By one-third? By one-sixth? By one-ninth? When the multiplier is less than 1, how much of the multiplicand is taken? Does the multiplicand by a proper fraction imply increase? What part of the multiplicand is the product?

CASE II.

156. To multiply one fraction by another.

Reduce all the mixed numbers to improper fractions, and all compound fractions to simple ones: then multiply the numerators together for a numerator, and the denominators together for a denominator.

EXAMPLES ILLUSTRATING PRINCIPLES.

1. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

OPERATION.

$$\frac{3}{4}\times\frac{5}{7}=\frac{3}{4}\times 5\times\frac{1}{7}=\frac{15}{28}.$$

In this example $\frac{3}{4}$ is to be taken $\frac{5}{7}$ times. That is, $\frac{3}{4}$ is first to be multiplied by 5 and the product divided by 7, a result which is obtained by multiplying the numerators and denominators together.

2. Multiply $\frac{1}{6}$ of $\frac{3}{7}$ by $8\frac{1}{3}$.

OPERATION.

$$\frac{1}{6} \text{ of } \frac{3}{7}=\frac{3}{42},$$
$$8\frac{1}{3}=\frac{25}{3}:$$
$$\text{Hence, } \frac{3}{42}\times\frac{25}{3}=\frac{75}{126}=\frac{25}{42}.$$
Ans. $\frac{25}{42}$.

We first reduce the compound fraction to the simple one $\frac{3}{42}$, and then the mixed number to the equivalent fraction $\frac{25}{3}$; after which, we multiply the numerators and denominators together.

3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. *Ans.* $\frac{21}{24}=\frac{7}{8}$.
4. Multiply $\frac{12}{10}$ by $\frac{3}{4}$ of 9. *Ans.* $8\frac{1}{10}$.
5. Multiply $\frac{1}{8}$ of 3 of $\frac{1}{6}$ by $15\frac{1}{9}$. *Ans.* $\frac{17}{18}$.
6. Multiply $\frac{5}{6}$ by $\frac{2}{3}$ of $\frac{6}{7}$. *Ans.* ——
7. Required the product of 6 by $\frac{2}{3}$ of 5. *Ans.* 20.
8. Required the product of $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$. *Ans.* $\frac{23}{84}$.
9. Required the product of $3\frac{2}{7}$ by $4\frac{14}{33}$. *Ans.* $14\frac{124}{231}$.
10. Required the product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$, and $4\frac{1}{6}$.
11. Required the product of $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$, and $18\frac{4}{5}$. *Ans.* $9\frac{9}{140}$.
12. Required the product of 14, $\frac{5}{6}$, $\frac{4}{5}$ of 9, and $6\frac{3}{7}$.

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156. What is the product of one-sixth by one-seventh? Of three-fourths by one-half? Of six-ninths by three-fifths? Give the general rule for the multiplication of fractions.

157. In multiplying by a mixed number, we may first multiply by the integer, then multiply by the fraction, and then add the two products together. This is the best method when the numerator of the fraction is 1.

EXAMPLES.

1. Multiply 26 by $3\frac{1}{2}$.

We first multiply 26 by 3: the product is 78. Afterwards we multiply 26 by $\frac{1}{2}$: the product is 13: hence the entire product is 91.

OPERATION.

$$\begin{array}{r} 26 \\ 3 \\ \hline 78 \\ 26\times\frac{1}{2}=13 \\ \hline 91 \end{array}$$ *Ans.*

2. Multiply 48 by $8\frac{1}{6}$.

We first multiply by 8, and then add a sixth.

OPERATION.

$$\begin{array}{r} 48\times8=384 \\ 48\times\frac{1}{6}=\ \ 8 \\ \hline \textit{Ans.}\ 392 \end{array}$$

3. Multiply 67 by $9\frac{1}{12}$. *Ans.* $608\frac{7}{12}$.
4. Multiply 842 by $7\frac{1}{8}$. *Ans.* $5987\frac{5}{8}$.
5. Multiply 3756 by $3\frac{1}{5}$. *Ans.* $12019\frac{1}{5}$.
6. Multiply 2056 by $5\frac{1}{6}$. *Ans.* ——

APPLICATIONS.

1. What will 7 yards of cloth cost at $\$\frac{3}{4}$ per yard? *Ans.* $\$5\frac{1}{4}$.
2. What will 32 gallons of brandy cost at $\$1\frac{1}{8}$ per gallon? *Ans.* \$36.
3. If 1*lb.* of tea cost $\$1\frac{1}{4}$, what will $6\frac{1}{4}$*lb.* cost?
4. What will be the cost of $17\frac{1}{2}$ yards of cambric at $2\frac{1}{2}$ shillings per yard? *Ans.* £2 3*s.* 9*d.*
5. What will $15\frac{1}{16}$ barrels of cider come to at \$3 per barrel? *Ans.* $\$45\frac{3}{16}$.
6. What will $3\frac{3}{8}$ boxes of raisins cost at $\$2\frac{1}{2}$ per box? *Ans.* $\$8\frac{7}{16}$.

157. How may you multiply by a mixed number? When is this best method?

7. What will $15\frac{1}{2}$ barrels of sugar cost at $17\frac{1}{4}$ dollars per barrel? *Ans.* ——

8. What will $3\frac{3}{4}$ cords of wood cost at $3\frac{3}{4}$ per cord? *Ans.* $14\frac{1}{16}$.

9. Multiply $\frac{4}{5}$ bushels by $\frac{2}{3}$ of 7. *Ans.* $3\frac{11}{15}$*bu.*

10. A man bought $\frac{3}{5}$ of $\frac{3}{5}$ of a ship: what part did he buy? *Ans.* $\frac{9}{25}$.

11. How much is $\frac{1}{6}$ of $2\frac{1}{2}$ times 8 dollars? *Ans.* $3\frac{1}{3}$.

12. How far will a man travel in $17\frac{3}{11}$ hours if he goes at the rate of $9\frac{5}{9}$ miles an hour? *Ans.* ——

13. How many miles are $\frac{9}{10}$ of 7 miles, multiplied by $\frac{11}{15}$ of $87\frac{3}{11}$? *Ans.* $403\frac{1}{5}$*mi.*

14. What will $29\frac{3}{7}$ tons of gravel cost at $1\frac{1}{8}$ dollar a ton? *Ans.* ——

15. I own $\frac{6}{7}$ of a ship, and sell $\frac{1}{3}$ of $\frac{1}{9}$ of my share: what part is it of the whole? *Ans.* $\frac{2}{63}$.

16. What will $23\frac{7}{8}$ pounds of lead cost at $\frac{2}{25}$ dollar a pound? *Ans.* $1\frac{91}{100}$.

17. What will $\frac{7}{8}$ cords of wood cost at $5\frac{3}{4}$ dollars a cord? *Ans.* $5\frac{1}{32}$.

18. A merchant sold $37\frac{11}{63}$ hogsheads of vinegar for $17\frac{5}{8}$ dollars a hogshead: what did it come to?

19. Sold $\frac{3}{4}$ of $9\frac{1}{4}$*cwt.* of sugar for $\frac{2}{3}$ of 17 dollars a *cwt.*: what did it come to? *Ans.* $78\frac{5}{8}$.

20. Sold $\frac{2}{3}$ of $\frac{3}{4}$ of $26\frac{1}{3}$*lb.* of rice for $\frac{1}{4}$ of $2\frac{1}{2}$ of $10\frac{2}{5}$ cents a pound: what did it come to? *Ans.* $83\frac{11}{16}$.

DIVISION OF VULGAR FRACTIONS.

158. We have seen that division of entire numbers explains the manner of finding how many times a less number is contained in a greater.

In division of fractions the divisor may be larger than the dividend, in which case the quotient will be less than 1. For example, divide 1 apple into 4 equal parts.

Here it is plain that each part will be $\frac{1}{4}$; or that the dividend will *contain the divisor* but $\frac{1}{4}$ times.

Again, divide $\frac{1}{2}$ of a pear into 6 equal parts.

If a whole pear were divided into 6 equal parts, each part would be expressed by $\frac{1}{6}$. But since the half of the pear was divided, each part will be expressed by $\frac{1}{2}$ of $\frac{1}{6}$, or $\frac{1}{12}$.

In the division of fractions we should note the following principles:

1st. When the dividend is just equal to the divisor, the quotient will be 1.

2d. When the dividend is greater than the divisor, the quotient will be greater than 1.

3d. When the dividend is less than the divisor, the quotient will be less than 1.

4th. The quotient will be as many times greater than 1, as the dividend is greater than the divisor.

5th. The quotient will be as many times less than 1, as the dividend is less than the divisor.

CASE I.

159. To divide a fraction by a whole number.

Divide the numerator or multiply the denominator by the whole number.

EXAMPLES.

1. Divide $\frac{4}{3}$ by 2.

In the first operation we divide the fraction by multiplying the denominator (ART. 120): in the second we divide the numerator (ART. 119), giving the same result in both cases.

OPERATION.

$$\frac{4}{3}\div 2=\frac{4}{3\times 2}=\frac{4}{6}=\frac{2}{3}.$$

$$\text{or } \frac{4}{3}\div 2=\frac{2)4}{3}=\frac{2}{3}.$$

158. What does division of whole numbers explain? In division of fractions, may the divisor exceed the dividend? How will the quotient then compare with 1? If an apple be divided into 2 equal parts, what will express each part? If half an apple be divided into 4 equal parts, what will express one of the parts? What is one-half of one-half? What is one-sixth of one-half? What principles do you note in the division of fractions? When will the quotient be 1? When greater than 1? When will the quotient be less than 1? When greater than 1, how many times greater? When less than 1, how many times less?

2. Divide $\frac{18}{37}$ by 9. *Ans.* $\frac{18}{333}=\frac{2}{37}$.
3. Divide $\frac{405}{19}$ by 15. *Ans.* $\frac{405}{285}=\frac{27}{19}$.
4. Divide $\frac{2755}{3758}$ by 19. *Ans.* $\frac{145}{3758}$.
5. Divide $\frac{379}{1267}$ by 15. *Ans.* ——
6. Divide $\frac{37}{19}$ by 8. *Ans.* $\frac{37}{152}$.
7. Divide $\frac{61}{21}$ by 37. *Ans.* $\frac{61}{777}$.

CASE II.

160. To divide one fraction by another.

FIRST METHOD OF PROOF.

Let it be required to divide $\frac{10}{24}$ by $\frac{5}{8}$.

The true quotient will be expressed by the complex fraction $\dfrac{\frac{10}{24}}{\frac{5}{8}}$.

Let the terms of this fraction be now multiplied by the denominator with its terms inverted: this will not alter the value of the fraction (ART. 122), and we shall then have,

$$\frac{\frac{10}{24}}{\frac{5}{8}}=\frac{\frac{10}{24}\times\frac{8}{5}}{\frac{5}{8}\times\frac{8}{5}}=\frac{\frac{10}{24}\times\frac{8}{5}}{1}=\frac{10}{24}\times\frac{8}{5}=\frac{2}{3}=\text{quotient.}$$

It will be seen that the quotient is obtained by simply multiplying the numerator by the denominator with its terms inverted. This quotient may be further simplified by cancelling the common factors 5 and 8, giving $\frac{2}{3}$ for the true quotient.

SECOND METHOD OF PROOF.

OPERATION.
$\frac{10}{24}\div 5=\frac{10}{120}$
$\frac{10}{120}\times 8=\frac{80}{120}$.

Let us first divide the dividend by 5. This is done by multiplying the denominator (ART. 120), which gives $\frac{10}{120}$. But the divisor being but $\frac{1}{8}$ of

159. In how many ways may a fraction be divided by a whole number?

160. How do you divide one fraction by another? How may the quotient of one fraction divided by another be expressed? If any fraction be multiplied by the fraction which arises from inverting its terms, to what will the product be equal? In the second method of proof, after dividing by 5, is the quotient too small or too large, and how much? *How then do you find the true quotient?*

5, this quotient is 8 times too small, since the eighth of a number will be contained in the dividend 8 times more than the number itself. Therefore, by multiplying $\frac{10}{120}$ by 8, we obtain $\frac{80}{120}$ for the true quotient.

Hence, to divide one fraction by another,

Reduce compound and complex fractions to simple ones, also whole and mixed numbers to improper fractions: then multiply the dividend by the divisor with its terms inverted, and the product reduced to its simplest terms will be the quotient sought.

EXAMPLES.

1. Divide $\frac{1}{8}$ by $\frac{1}{7}$.	*Ans.*	$\frac{7}{8}$.
2. Divide $3\frac{1}{4}$ by $\frac{1}{9}$.	*Ans.*	$29\frac{1}{4}$.
3. Divide $16\frac{1}{2}$ of $\frac{1}{3}$ by $4\frac{1}{7}$.	*Ans.*	$1\frac{19}{58}$.
4. Divide $44\frac{1}{33}$ by $\frac{213}{333}$.	*Ans.*	———
5. Divide $371\frac{1}{2}$ by $\frac{1}{414}$.	*Ans.*	153801.
6. Divide $\frac{64}{111}$ by $\frac{23}{1213}$.	*Ans.*	$30\frac{1942}{2553}$.
7. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$.	*Ans.*	$\frac{2}{3}$.
8. Divide 5 by $\frac{7}{16}$.	*Ans.*	———
9. Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 91.	*Ans.*	$71\frac{1}{2}$.
10. Divide 100 by $4\frac{7}{8}$.	*Ans.*	$20\frac{20}{39}$.
11. Divide $\frac{3}{4}$ of $\frac{7}{8}$ by $\frac{2}{3}$.	*Ans.*	$\frac{63}{64}$.
12. Divide $\frac{5}{6}$ of 50 by $4\frac{1}{3}$.	*Ans.*	———
13. Divide $14\frac{1}{8}$ of $\frac{1}{9}$ by $3\frac{1}{3}$ of 6.	*Ans.*	$\frac{113}{1440}$.
14. Divide $34\frac{1}{7}$ by $\dfrac{54\frac{1}{8}}{93\frac{5}{11}}$.	*Ans.*	$58\frac{31758}{33341}$.

APPLICATIONS.

1. If 7*lb.* of sugar cost $\frac{44}{75}$ of a dollar, what is the price per pound?

$\frac{44}{75} \div 7 = \frac{44}{525}$ of \$1; or $\frac{44}{525}$ of $\frac{100}{1}$ cents $= \frac{4400}{525} = 8\frac{8}{21}$.

Ans. $8\frac{8}{21}$ *cents.*

2. If $\frac{3}{7}$ of a dollar will pay for $10\frac{1}{2}$*lb.* of nails, how much is the price per pound? *Ans.* \$

3. If $\frac{4}{7}$ of a yard of cloth cost \$3, what is the price per yard? *Ans.* $\$5\frac{1}{4}$.

4. If $\$21\frac{1}{2}$ will buy $7\frac{13}{16}$ barrels of apples, how much are they per barrel? *Ans.* $\$2\frac{94}{125}$.

5. If $4\frac{1}{7}$ gallons of molasses cost $\$2\frac{5}{8}$, how much is it per quart? *Ans.* ——

6. If $1\frac{1}{8}$*hhd.* of wine cost $\$250\frac{1}{3}$, how much is the wine per quart? *Ans.* $\$\frac{1502}{1701}=88\frac{512}{1701}$*cts.*

7. If eight pounds of tea cost $7\frac{5}{8}$ of a dollar, how much is it per pound? *Ans.* $95\frac{5}{16}$*cts.*

8. In $8\frac{1}{2}$ weeks a family consumes $165\frac{3}{9}$ pounds of butter: how much do they consume a week?

Ans. $19\frac{69}{153}$*lb.*

9. If a piece of cloth containing $176\frac{3}{4}$ yards costs $\$375\frac{6}{5}$, what does it cost per yard? *Ans.* ——

10. If I pay $\frac{7}{8}$ dollar a pound for tea, how many pounds can I have for 4284 dollars? *Ans.* 4896*lb.*

11. Bought flour at $7\frac{6}{7}$ dollars a barrel, and laid out 129 dollars for the article: how many barrels did I buy?

Ans. $16\frac{23}{55}$.

12. Paid $666\frac{2}{3}$ cents for marbles at 6 cents apiece: how many did I buy? *Ans.* $111\frac{1}{9}$.

13. If raisins are $28\frac{11}{26}$ cents a pound, how much can I have for $17\frac{3}{11}$ cents? *Ans.* ——

14. How many barrels of flour can I buy for $161\frac{3}{17}$ dollars if I pay $14\frac{3}{5}$ dollars a barrel? *Ans.* $11\frac{49}{1241}$*bar.*

15. Divide $5205\frac{1}{5}$ dollars among $\frac{4}{5}$ of 90 persons: what will each have? *Ans.* $72\frac{53}{180}$.

16. At 27 dollars an acre, how much land can I buy for $\frac{1}{17}$ of a dollar? *Ans.* $\frac{1}{459}$ *acre.*

17. How many apples can I buy for $2\frac{1}{3}$ of $\frac{1}{3}$ of 2 cents, if I pay $\frac{1}{3}$ of $2\frac{2}{3}$ of $\frac{9}{11}$ cents apiece? *Ans.* $2\frac{5}{36}$.

18. Bought $\frac{7}{9}$ of a lot of land for 5040 dollars, and having sold $\frac{5}{6}$ of what was bought, I gave $\frac{3}{5}$ of the remainder to a charitable society, and divided the residue among 9 poor persons: what was the share of each?

Ans. $37\frac{1}{3}$.

19. Of an estate valued 15000 dollars, the widow has $\frac{1}{3}$, the oldest son $\frac{2}{3}$ of remainder, and the residue was divided among 9 children: what was the share of each of the 9 children? Ans. $\$370\frac{10}{27}$.

DECIMAL FRACTIONS.

161. If the unit 1 be divided into 10 equal parts, the parts are called *tenths*, because each part is one-tenth of unity.

If the unit 1 be divided into one hundred equal parts, the parts are called *hundredths*, because each part is one hundredth of unity.

If the unit 1 be divided into one thousand equal parts, the parts are called *thousandths*, because each part is one thousandth of unity: and we have similar expressions for the parts, when the unit is divided into ten thousand, one hundred thousand, &c., equal parts.

The division of the unit into tenths, hundredths, thousandths, &c., forms a system of numbers called *Decimal Fractions*. They may be written thus:

Four-tenths, - - - - - -	$\frac{4}{10}$.
Six-tenths, - - - - - -	$\frac{6}{10}$.
Forty-five hundredths, - - - - -	$\frac{45}{100}$.
125 thousandths, - - - - - -	$\frac{125}{1000}$.
1047 ten thousandths, - - - - -	$\frac{1047}{10000}$.

From which we see, that in each case the denominator gives denomination or name to the fraction; that is, determines whether the parts are tenths, hundredths, thousandths, &c.

162. The denominators of decimal fractions are seldom set down. The fractions are usually expressed by means of a comma, or period, placed at the left of the numerator.

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161. When the unit 1 is divided into 10 equal parts, what is each part called? What is each part called when it is divided into 100 equal parts? When into 1000? Into 10,000, &c.? How are decimal fractions formed? What gives denomination to the fraction?

162. Are the denominators of decimal fractions generally set down? How are the fractions expressed? Is the denominator understood? What is it? What is the place next the decimal point called? The next? The third, &c.? Which way are decimals numerated?

Thus, $\frac{4}{10}$ . . . is written 4
$\frac{45}{100}$ 45
$\frac{125}{1000}$ 125
$\frac{1047}{10000}$ . . . . .1047.

This manner of writing decimal fractions is a mere language, and is used to avoid the inconvenience of writing the denominators. The denominator, however, of every decimal fraction is always understood. *It is a unit 1 with as many ciphers annexed as there are places of figures in the numerator.*

The place next to the decimal point is called tenths; the next place to the right, the place of hundredths; the next, the place of thousandths; and so on, for places further to the right, according to the following table.

DECIMAL NUMERATION TABLE.

Tenths.	Hundredths.	Thousandths.	Tens of thousandths.	Hundreds of thousandths.	Millionths.	Tens of millionths.		
.4							is read	4 tenths.
.6	4						- -	64 hundredths.
.0	6	4					- -	64 thousandths.
.6	7	5	4				- -	6754 ten thousandths.
.0	1	2	3	4			- -	1234 hundred thousandths.
.0	0	7	6	5	4		- -	7654 millionths.
.0	0	4	3	6	0	4	- -	43604 ten-millionths.

163. Let us now write and numerate the following decimals.

163. Does the value of a figure depend upon the place which it occupies? How does the value change from the left towards the right? What do ten parts of any one place make? How do they increase from the right towards the left? How may whole numbers be joined with decimals? What is a number called when composed partly of whole numbers and partly of decimals?

Four-tenths, - - - -	.4
Four hundredths, - - - -	.0 4
Four thousandths, - - -	.0 0 4
Four ten-thousandths, - -	.0 0 0 4
Four hundred thousandths, -	.0 0 0 0 4
Four millionths, - - -	.0 0 0 0 0 4
Four ten-millionths, - -	.0 0 0 0 0 0 4.

Here we see, that the same figure expresses different values, according to the place which it occupies.

But $\frac{1}{10}$ of $\frac{4}{10}$ is equal to $\frac{4}{100}=.04$
- $\frac{1}{10}$ of $\frac{4}{100}$ - - - $\frac{4}{1000}=.004$
- $\frac{1}{10}$ of $\frac{4}{1000}$ - - - $\frac{4}{10000}=.0004$
- $\frac{1}{10}$ of $\frac{4}{10000}$ - - - $\frac{4}{100000}=.00004$
- $\frac{1}{10}$ of $\frac{4}{100000}$ - - - $\frac{4}{1000000}=.000004$
- $\frac{1}{10}$ of $\frac{4}{1000000}$ - - - $\frac{4}{10000000}=.0000004.$

Therefore the value of the parts of a unit, expressed by the different figures in passing from the left to the right, diminishes in a tenfold proportion.

Hence, ten of the parts in any one of the places, are equal to one of the parts in the place next to the left; that is, ten thousandths make one hundredth, ten hundredths make one-tenth, and ten-tenths a unit 1.

This law of increase from the right hand towards the left, is the same as in whole numbers; therefore,

Whole numbers and decimal fractions may be united by placing the decimal point between them. Thus,

Whole numbers.								Decimals.						
Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	Thousandths.	Tens of thousandths.	Hundreds of thousandths.	Millionths.	Tens of millionths.
8	3	6	3	0	6	4	1	.0	4	7	8	9	7	6

A number composed partly of a whole number and partly of a decimal, is called a mixed number.

Write the following numbers in figures, and numerate them.

1. Forty-one, and three-tenths. 41.3.
2. Sixteen, and three millionths. 16.000003.
3. Five, and nine hundredths. 5.09.
4. Sixty-five, and fifteen thousandths. 65.015.
5. Eighty, and three millionths. 80.000003.
6. Two, and three hundred millionths.
7. Four hundred and ninety-two thousandths.
8. Three thousand, and twenty-one ten thousandths.
9. Forty-seven, and twenty-one ten thousandths.
10. Fifteen hundred and three millionths.
11. Thirty-nine, and six hundred and forty thousandths.
12. Three thousand, eight hundred and forty millionths.
13. Six hundred and fifty thousandths.
14. Fifty thousand, and four hundredths.
15. Six hundred, and eighteen ten thousandths.
16. Three millionths.
17. Thirty-nine hundred thousandths.

164. The denominations of Federal Money will correspond to the decimal division, if we regard 1 dollar as the unit. For, the dimes are tenths of the dollar, the cents are hundredths of the dollar, and the mills, being tenths of the cent, are thousandths of the dollar.

EXAMPLES.

1. Express $16, 3 dimes 8 cents and 9 mills decimally. *Ans.* $16.389.
2. Express $95, 8 dimes 9 cents 5 mills decimally. *Ans.* ——
3. Express $107, 9 dimes 6 cents 8 mills decimally. *Ans.* $107.968.

164. If the denominations of Federal Money be expressed decimally what is the unit? What part of a dollar is 1 dime? What part of a dime is a cent? What part of a cent is a mill? What part of a dollar is 1 cent? 1 mill?

4. Express $47 and 25 cents decimally.

Ans. $47.25.

5. Express $39,39 cents and 7 mills decimally.

Ans. $39.397.

6. Express $12 and 3 mills decimally. *Ans.* ——

7. Express $147 and 4 cents decimally.

Ans. $147.04.

8. Express $148, 4 mills decimally. *Ans.* $148.004.

9. Express four dollars, six mills decimally.

165. A cipher is annexed to a number, when it is placed on the right of it. If ciphers be annexed to the numerator of a decimal fraction, the same number of ciphers must also be annexed to the denominator; for there must be as many ciphers in the denominator as there are places of figures in the numerator (ART. 162). The numerator and denominator will therefore have been multiplied by the same number, and consequently the value of the fraction will not be changed (ART. 122). Hence,

Annexing ciphers to a decimal fraction does not alter its value.

We may take as an example, $.3=\frac{3}{10}$. If now we annex a cipher to the numerator, we must, at the same time, annex one to the denominator, which gives,

$.30 = \frac{30}{100}$ by annexing one cipher,

$.300 = \frac{300}{1000}$ by annexing two ciphers,

$.3000=\frac{3000}{10000}$, all of which are equal to $\frac{3}{10}=.3$.

Also, $.5=\frac{5}{10}=.50=\frac{50}{100}=.500=\frac{500}{1000}$.

Also, $.8=.80=.800=.8000=.80000$.

166. Prefixing a cipher is placing it on the left of a number. If ciphers be prefixed to the numerator of a decimal fraction, the same number of ciphers must be annexed to the denominator. Now, the numerator will

165. When is a cipher annexed to a number? Does the annexing of ciphers to a decimal alter its value? Why not? What does three-tenths become by annexing a cipher? What by annexing two ciphers? Three ciphers? What does .8 become by annexing a cipher? By annexing two ciphers? By annexing three ciphers?

remain unchanged while the denominator will be increased ten times for every cipher which is annexed, and the value of the fraction will be decreased in the same proportion (ART. 120). Hence,

Prefixing ciphers to a decimal fraction diminishes its value ten times for every cipher prefixed.

Take, for example, the fraction $.2=\frac{2}{10}$.

$.02 = \frac{02}{100}$ by prefixing one cipher,

$.002 = \frac{002}{1000}$ by prefixing two ciphers,

$.0002=\frac{0002}{10000}$ by prefixing three ciphers:

in which the fraction is diminished ten times for every cipher prefixed.

ADDITION OF DECIMAL FRACTIONS.

167. It must be recollected that only like parts of the same unit can be added together, and therefore in setting down the numbers for addition the figures occupying places of the same value must be placed directly under each other.

The addition of decimal fractions is then made in the same manner as that of whole numbers.

For example, add 37.04, 704.3, and .0376 together.

In this example, we place the tenths under tenths, the hundredths under hundredths, and this brings the decimal points and the like parts of the unit directly under each other. We then add as in whole numbers.

OPERATION.

```
  37.04
 704.3
    .0376
 --------
 741.3776
```

166. When is a cipher prefixed to a number? When prefixed to a decimal, does it increase the numerator? Does it increase the denominator? What effect then has it on the value of the fraction? What do .5 become by prefixing a cipher? By prefixing two ciphers? By prefixing three? What do .07 become by prefixing a cipher? By prefixing two? By prefixing three? By prefixing four?

167. What parts of unity may be added together? How do you set down the numbers for addition? How will the decimal points fall? How do you then add? How many decimal places do you point off in the sum?

Hence, for the addition of decimal numbers,

I. *Set down the numbers to be added so that figures occupying places of the same value shall fall directly under each other.*

II. *Then add as in simple numbers and point off in the sum, from the right hand, so many places for decimals as are equal to the greatest number of places in any of the added numbers.*

EXAMPLES.

1. Add 4.035, 763.196, 445.3741, and 91.3754 together. *Ans.* ——

2. Add 365.103113, .76012, 1.34976, .3549, and 61.11 together. *Ans.* 428.677893.

3. 67.407+97.004+4+.6+.06+.3=169.371.

4. .0007+1.0436+.4+.05+.047=1.5413.

5. .0049+47.0426+37.0410+360.0039=444.0924.

6. What is the sum of 27, 14, 49, 126, 999, .469, and .2614? *Ans.* 1215.7304.

7. Add 15, 100, 67, 1, 5, 33, .467, and 24.6 together.

8. What is the sum of 99, 99, 31, .25, 60.102, .29, and 100.347? *Ans.* ——

9. Add together .7509, .0074, 69.8408, and .6109.

10. Required the sum of twenty-nine and 3 tenths, four hundred and sixty-five, and two hundred and twenty-one thousandths. *Ans.* ——

11. Required the sum of two hundred dollars one dime three cents and nine mills, four hundred and forty dollars nine mills, and one dollar one dime and one mill?

Ans. $641.249, or 641 dollars 2 dimes 4 cents 9 mills.

12. What is the sum of one-tenth, one hundredth, and one thousandth? *Ans.* ——

13. What is the sum of 4, and 6 ten-thousandths?

Ans. 4.0006.

14. Required in dollars and decimals, the sum of one dollar one dime one cent one mill, six dollars three mills, four dollars eight cents, nine dollars six mills, one hundred dollars six dimes, nine dimes one mill, and eight dollars six cents. *Ans.* ——

15. What is the sum of 4 dollars 6 cents, 9 dollars 3 mills, 14 dollars 3 dimes 9 cents 1 mill, 104 dollars 9 dimes 9 cents 9 mills, 999 dollars 9 dimes 1 mill, 4 mills, 6 mills, and 1 mill? *Ans.* $1132,365.

16. If you sell one piece of cloth for $4,25, another for $5,075, and another for $7,0025, how much do you get for all? *Ans.* $16,3275.

17. What is the amount of $151,7, $70,602, $4,06, and $807,2659? *Ans.* $1033,6279.

18. A man received at one time $13,25; at another $8,4; at another $23,051; at another $6; and at another $0,75: how much did he receive in all?

Ans. $51,451.

19. Find the sum of twenty-five hundredths, three hundred and sixty-five thousandths, six tenths, and nine millionths. *Ans.* ——

20. What is the sum of twenty-three millions and ten, one thousand, four hundred thousandths, twenty-seven, and nineteen millionths, seven, and five tenths?

Ans. 23001044.500059.

21. What is the sum of six millionths, four ten-thousandths, 19 hundred thousandths, sixteen hundredths, and four tenths? *Ans.* ——

SUBTRACTION OF DECIMAL FRACTIONS.

168. Subtraction of Decimal Fractions is the process of finding the difference between two decimal numbers.

Let it be required from 3.275 to take .0879.

In this example a cipher is annexed to the minuend to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend (Art. 122).

OPERATION.

```
3.2750
 .0879
------
3.1871
```

Hence, for the subtraction of decimal numbers,

168. What does subtraction teach? How do you set down the numbers for subtraction? How do you then subtract? How many decimal places do you point off in the remainder?

I. *Set down the less number under the greater, so that figures occupying places of the same value shall fall directly under each other.*

II. *Then subtract as in simple numbers, and point off in the remainder, from the right hand, as many places for decimals as are equal to the greatest number of places in either of the given numbers.*

EXAMPLES.

1. From 3295 take .0879. *Ans.* 3294.9121.
2. From 291.10001 take 41.375. *Ans.* 249.72501.
3. From 10.000001 take .111111. *Ans.* 9.888890.
4. From three hundred and ninety-six, take 8 ten-thousandths. *Ans.* ——
5. From 1 take one thousandth. *Ans.* .999.
6. From 6378 take one tenth. *Ans.* 6377.9.
7. From 365.0075 take 3 millionths. *Ans.* 365.007497.
8. From 21.004 take 97 ten-thousandths. *Ans.* ——
9. From 260.4709 take 47 ten-millionths. *Ans.* 260.4708953.
10. From 10.0302 take 19 millionths. *Ans.* 10.030181.
11. From 2.01 take 6 ten-thousandths. *Ans.* ——
12. From thirty-five thousands, take thirty-five thousandths. *Ans.* 34999.965.
13. What is the difference between 4262.0246 and 23.41653? *Ans.* ——
14. From 346.523120 take 219.691245943. *Ans.* 126.831874057.
15. From 64.075 take .195326. *Ans.* ——
16. What is the difference between 107 and .0007? *Ans.* 106.9993.
17. What is the difference between 1.5 and .3785?
18. From 96.71 take 96.709. *Ans.* .001.
19. From forty-three, and seventy-five thousandths, take eight, and twenty-three millionths. *Ans.* ——

MULTIPLICATION OF DECIMAL FRACTIONS.

169.—1. Multiply .37 by .8.

We may first write $.37=\frac{37}{100}$, and $.8=\frac{8}{10}$.

If, now, we multiply the fraction $\frac{37}{100}$ by $\frac{8}{10}$, we find the product to be $\frac{296}{1000}$; the number of ciphers in the denominator of this product is equal to the number of decimal places in the two factors, and the same will be true for any two factors whatever.

OPERATION.

$$.37=\frac{37}{100}$$
$$.8=\frac{8}{10}$$
$$.296=\frac{296}{1000}=.296.$$

2. Multiply .3 by .02.

OPERATION.

$$.3\times.02=\frac{3}{10}\times\frac{02}{100}=\frac{6}{1000}=.006 \text{ answer.}$$

Now, to express the 6 thousandths decimally, we have to prefix two ciphers to the 6, and this makes as many decimal places in the product as there are in both multiplicand and multiplier.

Therefore, to multiply one decimal by another,

Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as are equal to the number of decimal places in the multiplicand and multiplier; and if there be not so many in the product, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Multiply 3.049 by .012. *Ans.* .036588.

(2.)		(3.)	
Multiply	365.491	Multiply	496.0135
by	.001	by	1.496
Ans.	.365491	*Ans.*	742.0361960

4. Multiply one and one millionth by one thousandth. *Ans.* .001000001.

---

169. After multiplying, how many decimal places will you point off in the product? When there are not so many in the product, what do you do? Give the rule for the multiplication of decimals.

5. Multiply one hundred and forty-seven millionths by one millionth. *Ans.* ——

6. Multiply three hundred, and twenty-seven hundredths by 31. *Ans.* 9308.37.

7. Multiply 31.00467 by 10.03962.
Ans. 311.2751050254

8. What is the product of five-tenths by five-tenths?

9. What is the product of five-tenths by five-thousandths? *Ans.* .0025.

10. Multiply 596.04 by 0.00004. *Ans.* ——

11. Multiply 38049.079 by 0.00008.
Ans. 3.04392632.

12. What will 6.29 weeks' board come to at 2,75 dollars per week? *Ans.* $17,2975.

13. What will 61 pounds of sugar come to at $0,234 per pound? *Ans.* ——

14. If 12.836 dollars are paid for one barrel of flour, what will .354 barrels cost? *Ans.* $4,543944.

15. What is the content of a board, .06 feet long and .06 wide? *Ans.* ——

16. Multiply 49000 by .0049. *Ans.* 240.1.

17. Bought .1234 oranges for 4.6 cents apiece: how much did they cost? *Ans.* ——

18. What will 375.6 pounds of coffee cost at .125 dollars per pound? *Ans.* $46,95.

19. If I buy 36.251 pounds of indigo at $0,029 per pound, what will it come to? *Ans.* $1,051279.

20. Multiply $89,3421001 by .0000028.
Ans. $0,00025015788028.

21. Multiply $341,45 by .007. *Ans.* ——

22. What is the content of a lot which is .004 miles long and .004 miles wide? *Ans.* .000016 *sq. miles.*

23. Multiply .007853 by .035. *Ans.* ——

24. What is the product of $26,000375 multiplied by .00007? *Ans.* $0,00182002625.

170. When a decimal number is to be multiplied by 10, 100, 1000, &c., the multiplication may be made by *removing* the decimal point as many places to the right *hand* as there are ciphers in the multiplier, and if there be not so many figures on the right of the decimal point, supply the deficiency by annexing ciphers.

$$\text{Thus, 6.79 multiplied by} \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}67.9\\679.\\6790.\\67900.\\679000.\end{array}\right.$$

$$\text{Also, 370.036 multiplied by} \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}3700.36\\37003.6\\370036.\\3700360.\\37003600.\end{array}\right.$$

DIVISION OF DECIMAL FRACTIONS.

171. Division of Decimal Fractions is similar to that of simple numbers.

We have just seen that, if one decimal fraction be multiplied by another, the product will contain as many places of decimals as there are in both the factors. Now, if this product be divided by one of the factors, the quotient will be the other factor. Hence, in division, the dividend must contain just as many decimal places as the divisor and quotient together. *The quotient, therefore, will contain as many places as the dividend, less the number in the divisor.*

Let it be required to divide 1.38483 by 60.21

There are five decimal places in the dividend, and two in the divisor: there must therefore be three places in the quotient: hence, one 0 must be prefixed to the 23, and the decimal point placed before it.

OPERATION.

```
60.21)1.38483(23
      1.2042
      ------
       18063
       18063
      ------
   Ans. .023.
```

Hence, for the division of decimals,

170. How do you multiply a decimal number by 10, 100, 1000, &c.? If there are not as many decimal figures as there are ciphers in the multiplier, what do you do?

Divide as in simple numbers, and point off in the quotient, from the right hand, so many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Divide 2.3421 by 2.11. *Ans.* 1.11.
2. Divide 12.82561 by 3.01. *Ans.* ——
3. Divide 33.66431 by 1.01. *Ans.* 33.331.
4. Divide .010001 by .01. *Ans.* ——
5. Divide 8.2470 by .002. *Ans.* 4123.5.
6. What is the quotient of 37.57602, divided by 3? By .3? By .03? By .003? By .0003?
7. What is the quotient of 129.75896, divided by 8? By .08? By .008? By .0008? By .00008?
8. What is the quotient of 187,29900, divided by 9? By .9? By .09? By .009? By .0009? By .00009?
9. What is the quotient of 764.2043244, divided by 6? By .06? By .006? By .0006? By .00006? By .000006?

172. When any decimal number is to be divided by 10, 100, 1000, &c., the division is made by removing the decimal point as many places to the left as there are 0's in the divisor; and if there be not so many figures on the left of the decimal point, the deficiency must be supplied by prefixing ciphers.

$$27.69 \text{ divided by} \begin{Bmatrix} 10 \\ 100 \\ 1000 \\ 10000 \end{Bmatrix} = \begin{cases} 2.769 \\ .2769 \\ .02769 \\ .002769 \end{cases}$$

171. If one decimal fraction be multiplied by another, how many decimal places will there be in the product? How does the number of decimal places in the dividend compare with that in the divisor and quotient? How do you determine the number of decimal places in the quotient? If the divisor contains four places and the dividend six, how many in the quotient? If the divisor contains three places and the dividend five, how many in the quotient? Give the rule for the division of decimals.

172. How do you divide a decimal number by 10, 100, 1000, &c.? If there be not as many figures to the left of the decimal point as there are ciphers in the divisor, what do you do?

$$642.89 \text{ divided by } \begin{Bmatrix} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \end{Bmatrix} = \begin{cases} 64.289 \\ 6.4289 \\ .64289 \\ .064289 \\ .0064289 \end{cases}$$

173. When there are more decimal places in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places equal to those of the divisor; *all the figures of the quotient will then be whole numbers.*

EXAMPLES.

1. Divide 4397.4 by 3.49.

We annex one 0 to the dividend. Had it contained no decimal place we should have annexed two.

OPERATION.

```
3.49)4397.40(1260
     349
     ---
      907
      698
      ---
      2094
      2094
      ----
  Ans. 1260
```

2. Divide 2194.02194 by .100001. *Ans.* 21940.
3. Divide 9811.0047 by .325947. *Ans.* ——
4. Divide .1 by .0001. *Ans.* 1000.
5. Divide 10 by .1. *Ans.* ——
6. Divide 6 by .6. By .06. By .006. By .2. By .3. By .003. By .5. By .05. By .005. By .000012.

174. When it is necessary to continue the division farther than the figures of the dividend will allow, we may annex ciphers and consider them as decimal places of the dividend.

---

173. If there are more decimal places in the divisor than in the dividend, what do you do? What will the figures of the quotient then be?

174. How do you continue the division after you have brought down all the figures of the dividend? What sign do you place after the quotient? *What does it show?*

EXAMPLES.

1. Divide 4.25 by 1.25.

In this example we annex one 0 and then the decimal places in the dividend will exceed those in the divisor by 1.

OPERATION.

```
1.25)4.25(3.4
     3.75
      500
      500
Ans. 3.4
```

2. Divide .2 by .06.

We see in this example that the division will never terminate. In such cases the division should be carried to the third or fourth place, which will give the answer true enough for all practical purposes, and the sign + should then be written, to show that the division may be still continued.

OPERATION.

```
.06).20(3.333+
     18
      20
      18
       20
       18
        20
Ans. 3.333+.
```

3. Divide 37.4 by 4.5. *Ans.* 8.3111+.

4. Divide 586.4 by 375. *Ans.* ——

5. Divide 94.0369 by 81.032. *Ans.* 1.160+.

ADDITIONAL EXAMPLES.

1. If I divide .6 dollars among 94 men, how much will each receive? *Ans.* $0,00638+.

2. I gave 28 dollars to 267 persons: how much apiece? *Ans.* ——

3. Divide 6.35 by .425. *Ans.* 14.941+.

4. Tell the quotient of 36.2678 dollars divided by 2.25. *Ans.* ——

5. Divide a dollar into 12 parts. *Ans.* $,083333+.

6. Divide .25 of 3.26 into .034 of 3.04 parts. *Ans.* 7.885+.

7. How many times will .35 of 35 be contained in .024 of 24? *Ans.* ——

8. At .75 dollars a bushel, how many bushels of rye can be bought for 141 dollars? *Ans.* 188*bu.*

9. Bought .001 bushels of potatoes for .20341 dollars a bushel, and paid in rye at ,00044 dollars a bushel: how much rye did it take? *Ans.* ——

10. Bought 53.1 yards of cloth for 2 dollars: how much was it a yard? *Ans.* $0,037+.

11. Divide 125 by .1045. *Ans.* ——

12. Divide one millionth by one billionth. *Ans. one thousand.*

APPLICATIONS IN THE FOUR PRECEDING RULES.

1. A merchant sold 4 parcels of cloth, the first contained 127 and 3 thousandths yards; the 2d, 6 and 3 tenths yards; the 3d, 4 and one hundredth yards; the 4th, 90 and one millionth yards: how many yards did he sell in all? *Ans.* 227.313001*yd.*

2. A merchant buys three chests of tea, the first contains 60 and one thousandth *lb.*; the second, 39 and one ten thousandth *lb.*; the third, 26 and one tenth *lb.*: how much did he buy in all? *Ans.* *lb.*

3. What is the sum of $20 and three hundredths; $4 and one-tenth, $6 and one thousandth, and $18 and one hundredth? *Ans.* $48,141.

4. A puts in trade $504,342; B puts in $350,1965; C puts in $100,11; D puts in $99,334; and E puts in $9001,32: what is the whole amount put in?

5. B has $936, and A has $1, 3 dimes and 1 mill: how much more money has B than A? *A.* $934,699.

6. A merchant buys 37.5 yards of cloth, at one dollar twenty-five cents per yard: how much does the whole come to? *Ans.* $46,875.

7. A farmer sells to a merchant 13.12 cords of wood at $4,25 per cord, and 13 bushels of wheat at $1,06 per bushel: he is to take in payment 13 yards of broadcloth at $4,07 per yard, and the remainder in cash: how much money did he receive? *Ans.* $16,63

8. If 12 men had each $339 one dime 9 cents and 3 mills, what would be the total amount of their money?

9. If one man can remove 5.91 cubic yards of earth in a day, how much could nineteen men remove?

Ans. 112.29yd.

10. What is the cost of 8.3 yards of cloth at $5,47 per yard? *Ans.* $45,401.

11. If a man earns one dollar and one mill per day, how much will he earn in a year? *Ans.* ——

12. What will be the cost of 375 thousandths of a cord of wood, at $2 per cord? *Ans.* $0,75.

13. A man leaves an estate of $1473,194 to be equally divided among 12 heirs: what is each one's portion? *Ans.* $122,766$\frac{1}{6}$.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

175. The value of every vulgar fraction is equal to the quotient arising from dividing the numerator by the denominator (ART. **109**).

1. Let it be required to find the decimal value of $\frac{9}{2}$.

We first divide 9 by 2 which gives a quotient 4, and 1 for a remainder. Now 1 is equal to 10 tenths. If then we add a cipher, 2 will divide 10, giving the quotient 5 tenths. Hence the true quotient is 4.5.

OPERATION.

$\frac{9}{2}=4\frac{1}{2}$; but

$4\frac{1}{2}=4\frac{10}{2}=4.5$.

2. What is the value of $\frac{13}{4}$.

We first divide by 4 which gives a quotient 3 and a remainder 1. But 1 is equal to 100 hundredths. If then we add two ciphers, 4 will divide the 100, giving a quotient of 25 hundredths.

OPERATION.

$\frac{13}{4}=3\frac{1}{4}$; but

$3\frac{1}{4}=3\frac{100}{4}=3.25$.

Hence, to reduce a vulgar fraction to a decimal,

I. *Annex one or more ciphers to the numerator and then divide by the denominator.*

II. *If there is a remainder, annex a cipher or ciphers, and divide again, and continue to annex ciphers and to di-*

175. What is the value of a fraction equal to? What is the value of four-halves? What is the decimal value of one-half? Of three-halves? Of six-fourths? Of nine-halves? Of seven-halves? Of five-fourths? Of one-fourth? Give the rule for reducing a vulgar fraction to a decimal.

vide until there is no remainder, or until the quotient is sufficiently exact: the number of decimal places to be pointed off in the quotient is the same as the number of ciphers used; and when there are not so many, ciphers must be prefixed.

EXAMPLES.

1. Reduce $\frac{635}{125}$ to its equivalent decimal.

We here use two ciphers and therefore point off two decimal places in the quotient.

OPERATION.

```
125)635(5.08
    625
    ----
     1000
     1000
     ----
```

2. Reduce $\frac{1}{4}$ and $\frac{9}{1129}$ to decimals. *Ans.* .25 and .00797+.
3. Reduce $\frac{12}{480}$, $\frac{27}{39}$, $\frac{3}{1000}$, and $\frac{11}{60006}$ to decimals. *Ans.* .025; .692+; .003; .000183+.
4. Reduce $\frac{1}{2}$ and $\frac{5}{1785}$ to decimals. *Ans.* +.
5. Reduce $\frac{314957123}{210456891}$ to a decimal. *Ans.* 1.496+.
6. Reduce $\frac{8}{6}$, $\frac{1375}{8436}$, $\frac{3265}{4121}$, $\frac{574}{123}$ to decimals. *Ans.* 1.333+; 0.162+; 0.792+; 4.666+.
7. Reduce $\frac{17}{20}$ to a decimal. *Ans.* 0.85.
8. Reduce $\frac{3}{40}$ to a decimal. *Ans.* 0.075.
9. Reduce $\frac{17}{125}$ to a decimal. *Ans.* 0.136.
10. Reduce $\frac{7}{800}$ to a decimal. *Ans.* ——
11. Reduce $\frac{372}{1250}$ to a decimal. *Ans.* 0.2976.
12. Reduce $\frac{11}{1600}$ to a decimal. *Ans.* 0.006875.
13. Reduce $\frac{15}{1280}$ to a decimal. *Ans.* 0.01171875.
14. Reduce $\frac{347}{2560}$ to a decimal. *Ans.* ——
15. Reduce $\frac{1}{10000}$ to a decimal. *Ans.* 0.0001.
16. Reduce $\frac{3476}{15625}$ to a decimal. *Ans.* 0.222464.
17. Reduce $\frac{1}{2048000}$ to a decimal. *Ans.* 0.00000048828125.
18. Reduce $\frac{3}{7}$ to a decimal. *Ans.* ——
19. Reduce $\frac{15}{17}$ to a decimal. *Ans.* 0.88235+.
20. Reduce $\frac{3}{35}$ to a decimal. *Ans.* 0.0857142.

21. Reduce $\frac{17}{19}$ to a decimal. *Ans.* 0.89473684+

22. Reduce $\frac{19}{1365}$ to a decimal. *Ans.* ——

23. Reduce $\frac{706}{5907}$ to a decimal. *Ans.* 0.119519+.

24. What is the decimal value of $\frac{2}{5}$ of $\frac{3}{5}$ multiplied by $\frac{5}{6}$ of $3\frac{2}{7}$? *Ans.* .65714+.

25. What is the value in decimals of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{8}$ divided by $\frac{1}{8}$ of $\frac{2}{4}$? *Ans.* 4.6666+

26. A man owns $\frac{7}{8}$ of a ship; he sells $\frac{4}{11}$ of his share: what part is that of the whole, expressed in decimals? *Ans.* .3181818+

27. Bought $\frac{11}{15}$ of $87\frac{3}{11}$ bushels of wheat for $\frac{9}{10}$ of 7 dollars a bushel: how much did it come to, expressed in decimals? *Ans.* ——

28. If a man receive $\frac{4}{5}$ of a dollar at one time, $7\frac{1}{2}$ at another, and $8\frac{3}{4}$ at a third: how many in all, expressed in decimals? *Ans.* $17,05.

29. What decimal is equal to $\frac{5}{8}$ of 18, and $\frac{4}{11}$ of $\frac{11}{12}$ of $7\frac{4}{11}$, added together? *Ans.* ——

30. What decimal is equal to $\frac{2}{3}$ of 5 taken from $\frac{3}{5}$ of $8\frac{3}{4}$? *Ans.* 1.91666+.

31. What decimal is equal to $\frac{21}{44}$, $\frac{6}{7}$, $\frac{2}{18}$, added together? *Ans.* 1.4455+.

32. What decimal is equal to the difference between $\frac{1}{3}$ of $\frac{1}{7}$ and $14\frac{1}{3}$ of $\frac{1}{12}$? *Ans.* ——

REDUCTION OF DENOMINATE DECIMALS.

176. We have seen that a denominate number is one in which the *kind* of unit is denominated or expressed.

A denominate decimal is a decimal fraction in which the kind of unit that has been divided is expressed. Thus, .5 of a £, and .6 of a shilling, are denominate decimals. The unit that was divided in the first fraction being £1, and that in the second 1 shilling.

---

176. What is a denominate number? What is a denominate decimal? In the decimal five-tenths of a £, what is the unit? In the decimal six-tenths of a shilling, what is the unit?

CASE I.

177. To find the value of a denominate number in decimals of a higher denomination.

Let it be required to reduce 9*d.* to the decimal of a £.

We first find that there are 240 pence in £1. We then divide 9*d.* by 240, which gives the quotient .0375 of a £. This is the true value of 9*d.* in the decimal of a £.

OPERATION.

240*d.*=£1
240)9(.0375
Ans. £.0375.

Hence, to make the reduction,

I. *Consider how many units of the given denomination make one unit of the denomination to which you would reduce.*

II. *Divide the given denominate number by the number so found, and the quotient will be the value in the required denomination.*

EXAMPLES.

1. Reduce 7 drams to the decimal of a *lb.* avoirdupois. *Ans.* .02734375*lb.*
2. Reduce 26*d.* to the decimal of a £. *Ans.* .1083333+.
3. Reduce .056 poles to the decimal of an acre.
4. Reduce 14 minutes to the decimal of a day. *Ans.* .0097222*da.*+.
5. Reduce .21 pints to the decimal of a peck.
6. Reduce 3 hours to the decimal of a day. *Ans.* .125.
7. Reduce 375678 feet to the decimal of a mile. *Ans.* 71.151+.
8. Reduce 36 yards to the decimal of a rod.
9. Reduce .5 quarts to the decimal of a barrel.
10. Reduce .7 of an ounce, avoirdupois, to the decimal of a hundred. *Ans.* ——

---

177. How do you find the value of a denominate number in a decimal of a higher denomination?

CASE II.

178 To reduce denominate fractions of different denominations to an equivalent decimal of a given denomination.

Reduce £1 4*s*. 9$\frac{3}{4}$*d*. to the denomination of pounds.

We first reduce 3 farthings to the decimal of a penny, by dividing by 4. We then annex the quotient .75 to the 9 pence. We next divide by 12, giving .8125, which is the decimal of a shilling. This we annex to the shillings, and then divide by 20.

OPERATION.

$\frac{3}{4}d. = .75d.$; hence,
$9\frac{3}{4}d. = 9.75d.$

12)9.75*d*.
.8125*s*., and

20)4.8125*s*.
£.240625 ; therefore,
£1 4*s*. 9$\frac{3}{4}$*d*. = £1.240625.

Hence, to make the reduction,

Divide the lowest denomination named, by that number which makes one of the denomination next higher, annexing ciphers if necessary: then annex this quotient to the next higher denomination, and divide as before; proceed in the same manner through all the denominations to the last: the last result will be the answer sought.

EXAMPLES.

1. Reduce £19 17*s*. 3$\frac{1}{4}$*d*. to the decimal of a £. *Ans*. £19.863+.
2. Reduce 15*s*. 6*d*. to the decimal of a £. *A*. £.775.
3. Reduce 7$\frac{1}{2}$*d*. to the denomination of shillings.
4. Reduce 2*lb*. 5*oz*. 12*pwt*. 16*gr*., Troy, to the decimal of a *lb*. *Ans*. 2.469444*lb*.+.
5. Reduce 3 feet 9 inches to the denomination of yards. *Ans*. 1.25*yd*.
6. Reduce 1*lb*. 12*dr*., avoirdupois, to the denomination of pounds. *Ans*. 1.046875*lb*.
7. Reduce 5 leagues 2 furlongs to the denomination of leagues. *Ans*. ——

178. How do you reduce denominate numbers of different denominations to equivalent decimals of a given denomination?

CASE III.

179. To find the value of a denominate decimal in terms of integers of inferior denominations.

What is the value of .832296 of a £.?

We first multiply the decimal by 20, which brings it to shillings, and after cutting off from the right as many places for decimals as in the given number, we have 16*s*. and the decimal .645920 over. This we reduce to pence by multiplying by 12, and then reduce to farthings by multiplying by 4.

OPERATION.

```
     .832296
          20
   16.645920
          12
    7.751040
           4
    3.004160
Ans. 16s. 7d. 3far.
```

Hence, to make the reduction,

I. *Consider how many in the next less denomination make one of the given denomination, and multiply the decimal by this number; then cut off from the right hand as many places as there are in the given decimal.*

II. *Multiply the figures so cut off by the number which it takes in the next less denomination to make one of a higher, and cut off as before. Proceed in the same way to the lowest denomination: the figures to the left will form the answer sought.*

EXAMPLES.

1. What is the value of .002084*lb*. Troy?
Ans. 12.00384*gr*.
2. What is the value of .625 of a *cwt*.?
Ans. 2*qr*. 12*lb*. 8*oz*.
3. What is the value of .625 of a gallon?
Ans. 2*qt*. 1*pt*.
4. What is the value of £.3375? *Ans*. ——
5. What is the value of .3375 of a ton?
Ans. 6*cwt*. 3*qr*.
6. What is the value of .05 of an acre? *Ans*. 8*P*.

179. How do you find the value of a denominate decimal in integers of inferior denominations? What is the value in shillings of one-half of a £? In pence of one-half of a shilling?

7. What is the value of .875 pipes of wine?

8. What is the value of .125 hogsheads of beer? *Ans.* 6*gal.* 3*qt.*

9. What is the value of .375 of a year of 365 days? *Ans.* 136*da.* 21*hr.*

10. What is the value of .085 of a £? *Ans.* ——

11. What is the value of .86 of a *cwt.*? *Ans.* 3*qr.* 11*lb.*

12. What is the difference between .82 of a day and .32 of an hour? *Ans.* 19*hr.* 21*m.* 36*sec.*

13. What is the value of 1.089 miles? *Ans.* 1*mi.* 28*rd.* 7*ft.* 11.04*in.*

14. What is the value of .09375 of a pound, avoirdupois weight? *Ans.* ——

15. What is the value of .28493 of a year of 365 days? *Ans.* 103*da.* 23*hr.* 59*m.* 12.48*sec.*

16. What is the value of £1.046? *Ans.* £1 11*d.*+.

17. What is the value of £1.88? *Ans.* ——

PROMISCUOUS QUESTIONS.

1. What will 11$\frac{5}{8}$ tons of hay cost at $17,37 a ton? *Ans.* $201,92625.

2. What will 12*gal.* 3*qt.* 1*pt.* of wine cost at $0,28 a quart? *Ans.* ——

3. Bought a load of potash for $9,17, paying at the rate of $16 a ton: what was the weight of the potash? *Ans.* 11*cwt.* 1*qr.* 21*lb.*

4. What will 57*yd.* 2*qr.* 3*na.* of cloth cost at $6,78 a yard? *Ans.* ——

5. What will 7*A.* 2*R.* 38*P.* of land cost at $64,50 per acre? *Ans.* $499,06875.

6. Suppose a farmer had 4 granaries of rye: the first contained 4.67 bushels; the second 9.87; the third 10.01; and the fourth 11.0012; after using 18.0679 bushels he sold the remainder for $1,03 per bushel, and divided the money among nine persons: what did each receive?

7. What is the cost of 693 yards of cloth at $3,4775 per yard? *Ans.* $2409.9075.

8. What is the cost of 917 bushels of wheat at $1,125 per bushel? *Ans.* ——

OF THE RATIO AND PROPORTION OF NUMBERS.

180. Two numbers having the same unit may be compared together in two ways.

1st. By considering *how much* one is greater or less than the other, which is shown by their difference; and

2d. By considering *how many times* one is greater or less than the other, which is shown by their quotient.

Thus, in comparing the numbers 3 and 12 together with respect to their difference, we find that 12 *exceeds* 3 by 9; and in comparing them together with respect to their quotient, we find that 12 contains 3 four times, or that 12 is four times greater than 3.

The quotient which arises from dividing the second number by the first, is called the *ratio* of the numbers, and shows how many times the second number is greater than the first, or how many times it is less.

Thus, the ratio of 3 to 9 is 3, since $9 \div 3 = 3$. The ratio of 2 to 4 is 2, since $4 \div 2 = 2$.

We may also compare a larger number with a less. For example, the ratio of 4 to 2 is $\frac{1}{2}$; for, $2 \div 4 = \frac{1}{2}$. The ratio of 9 to 3 is $\frac{1}{3}$, since $3 \div 9 = \frac{1}{3}$.

EXAMPLES.

1. What is the ratio of 9 to 18? *Ans.* 2.
2. What is the ratio of 6 to 24? *Ans.* 4.
3. What is the ratio of 12 to 48? *Ans.* 4.
4. What is the ratio of 11 to 13? *Ans.* ——
5. What part of 20 is 4? Or what is the ratio of 20 to 4? *Ans.* $\frac{1}{5}$.

---

180. In how many ways may two numbers having the same unit be compared? How do you determine how much one number is greater than another? How do you determine how many times it is greater or less? How much does 12 exceed 3? How many times is 12 greater than 3? What is the quotient called which arises from dividing the second number by the first? What does it show? When the second number is the least, what does it show?

6. What part of 100 is 30 ? Or what is the ratio of 100 to 30 ? *Ans.* $\frac{3}{10}$.

7. What part of 6 is 3 ? *Ans.* $\frac{1}{2}$.

8. What part of 9 is 3 ? *Ans.* $\frac{1}{3}$.

9. What part of 12 is 4 ? *Ans.* ——

10. What part of 50 is 5 ? *Ans.* $\frac{1}{10}$.

11. What part of 75 is 3 ? *Ans.* $\frac{1}{25}$.

181. In determining *what part* one number is of another, it is plain that the number which makes *the part* must be written in the numerator, and the number of which it is a part, in the denominator, and that this *fraction reduced to its lowest terms will express the part.*

182. If one yard of cloth cost $2, how many dollars will 6 yards of cloth cost at the same rate ?

It is plain that 6 yards of cloth will cost 6 times as much as one yard; that is, the cost will contain $2 as many times as 6 contains 1. Hence the cost will be $12.

In this example there are four numbers considered, viz., 1 yard of cloth, 6 yards of cloth, $2, and $12: these numbers are called *terms*.

1 yard of cloth is the	1st term,
6 yards of cloth is the	2d term,
$2 is the - - -	3d term,
$12 is the - - -	4th term.

Now the ratio of the first term to the second is the same as the ratio of the third to the fourth.

This relation between four numbers is called *proportion;* and generally,

Four numbers are said to be in proportion when the ratio of the first to the second is the same as that of the third to the fourth. Hence,

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181. How do you determine what part one number is of another?

182. If one yard of cloth cost $2, what will 6 yards cost? How many numbers are here considered? What are they called? What is the ratio of the first to the second equal to? What is this relation between numbers called? When are four numbers said to be in proportion? How do you define proportion?

PROPORTION *is an equality of ratios between numbers compared together two and two.*

183. We express that four numbers are in proportion thus:

1 : 6 : : 2 : 12.

That is, we write the numbers in the same line and place two dots between the 1st and 2d terms, four between the 2d and 3d, and two between the 3d and 4th terms. We read the proportion thus,

as 1 is to 6, so is 2 to 12.

The 1st and 2d terms of a proportion always express quantities of the same kind, and so likewise do the 3d and 4th terms. As in the example,

yd.		yd.		$		$
1	:	6	: :	2	:	12.

This is implied by the definition of a ratio; for, it is only quantities of the same kind which can be divided the one by the other. The ratio of the first term to the second, or of the third to the fourth, is called the ratio of the proportion.

What are the ratios of the proportions

3	:	9	: :	12	:	36?	*Ans.* 3.
2	:	10	: :	12	:	60?	*Ans.* 5.
4	:	2	: :	8	:	4?	*Ans.* $\frac{1}{2}$.
9	:	1	: :	90	:	10?	*Ans.* $\frac{1}{9}$.
16	:	15	: :	48	:	45?	*Ans.* $\frac{15}{16}$.

184. When two numbers are compared together, the first is called the *antecedent*, and the second the *consequent*; and when four numbers are compared, the first antecedent and consequent are called the *first couplet*, and the second antecedent and consequent the *second couplet*. Thus, in the last proportion, 16 and 48 are the

---

183. How do you indicate that four numbers are in proportion? How is the proportion read? What do you remark of the first and second terms? Also of the third and fourth?

184. When two numbers are compared together, what is the first called? What the second? When four numbers are compared, what are the *two first called*? What the two second?

antecedents, and 15 and 45 the consequents; also, 16 and 15 make the first couplet, and 48 and 45 the second.

185. If 4*lb.* of tea cost \$8, what will 12*lb.* cost at the same rate?

OPERATION.

	lb.	*lb.*	\$: \$
As	4 :	12 : :	8 : *Ans.*

$$\begin{array}{r} 12 \\ \hline 4)96 \\ \hline \$24 \end{array} \text{ the cost of 12lb. of tea.}$$

$$\frac{12}{4}\times 8 = 3\times 8 = 24.$$

Ans. \$24.

It is evident that the 4th term, or cost of 12*lb.* of tea, must be as many times greater than \$8, as 12*lb.* is greater than 4*lb.* But the ratio of 4*lb.* to 12*lb.* is 3; hence, 3 is the number of times which the cost exceeds \$8: that is, the cost is equal to $\$8\times 3=\$24$. But instead of writing the numbers

$$\frac{12}{4}\times 8 = 24,$$

we may write them

$$(12\times 8)\div 4 = 24:$$

and as the same may be shown for every proportion, we conclude,

That the 4th term of every proportion may be found by multiplying the 2d and 3d terms together, and dividing their product by the 1st term.

EXAMPLES.

1. The first three terms of a proportion are 1, 2, and 3: what is the fourth? *Ans.* 6.

2. The first three terms are 6, 2, and 1: what is the 4th? *Ans.* $\frac{1}{3}$.

3. The first three terms are 10, 3, and 1: what is the 4th? *Ans.* $\frac{3}{10}$.

---

185. Explain this example orally. How may the fourth term of every proportion be found?

186. The 1st and 4th terms of a proportion are called the two extremes, and the 2d and 3d terms are called the two means.

Now, since the 4th term is obtained by dividing the product of the 2d and 3d terms by the 1st term, and since the product of the divisor by the quotient is equal to the dividend, it follows,

That in every proportion the product of the two extremes is equal to the product of the two means.

Thus, in the following examples, we have

	$1 : 6 :: 2 : 12;$	and	$1\times 12 = 2\times 6;$
also,	$4 : 12 :: 8 : 24;$	and	$4\times 24 = 12\times 8;$
"	$6 : 9 :: 10 : 15;$	and	$6\times 15 = 9\times 10;$
"	$7 : 15 :: 14 : 30;$	and	$7\times 30 = 15\times 14.$

OF CANCELLING.

187. When one number is to be divided by another, the operation may often be shortened by striking out or cancelling the factors common to both, before the division is made.

1. For example, suppose it were required to divide 360 by 120.

We first write the dividend above a horizontal line, and the divisor beneath it, after the form of a fraction. We next separate both of them into factors, and then cancel the factors which are alike.

OPERATION.

$$\frac{360}{120}=\frac{12\times 30}{12\times 10}=\frac{\not{12}\times 3\times \not{10}}{\not{12}\times \not{10}}=3.$$

2. Divide 630 by 35.

We separate the dividend and divisor into like factors, and then cancel those which are common in both.

OPERATION.

$$\frac{630}{35}=\frac{3\times \not{5}\times 6\times \not{7}}{\not{5}\times \not{7}}=18.$$

---

186. What are the first and fourth terms of a proportion called? What are the second and third terms called? In every proportion, what is the product of the extremes equal to?

187. How may the division of two numbers be often abridged? Explain the *example orally*. Also the second example.

3. Divide 1860 by 36. *Ans.* ——
4. Divide 7920 by 720. *Ans.* 11.
5. Divide 1890 by 210. *Ans.* 9.
6. Divide 1260 by 504. *Ans.* $2\frac{1}{2}$.
7. Divide 1768 by 221. *Ans.* 8.
8. Divide 2856 by 238. *Ans.* ——

188. If two or more numbers are to be multiplied together and their product divided by the product of other numbers, the operation may be abridged by *striking out* or *cancelling* any factor which is common to the dividend and divisor. For example, if 6 is to be multiplied by 8 and the product divided by 4, we have

$$\frac{6\times8}{4}=\frac{48}{4}=12;\ \text{or,}\ \frac{6\times8}{4}=6\times2=12:$$

in the latter case we cancelled the factor 4 in the numerator and denominator, and multiplied 6 by the quotient 2.

1. Let it be required to multiply 24 by 16 and divide the product by 12.

Having written the product of the figures, which form the dividend, above the line, and the product of the figures which form the divisor below it, then

OPERATION.

$$\frac{\overset{2}{\cancel{24}}\times16}{\underset{1}{\cancel{12}}}=32.$$

We cancel the common factors in the numerator and denominator, and write the quotients over and under the numbers in which such common factors are found, and if the quotients still have a common factor, they may be again divided.

2. Reduce the compound fraction $\frac{4}{5}$ of $\frac{6}{9}$ of $\frac{3}{12}$ of $\frac{5}{16}$ to a simple fraction.

Beginning with the first numerator, we find it is once a factor of itself and 4 times in 16; 6 is twice a factor in 12; 3, three times a factor in 9; and 5, once a factor in the denominator 5.

OPERATION.

$$\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{6}}}{\underset{3}{\cancel{9}}}\times\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{12}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{16}}}=\frac{1}{24}.$$

---

188. When two numbers are multiplied together and their product divided by a third, how may the operation be abridged?

3. What is the product of $3\times8\times9\times7\times15$ divided by $63\times24\times3\times5$?

This example presents a case that often arises, in which the *product* of two factors may be cancelled.

OPERATION.

$$\frac{\not3\times\not8\times\not9\times\not7\times\not{15}}{\not{63}\times\not{24}\times\not3\times\not5}=1.$$

Thus, 3×8 is 24: then cancel the 3 and 8 in the numerator and the 24 in the denominator. Again, 9 times 7 are 63; therefore cancel the 9 and 7 in the numerator and the 63 in the denominator. Also, 3×5 in the denominator cancels the 15 remaining in the numerator: hence, the quotient is unity.

4. What is the product of $126\times16\times3$ divided by 7×12?

We see that 7 is a factor of 126, giving a quotient 18, which we place over 126, crossing at the same time 126 and the 7 below. We then divide 18 and 12 by 6, crossing them both and writing down the quotients 3 and 2. We next divide 16 and 2 by 2, giving the quotients 8 and 1. Hence, the result is 72.

OPERATION.

$$\begin{array}{l} 3 \\ \not{18}\quad 8 \\ \dfrac{\not{126}\times\not{16}\times3}{\not7\times\not{12}}=72. \\ \quad\;\;\not2 \\ \quad\;\;1 \end{array}$$

EXAMPLES.

1. What is the product of $1\times6\times9\times14\times15\times7\times8$ divided by $36\times128\times56\times20$? *Ans.* $\frac{63}{512}$.

2. What is the value of $18\times36\times72\times144$ divided by $6\times6\times8\times9\times12\times8$? *Ans.* 27.

189. The process of cancelling may be applied to the terms of a proportion.

If we have any proportion, as

6 : 15 : : 28 : 70,

We may always cancel like factors in either couplet.

Thus, 6 : 15 : : 28 : 70,

or 2 5 14 35;

in which we divide the terms of the first couplet by 3, and those of the second by 2, and write the quotients below.

189. How else may the process of cancelling be applied? What may be *cancelled in each* couplet?

RULE OF THREE.

190. The Rule of Three takes its name from the circumstance that three numbers are always given to find a fourth, which shall bear the same proportion to one of the given numbers as exists between the other two.

The following is the manner of finding the fourth term:

I. *Reduce the two numbers which have different names from the answer sought, to the lowest denomination named in either of them.*

II. *Set the number which is of the same kind with the answer sought in the third place, and then consider from the nature of the question whether the answer will be greater or less than the third term.*

III. *When the answer is greater than the third term, write the least of the remaining numbers in the first place, but when it is less place the greater there.*

IV. *Then multiply the second and third terms together, and divide their product by the first term: the quotient will be the fourth term or answer sought, and will be of the same denomination as the third term.*

EXAMPLES.

1. If 48 yards of cloth cost $67,25, what will 144 yards cost at the same rate?

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190. From what does the Rule of Three take its name? What is the first thing to be done in stating the question? Which number do you make the third term? How do you determine which to put in the first? After stating the question, how do you find the fourth term? What will be its denomination? In the first example which is greater, the third or fourth term? Which number must then be in the first term? How many times will the fourth term be greater or less than the third?

In this example, as the answer is to be dollars, we place the $67,25 in the 3d term. Then, as 144 yards of cloth will cost more than 48 yards, the fourth term must be greater than the third, and therefore, we write the least of the two remaining numbers in the first place. The product of the 2d and 3d terms is $9684,00: then dividing by the 1st term we obtain $201,75 for the cost of 144 yards of cloth.

OPERATION.

```
yd.  yd.         $        $
48 : 144  : :  67,25  : Ans.
                 144
               -----
               26900
              26900
              6725
             --------
         48)9684,00($201,75
            96
            --
             84
             48
             ---
             360
             336
             ---
              240
              240
              ---
```

2. If 6 men can dig a certain ditch in 40 days, how many days would 30 men be employed in digging it?

As the answer must be days, the 40 days are written in the 3d place. Then as it is evident that 30 men will do the same work in a shorter time than 6 men, it is plain that the fourth term must be less than the third; therefore, 30 men, the greater of the remaining numbers, is written in the first term. Besides, it is plain that the fourth term must be just so many times less than 40, as 6 is less than 30.

OPERATION.

```
men  men    days   days.
30 : 6  : :  40  : Ans.
              6
          ---------
          3|0)24|0 days.
          Ans.  8 days.
```

PROOF.

191. The product of the two means is equal to the product of the extremes (Art. **186**). Hence, if either

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191. What is the product of the two means equal to? If the product of the extremes be divided by one of them, what will the quotient be?

of these equal products be divided by one of the mean terms, the quotient will be the other. Therefore,

Divide the product of the extremes by one of the mean terms, and if the work is right the quotient will be the other mean term.

EXAMPLE.

The 1st term is 4, the 2d 8, the 3d 12, and the answer 24: is the answer true?

The product of the extremes is 96. If this be divided by 8 the quotient is 12; if by 12 the quotient is 8: hence, the answer was true.

OPERATION OF PROOF.

$24\times4=96$

8)96(12; or

12)96(8

RULE OF THREE BY CANCELLING.

192. If two numbers are to be multiplied together and their product divided by a third, the operations may be abridged by *striking* out or *cancelling* any factor which is common to the divisor and either of the other numbers (Art. 188).

EXAMPLES.

1. Multiply 24 by 16 and divide the product by 12.

The greatest common factor of 12 and 24 is 12.

OPERATION.

$$\frac{\cancel{24}\times16}{\cancel{12}}=2\times16=32.$$

2. What is the 4th term of the proportion

16 : 15 : : 48?

Here 16 is the greatest common divisor between 16 and 48, and gives 3 for the quotient on the right.

OPERATION.

$$\frac{\cancel{48}\times15}{\cancel{16}}=3\times15=45.$$

3. If 4 pounds of tea cost $8, what will 12 pounds cost at the same rate?

OPERATION.

$$\frac{\cancel{8}\times12}{\cancel{4}}=2\times12=\$24.$$

---

192. How do you write the numbers before cancelling? If there are equal numbers above and below the line, what do you do with them?

4. If 48 yards of cloth cost \$67,25, what will 144 yards cost?

OPERATION.

$$\frac{\overset{3}{\not{1}\not{4}\not{4}} \times \$67,25}{\not{4}\not{8}} = 3 \times 67.25 = \$201,75.$$

5. If 25 yards of cloth cost £2 3*s.* 4*d.*, what will 5 yards cost?

In this example we have 5 left below the line, by which the product above it must be divided.

OPERATION.

$$\frac{\overset{1}{\not{5}} \times (£2\ 3s.\ 4d.)}{\underset{5}{\not{2}\not{5}}} = 8s.\ 8d.$$

6. If 12 hats cost \$60, how much will 40 cost?

OPERATION.

$$\frac{\$\not{6}\not{0} \times 40}{\not{1}\not{2}} = 5 \times 40 = \$200.$$

7. If 30 barrels of flour will subsist 100 men for 40 days, how long will it subsist 25?

OPERATION.

$$\frac{\not{1}\not{0}\not{0} \times 40}{\not{2}\not{5}} = 4 \times 40 = 160\ days.$$

8. If 120 sheep yield 360*lb.* of wool, how many pounds will be obtained from 600?

OPERATION.

$$\frac{600 \times \not{3}\not{6}\not{0}}{\not{1}\not{2}\not{0}} = 600 \times 3 = 1800lb.$$

9. If a man travel 210 miles in 6 days, how far will he travel in 40 days?

OPERATION.

$$\frac{\not{2}\not{1}\not{0} \times 40}{\not{6}} = 35 \times 40 = 1400miles.$$

RULE OF THREE BY ANALYSIS.

193. The solution of questions in the Rule of Three by Analysis consists in finding the ratio of two of the given terms, and multiplying this ratio by the other term.

The ratio of two of the terms will generally express the value or cost of a single thing.

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193. In what does the solution of questions by analysis consist? What does the ratio of two of the terms express? If this ratio be multiplied by the other term, what is the product? If 6 oranges cost 12 cents, how much will 8 cost?

EXAMPLES.

1. If 3 barrels of flour cost $24, what will 7 barrels cost?

By dividing the $24 by 3 we get the cost of 1 barrel. For, if $24 will buy 3 barrels, it is plain that $\frac{1}{3}$ of it will buy 1 barrel. This, multiplied by 7, gives $56, the cost of 7 barrels.

OPERATION.

```
3)24
   8
8×7=56
Ans. $56.
```

2. If in 29 days a man travels 58 miles, how far will he travel in 30 days? *Ans.* 60.

3. If 6 men consume 1 barrel of flour in 30 days, how much would 48 men consume?

It is evident that $\frac{1}{6}$ of a barrel would be the amount consumed by 1 man; hence, 48 times $\frac{1}{6}$ is the amount consumed by 48 men.

OPERATION.

$\frac{1}{6}\times 48=8.$

Ans. 8.

4. If $\frac{1}{8}$ of a barrel of flour cost $\frac{2}{3}$ of a dollar, what will $\frac{5}{16}$ cost? *Ans.* $1.

5. If I walk 84 miles in 3 days, how far should I walk at the same rate in 9? *Ans.* 252.

6. If 8*lb.* of sugar cost $1,28, how much will 13*lb.* cost? What is 16×13? *Ans.* ——

7. If $\frac{3}{4}$ of a piece of cloth cost $8,25, what will $\frac{9}{4}$ cost? *Ans.* $24,75.

8. If 300 barrels of flour cost $570, what will 200 cost? What is $\frac{2}{3}\times 570$? *Ans.* ——

TO FIND THE COST OF THINGS BY THE 100, 1000, &c.

194. What will be the cost of 895 feet of timber, at $6 per hundred feet?

In this example, 100 feet of timber, is to the given quantity 895 feet, as $6, the cost of 100 feet, is to $53,70, the cost of 895 feet. If the timber had been sold at the rate of $6 per thousand feet, the cost of 895 feet

OPERATION.

```
100 : 895 : : 6 : Ans.
        6
100)5370
   53,70
          Ans. $53,70.
```

would have been $5,37, for we should have divided by 1000, instead of 100; that is, we should have removed the separating point in the product three places to the left. Hence, to find the cost of things sold by the 100, or 1000,

Multiply the number of things by the price, and if the things be reckoned by the 100, *cut off two places from the right, and if reckoned by the* 1000, *cut off three, and the figures to the left will be the answer in the same denomination as the given price.*

EXAMPLES.

1. What will be the cost of 1350 feet of boards at $11 per hundred? *Ans.* $148,50.
2. What will be the cost of 36578 bricks at $6,50 per thousand? *Ans.* $237,75 7.
3. What will be the cost of 6359 feet of boards at $9,25 per 100 feet? *Ans.* ——
4. What will be the cost of 13918 feet of timber at $14,37 per thousand? *Ans.* $200,00+.
5. What will 18759 oranges cost at $5,50 per hundred? *Ans.* $1031,74+.
6. What is the cost of 6559 feet of round timber at $9,25 per 100 feet? *Ans.* ——
7. What is the cost of 37032 feet of square timber at $85,72 per thousand feet? *Ans.* $3174,38+.

APPLICATIONS IN THE RULE OF THREE.

1. If 4 hats cost $12, what will 55 cost at the same rate? *Ans.* $165.
2. What is the value of 2*cwt.* of sugar at 5*d.* per *lb.*? *Ans.* £4 3*s.* 4*d.*
3. If 40 yards of cloth cost $170, what will 325 yards cost? *Ans.* $1381,25.
4. If 240 sheep yield 660 pounds of wool, how many pounds will be obtained from 1200? *Ans.* 3300*lb.*

---

194. **How do you find the cost of things sold by the hundred? How do you find the cost of things sold by the thousand?**

5. If two gallons of molasses cost 65 cents, what will 3 hogsheads cost? *Ans.* \$61,42½.

6. If a man travels at the rate of 210 miles in 6 days, how far will he travel in a year, supposing him not to travel on Sundays? *Ans.* 10955 *miles.*

7. If 1 yard of cloth cost \$3,25, what will be the cost of 3 pieces, each containing 25 yards? *Ans.* \$243,75.

8. If 30 barrels of flour will support 100 men for 40 days, how long would it subsist 25 men? *Ans.* 160*da.*

9. If 30 barrels of flour will support 100 men for 40 days, how long would it subsist 400 men? *Ans.* 10*da.*

10. A owes B £679 6*s.*, but compounds with him by paying 3*s.* 4*d.* on the pound: how much does B receive of his debt? *Ans.* £113 4*s.* 4*d.*

11. If 90 bushels of oats will feed 40 horses for 6 days, how long would 450 bushels last them?

Ans. 30 *days.*

12. If 5*cwt.* 3*qr.* 14*lb.* of sugar cost £6 1*s.* 8*d.*, what will 35*cwt.* 28*lb.* cost? *Ans.* £36 8*s.* 9*d.*

13. What is the cost of 3*cwt.* of coffee at 15*d.* per pound? *Ans.* £18 15*s.*

14. If 3 quarters of a yard of velvet cost 7*s.* 3*d.*, how many yards can be bought for £13 15*s.* 6*d.*?

Ans. 28*yd.* 2*qr.*

15. If an ingot of gold, weighing 9*lb.* 9*oz.* 12*pwt.*, be worth £470 8*s.*: what is that per grain? *Ans.* 2*d.*

16. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards, at £16 4*s.* per piece: what is the value of the whole, and the cost per yard?

Ans. £388 16*s.* at 12*s.* per yard.

17. What will be the cost of 72 yards of cloth, at the rate of £5 12*s.* for 9 yards? *Ans.* £44 16*s.*

18. A person's annual income is £146: how much is that per day? *Ans.* 8*s.*

19. If 3 paces or common steps of a person, be equal to 2 yards, how many yards will 160 paces make?

Ans. 106*yd.* 2*ft.*

20. If 750 men require 22500 rations of bread for a *month,* how many rations will a garrison of 1200 men *require?* *Ans.* 36000.

21. What length must be cut off from a board that is 9 inches wide, to make a square foot, that is, as much as is contained in 12 inches in length and 12 in breadth?

Ans. 16 *inches.*

22. If 7*cwt.* 1*qr.* of sugar cost $64,96, what will be the price of 4*cwt.* 2*qr.*? *Ans.* $40,32.

23. The clothing of a regiment of foot of 750 men amounts to £2831 5*s.*: what will it cost to clothe a body of 3500 men? *Ans.* £13212 10*s.*

24. How many yards of carpeting, that is 3 feet wide, will cover a floor that is 27 feet long and 20 feet broad?

Ans. 60 *yards.*

25. What is the cost of 6 bushels of coal at the rate of £1 14*s.* 6*d.* the chaldron? *Ans.* 5*s.* 9*d.*

26. If 6352 stones of 3 feet long will complete a certain quantity of wall, how many stones of 2 feet long will raise the like quantity? *Ans.* 9528.

27. If a person can count 300 in two minutes, how many can he count in a day? *Ans.* 216000.

28. A garrison of 536 men have provisions for 365 days: how long will those provisions last if the garrison be increased to 1124 men? *Ans.* $174\frac{16}{281}$ *days.*

29. What will be the tax upon £763 15*s.* at the rate of 3*s.* 6*d.* per pound sterling? *Ans.* £133 13*s.* $1\frac{1}{2}$*d.*

30. What will be the tax on $3758, at the rate of 4 mills on the dollar? *Ans.* $15,032.

31. A certain work can be raised in 12 days by working 4 hours each day: how long would it require to raise the work by working 6 hours per day? *Ans.* 8 *days.*

32. What quantity of corn can I buy for 90 guineas, at the rate of 6 shillings a bushel? *Ans.* 315 *bushels.*

33. A person failing in trade owes £977, at which time he has in money, goods, and recoverable debts £420 6*s.* $3\frac{1}{4}$*d.*: now, supposing an equal division among his creditors, how much will they get on the pound?

Ans. 8*s.* $7\frac{1}{4}$*d.*

34. A pasture of a certain extent having supplied a body of horse, consisting of 3000, with forage for 18 days, how many days would the same pasture have supplied a body of 2000 horse? *Ans.* 27 *days.*

35. Suppose a gentleman's income to be 600 guineas a year, and that he spends 25*s*. 6*d*. per day, one day with another: how much will he have at the end of the year? *Ans*. £164 12*s*. 6*d*.

36. What is the cost of 30 pieces of lead, each weighing 1*cwt*. 12*lb*., at the rate of 16*s*. 4*d*. the *cwt*.? *Ans*. £27 8*s*. $9\frac{3}{5}$*d*.

37. The governor of a besieged place has provisions for 54 days at the rate of 2*lb*. of bread per ration, but is desirous to prolong the siege to 80 days, in expectation of succor: in that case what must be the ration of bread? *Ans*. $1\frac{28}{80}$*lb*.

38. If a person pays half a guinea a week for his board, how long can he be boarded for £21? *Ans*. 40 *weeks*.

39. What is the value of a year's rent of 547 acres of land at the rate of 15*s*. 6*d*. the acre? *Ans*. £423 18*s*. 6*d*.

40. If a person drinks 20 bottles of wine per month, when it costs 2*s*. per bottle, how much can he drink without increasing the expense when it costs 2*s*. 6*d*. per bottle? *Ans*. 16 *bottles*.

41. A merchant bought 21 pieces of cloth, each containing 40 yards, for which he paid $1260; he sold the cloth at $1,75 per yard: did he make or lose by the bargain? *Ans*. he gained $210.

42. A cistern containing 200 gallons is filled by a pipe which discharges 3 gallons in 5 minutes: but the cistern has a leak which empties 1 gallon in 5 minutes. Now if the water begins to run in, when the cistern is empty, how long will it be in filling? *Ans*. 8*hr*. 20*m*.

43. If a man perform a journey in $22\frac{1}{2}$ days, when the days are 12 hours long, how many days will it take him to perform the same journey when the days are 15 hours long? *Ans*. 18 *days*.

44. If the freight of 40 tierces of sugar, each weighing $3\frac{1}{2}$*cwt*., 150 miles, cost $42, what must be paid for the freight of 10*hhd*. of sugar, each weighing 12*cwt*., 50 miles? *Ans*. $12

45. If a family of 14 persons spend \$1120 in 8 months, how much will 9 of the same family spend in 5 months? *Ans.* \$450.

46. Two persons A and B are on the opposite sides of a wood which is 536 yards in circumference; they begin to travel in the same direction at the same moment; A goes at the rate of 11 yards per minute, and B at the rate of 34 yards in 3 minutes: the question is, how many times the quicker one must go round the wood before he overtakes the slower? *Ans.* 17 *times.*

47. What will be the cost of a piece of silver weighing 73*lb.* 5*oz.* 15*pwt.*, at 5*s.* 9*d.* per ounce?

Ans. £253 10*s.* $0\frac{3}{4}$*d.*

48. If the penny loaf weighs 8 ounces when the bushel of wheat costs 7*s.* 3*d.*, what ought it to weigh when the wheat is 8*s.* 4*d.* per bushel? *Ans.* 6*oz.* $15\frac{36}{100}$*dr.*

49. If one acre of land costs £1 7*s.* 8*d.*, what will be the cost of 173*A.* 2*R.* 14*P.* at the same rate?

Ans. £240 2*s.* $6\frac{1}{20}$*d.*

50. A gentleman's estate is worth £2107 12*s.* a year: what may he spend per day and yet save £500 per annum? *Ans.* £4 8*s.* $1\frac{19}{365}$*d.*

51. If 18 men can build 72 rods of wall in 4 days, how many rods will 38 build in 22 days? *Ans.* ——

52. Four thousand soldiers were supplied with bread for 24 weeks, each man to receive 14*oz.* per day; but by some accident 210 barrels containing 200*lb.* each were spoiled: what must each man receive in order that the remainder may last the same time? *Ans.* 13*oz.*

53. Let us suppose the 4000 soldiers having one-fourteenth of their bread spoiled, to be put on an allowance of 13*oz.* of bread per day for 24 weeks: required the weight of their bread, good and spoiled, and the amount spoiled? *Ans.* { *whole weight* 588000*lb.*, *spoiled* 42000*lb.* }

54. Suppose 4000 soldiers after losing 210 barrels of bread, each containing 200*lb.*, were to subsist on 13*oz.* a day for 24 weeks; had none been lost they might have received 14*oz.* a day: what was the whole weight, and how much did they receive? Ans. 546000lb.

55. Let us now suppose 4000 soldiers to lose one-fourteenth of their bread, then to reeeive 13*oz.* per day for 24 weeks: what was the whole weight of their bread, including the lost, and how much would each have received per day had none been spoiled?

Ans. { *whole weight* 588000*lb.*, *less* . . . . 42000*lb.*, 14*oz. per day had none been lost.* }

56. A certain amount of provisions will subsist an army of 3000 men for 30 days: if the army be increased by 2000, how long would the same provisions subsist it?

57. A merchant bought 42 pieces of cloth, each containing 20 yards, for which he paid $2520: he sold the cloth at $3 per yard, did he make or lose by the bargain?

Ans. He neither made nor lost.

58. If 8 barrels of flour will supply 240 men for 6 days, how long will 14 barrels supply 126 men?

Ans. 20*da.*

59. The sum of $2500 is to be divided between two brothers, so that for every dollar received by the younger the older is to receive $4: how much will each receive? *Ans.* ——

60. If 50 persons consume 600 bushels of wheat in 1 year, how much will 278 persons consume in 7 years?

EXAMPLES INVOLVING FRACTIONS.

1. If $\frac{3}{8}$ of a yard of cloth cost $3,20, what will $2\frac{1}{2}$ yards cost?

OPERATION.

$$\tfrac{3}{8} : 2\tfrac{1}{2} :: 3,20 : Ans.$$

$$\begin{array}{lr} & 2\frac{1}{2} \\ \hline & 6,40 \\ \text{by multiplying by } \frac{1}{2} & 1,60 \\ \hline & 8,00 \end{array}$$

$$8,00 \div \tfrac{3}{8} = 8,00 \times \tfrac{8}{3} = \tfrac{64,00}{3} = \$21,33+.$$

We state the question exactly as in whole numbers. In multiplying the second and third terms together, we observe the rules for multiplying fractions, and in dividing by the 1st term, the rules for *division*. Thus, in this example, we invert the terms of *the divisor* and multiply.

2. If $\frac{5}{7}$*oz.* cost £$\frac{11}{12}$, what will 1*oz.* cost?
Ans. £1 5*s.* 8*d.*

3. If $\frac{3}{16}$ of a ship cost £273 2*s.* 6*d.*, what will $\frac{5}{32}$ of her cost? *Ans.* £227 12*s.* 1*d.*

4. A mercer bought $3\frac{1}{2}$ pieces of silk, each containing $24\frac{1}{3}$ yards. He paid 6*s.* $\frac{1}{2}$*d.* per yard: what does the whole come to? *Ans.* £25 14*s.* $6\frac{1}{2}$*d*+.

5. If 14*lb.* of sugar cost $\$1\frac{1}{2}$, what will 6*lb.* cost?
Ans. $\$\frac{9}{14}$.

6. If $\frac{2}{5}$ of a yard of cloth cost $\frac{7}{9}$ of a dollar, what will $2\frac{1}{2}$ yards cost? *Ans.* $\$4\frac{31}{36}$.

7. If 2*lb.* of beef cost $\frac{1}{8}$ of a dollar, what will 30*lb.* cost? *Ans.* ——

8. If $14\frac{1}{2}$ yards of cloth cost $\$19\frac{1}{3}$, how much will $19\frac{7}{8}$ yards cost? *Ans.* $\$26\frac{1}{2}$.

9. If .3 of a house cost \$100,75, what would .95 cost?
Ans. \$319,04+.

10. A man receives $\frac{3}{5}$ of his income and finds it equal to \$3724,16: how much is his whole income?

11. Suppose a cistern has two pipes, and that one can fill it in $8\frac{1}{2}$ hours, the other in $4\frac{3}{4}$: in what time can both fill it together? *Ans.* $3\frac{5}{106}$*hr.*

12. There are 1000 men besieged in a town with provisions for 5 weeks, allowing each man 16 ounces a day. If they are reinforced by 500 more and no relief can be offered till the end of 8 weeks, how many ounces must be given daily to each man? *Ans.* $6\frac{2}{3}$*oz.*

13. A reservoir has three pipes, the first can fill it in 12 days, the second in 11 days, and the third can empty it in 14 days: in what time will it be filled if they are all running together? *Ans.* ——

14. In the latitude of London, the distance round the earth measured on the parallel of latitude, is about 15550 miles. Now as the earth turns round in 23 hours 56 minutes, at what rate per hour does the city of London move from west to east? *Ans.* $649\frac{259}{359}$ *miles per hour.*

15. A father left his son a fortune, $\frac{1}{4}$ of which he ran through in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer, when he had barely £820 left: what sum did his *father leave him*? Ans. £1913 6s. 8d.

16. After laying out $\frac{1}{4}$ of my money, and $\frac{1}{5}$ of the remainder, I had 72 guineas left: how much had I at first?

Ans. 120 *guineas.*

17. A father divided $\frac{7}{18}$ of his estate to one son, and $\frac{7}{18}$ of the remainder to another, leaving the remainder to his widow. The difference of the childrens' legacies was £514 6*s.* 8*d.*: what was the widow's portion?

18. Two persons, A and B, depart at the same time, the one from Boston and the other from Hartford, distant about 100 miles. After 7 hours they meet on the road, when it appears that A had rode $1\frac{1}{2}$ miles per hour faster than B: at what rate per hour did each traveller ride?

Ans. A $7\frac{25}{28}$, B $6\frac{11}{28}$ *miles per hour.*

19. If $\frac{1}{9}$ of a pole stands in the mud, 1 foot in the water, and $\frac{5}{6}$ in the air, or above the water, what is the length of the pole? *Ans.* ——

DOUBLE RULE OF THREE.

195. It often happens that questions arise involving five, seven, or even nine terms. Such are classed under a separate rule called The Double Rule of Three, or Compound Proportion. Such questions may be stated and resolved by the following

RULE.

I. *Make the first statement as though the question were to be solved by two or more statements by the Single Rule of Three, and suppose the fourth term to be found.*

II. *If it is of the same name with the answer sought, mark its place blank under the third term; if not, mark its place under the second term, and in either case arrange the two remaining terms as though it were a question in the Single Rule of Three. If there are more than five terms in the question, suppose the fourth term of the second proportion to be found, and make the third statement in the same manner as the second was made.*

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195. How may questions involving five &c. terms be stated and *resolved?* Give the rule.

III. *Then multiply the second and third terms together, and divide their product by the product of the first terms and the quotient will be the answer sought.*

EXAMPLES.

1. If a family of 6 persons expend $300 in 8 months, how much will serve a family of 15 persons for 20 months?

OPERATION.

persons		*persons*		$		
6	:	15	: :	300	:	1*st answer.*
months		*months*				
8	:	20	: :	1*st ans.*	:	*true answer*

$$\frac{15\times\overset{5}{\not{2}\not{0}}\times\overset{25}{\not{3}\not{0}\not{0}}}{\not{6}\times\not{8}}=15\times5\times25=\$1875.$$

The answer, in the above example, will depend on two things—1st, the proportion between the *number of persons* fed; and 2dly, the proportion based on the difference of time. The first statement gives what would be the true answer, if the time were the same; and the second gives the true answer under the supposition of different times. A similar proportion obtains in all like examples. But since the first answer will always form the second or third term of the second statement, it follows, that the true answer will be equal to the product of the second and third terms divided by the product of the first terms —according to the rule.

2. If 16 men build 18 feet of wall in 12 days, how many men must be employed to build 72 feet in 8 days, working at the same rate?

OPERATION.

feet		*feet*		*days.*		
18	:	72	: :	12	:	1*st answer.*
days		*days*		*men.*		
8	:	—	: :	16	:	*true answer.*

Then, $$\frac{\overset{4}{\not{7}\not{2}}\times12\times\overset{2}{\not{1}\not{6}}}{\not{1}\not{8}\times\not{8}}=4\times12\times2=96\ \text{Ans.}$$

3. If a man travel 217 miles in 7 days, travelling 6 hours a day, how far would he travel in 9 days, if he travelled 11 hours a day?

OPERATION.

days		*days*		*miles*		*miles.*
		3				
7	:	$\not{9}$	: :	217	:	1*st answer.*
$\not{6}$	:	11	: :	——	:	*true answer.*
2						

Ans. 511½ *miles.*

4. If a pasture of 16 acres will feed 6 horses for 4 months, how many acres will feed 12 horses for 9 months? *Ans.* 72.

5. If the wages of 6 men for 14 days be $84, what will be the wages of 9 men for 11 days? *Ans.* 99.

6. If 154 bushels of oats serve 14 horses for 44 days, how long would 406 bushels last 7 horses? *Ans.* ——

7. If 25 men can earn $6250 in 2 years, how long will it take 5 men to earn $11250? *Ans.* 18*yr.*

8. If a barrel of beer last 7 persons 12 days, how much will be drank by 42 persons in a year?
Ans. 182*bar.* 18*gal.*

9. If 9 men can cut 36 acres of grass in 4 days, how many acres will 19 men cut in 11 days?
Ans. 209 *acres.*

10. If 25 persons consume 300 bushels of corn in 1 year, how much will 139 persons consume in 7 years at the same rate? *Ans.* ——

11. If 32 men build a wall 36 feet long, 8 feet high, and 4 feet wide in 4 days, in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet wide?
Ans. 36 *days.*

12. If £7½ be the wages of 13 men for 7½ days, what will be the wages of 22 men for 17⅓ days?
Ans. £29 6*s.* 8*d.*

13. If a footman travel 298 miles in 8⅔ days of 12½ *hours long*, in how many days of 10⅔ hours long each, *will he* travel 157⅛ miles? Ans. 5$\frac{27085}{76288}$*da.*

PRACTICE.

196. Practice is a short method of finding the answers to questions in the Rule of Three, when the first term is unity.

For example, if one yard of cloth cost half a dollar, what will 60 yards cost? This is a question which may be answered by the rule called Practice.

If the cloth had been $1 per yard, the cost of 60 yards would have been $60; but since it is only a part of a dollar per yard, the whole cost will be the same part of $60, that the cost of one yard is of $1; that is, ½ of 60. Hence the cost is ½ of $60 or $30. *Ans.* $30.

197. One number is said to be an aliquot part of another, when it forms an exact part of it; that is, when it is contained in that other an exact number of times. Hence, an aliquot part is an exact or even part.

For example, 25 cents is an aliquot part of a dollar. It is an exact fourth part, and is contained in the dollar four times. So also, 2 months, 3 months, 4 months, and 6 months, are all aliquot parts of a year.

TABLE OF ALIQUOT PARTS.

Cts.	*Parts of* $1.	*Mo.*	*Parts of a year.*	*Days.*	*Parts of 1 mo.*	*Parts of* £1.	*Parts of 1 shilling.*
50	½	6	½	15	½	10*s.* = ½	6 *d.* = ½
33⅓	⅓	4	⅓	10	⅓	6*s.* 8*d.* = ⅓	4 *d.* = ⅓
25	¼	3	¼	7½	¼	5*s.* = ¼	3 *d.* = ¼
20	⅕	2	⅙	6	⅕	4*s.* = ⅕	2 *d.* = ⅙
12½	⅛	1	$\frac{1}{12}$	5	⅙	3*s.* 4*d.* = ⅙	1½*d.* = ⅛
6¼	$\frac{1}{16}$		or ⅓ of	3	$\frac{1}{10}$	2*s.* 6*d.* = ⅛	1 *d.* = $\frac{1}{12}$
5	$\frac{1}{20}$		3 *mo.*			1*s.* 8*d.* = $\frac{1}{12}$	

196. What is practice? If one yard of cloth cost $8, what will half a yard cost? What will one quarter of a yard cost?

197. When is one number said to be an aliquot part of another? What is an aliquot part? What are the aliquot parts of a dollar expressed in the table? What the aliquot parts of a year? What the aliquot parts of a month? What the aliquot parts of a pound? What the *aliquot parts of a shilling*?

EXAMPLES.

1. What is the cost of 376 yards of cloth at \$0,75, or $\frac{3}{4}$ of a dollar per yard?

Had the cloth cost \$1 per yard, the cost of the 376 yards would have been \$376. Had it cost 50*cts.* per yard, the cost would have been $\frac{1}{2}$ of \$376, or \$188: had it been 25*cts.* per yard, the cost would have been $\frac{1}{4}$ of \$376 or \$94; but the price being 75*cts.* per yard, the cost is 188+94=\$282.

OPERATION.

cts.		\$	
50	$\frac{1}{2}$	376	
		188	cost at 50*cts.*
25	$\frac{1}{4}$	94	cost at 25*cts.*
75	$\frac{3}{4}$	\$282	cost at $\frac{3}{4}$*doll.*

2. What will $9\frac{1}{2}$ yards of cloth cost at £1 4*s.* 6*d.* per yard?

9 yards at £1=£9			
$\frac{1}{2}$ yard at £1=	10*s.*		
9 yards at 4*s.* =£1	16*s.*		
$\frac{1}{2}$ yard at 4*s.* =	2*s.*		
9 yards at 6*d.*=	4*s.*	6*d.*	
$\frac{1}{2}$ yard at 6*d.*=		3*d.*	
Total cost £11	12*s.*	9*d.*	

3. What is the cost of 196 yards of cotton, at 9*d.* per yard?

196*yd.* at 6*d.* or $\frac{1}{2}$*s.*= 98*s.*
196*yd.* at 3*d.* or $\frac{1}{4}$*s.*= 49*s.*

Therefore, 196*yd.* at 9*d.* or $\frac{3}{4}$*s.*=147*s.*=£7 7*s.* *Ans.*

4. What is the cost of 1000 quills at $\frac{1}{4}$ cent per quill? *Ans.* \$2,50.

5. What is the cost of 900 lead pencils at 6 cents apiece? *Ans.* \$54,00.

6. What is the cost of 20*lb.* of soap at $6\frac{3}{4}$*cts.* per *lb.*? *Ans.* \$1,35.

7. What is the cost of 140 yards of tape at $2\frac{1}{4}$*cts.* per yard? *Ans.* \$3,15.

8. What is the cost of 438 bushels of apples at $31\frac{1}{4}$*cts.* per bushel? *Ans.* \$136,87$\frac{1}{2}$.

9. What is the cost of 51½ tons of hay at \$12 per ton? *Ans.* \$618.

10. What is the cost of 231 yards of linen at 75*cts.* per yard? *Ans.* \$173,25.

11. What is the cost of 144*lb.* of rice at 3½*d.* per *lb.*? *Ans.* £2 2*s.*

12. What is the cost of 14¼ yards of cloth, at \$4¾ per yard? *Ans.* \$67,68¾.

13. What will 131*lb.* of cheese come to at 1*s.* 2*d.* per pound? *Ans.* £7 12*s.* 10*d.*

14. What will 144 dozen of eggs come to at 1*s.* 3*d.* per dozen? *Ans.* £9.

15. What will 6*gal.* 1*qt.* 1*pt.* 2*gi.* of wine come to at 5*s.* 4*d.* per quart? *Ans.* £6 17*s.* 4*d.*

16. What will 51 acres of land be worth at £3 2*s.* 2*d.* per acre? *Ans.* £158 10*s.* 6*d.*

17. What will 15*cwt.* 2*qr.* 17*lb.* of sugar come to at 1*s.* per pound? *Ans.* £87 13*s.*

18. What will 4*E. E.* 3*qr.* 2*na.* of broadcloth cost at £2 3*s.* 8*d.* per yard? *Ans.* £12 16*s.* 6½*d.*

19. What will 1*hhd.* 2*gal.* 3*qt.* 1*pt.* 1*gi.* of molasses come to at 12½*cts.* per quart? *Ans.* ——

20. What will be the cost of 27*bu.* 3*pk.* 6*qt.* 1*pt.* of wheat at 10*s.* 2*d.* 3*far.* per bushel?
Ans. £14 5*s.* 11*d.* 0$\frac{63}{64}$*far.*

21. What is the cost of 120 pounds of soap at 6¾ cents per pound? *Ans.* \$8,10.

22. What will be the cost of 2*hhd.* 5*gal.* 3*qt.* 2*gi.* of molasses at 12½ cents per quart? *Ans.* \$65,906+.

23. What will be the cost of 376 yards of cloth at 1¾ dollars per yard? *Ans.* ——

24. What will be the cost of 1*hhd.* 2*gal.* 3*qt.* 1*pt.* 1*gi.* of brandy at 56¼ cents per quart? *Ans.* \$148,289+.

25. What will be the cost of 85½ yards of cloth at \$9½ per yard? *Ans.* ——

26. What will be the cost of 27*bu.* 3*pk.* 6*qt.* 1*pt.* of wheat at \$1¾ per bushel? *Ans.* \$48,917+.

PERCENTAGE.

198. The term per cent comes from per centum, and means by the hundred. The term is generally used to express the interest on money, but may also be employed to designate hundredth parts of other things. Thus, when we say twenty per cent of a bushel of wheat, we mean twenty hundredths, or one-fifth of it.

199. The rate per cent may always be expressed by a decimal fraction. Thus, five per cent may be expressed by .05, eight per cent by .08, fifteen per cent by .15, &c.

Hence, to find the amount of percentage on any number,

Multiply the number by the rate per cent, expressed in a decimal fraction, and the product will be the percentage.

EXAMPLES.

1. A has $852 deposited in the bank, and wishes to draw out 5 per cent of it: how much must he draw for? *Ans.* ——

2. A merchant has 1200 barrels of flour: he shipped 64 per cent of it and sold the remainder: how much did he sell? *Ans.* 432*bar.*

3. A merchant bought 1200 hogsheads of molasses. On getting it into his store, he found it short 3½ per cent: how many hogsheads were wanting? *Ans.* 42*hhd.*

4. Two men had each $240. One of them spends 14 per cent, and the other 18½ per cent: how many dollars more did one spend than the other? *Ans.* $10,80.

5. What is the difference between 5½ per cent of $800 and 6½ per cent of $1050? *Ans.* $24,25.

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198. What do you understand by the term per cent? For what is the term generally used? What is its signification? What do you understand by twenty per cent? What by eight per cent? By fourteen per cent?

199. How may the rate per cent be expressed? How do you express five per cent? Eight per cent? How do you find the amount of percentage on any given number?

200. To find the per cent which one number is of another.

If I buy 6 hogsheads of molasses for $200 and sell them for $220, what do I gain per cent, on the money expended?

It is plain that $20 is the amount made. What per cent is $20 of $200; that is, how many hundredths of $200? If we add two ciphers to the first, and then divide it by the second, the quotient will express the hundredths. Thus,

$$\frac{2000}{200}=10;$$

that is, 20 is ten per cent of 200.

Hence, to determine the per cent which one number is of another,

I. *Bring the number which determines the per cent to hundredths by annexing two ciphers or removing the decimal point two places to the right.*

II. *Divide the number so formed by the number on which the percentage is estimated, and the quotient will express the per cent.*

EXAMPLES.

1. A man has $550 and purchases goods to the amount of $82,75: what per cent of his money does he expend? *Ans.* $15\frac{1}{22}$ *per cent.*

2. A merchant goes to New York with $1500; he first lays out 20 per cent, after which he expends $660: what per cent was his last purchase of the money that remained after his first? *Ans.* 55 *per cent.*

3. Out of a cask containing 300 gallons, 60 gallons are drawn: what per cent is this? *Ans.* ——

4. If I pay $698,33 for 3 hogsheads of molasses and sell them for $837,996, how much do I gain per cent on the money laid out? *Ans.* 20 *per cent.*

5. If I pay $698,33 for 3 hogsheads of sugar and sell them for $837,996, how much do I gain per cent on the amount received? *Ans.* $16\frac{2}{3}$ *per cent.*

---

200. How do you find the per cent which one number is of another?

TARE AND TRET.

201. *Tare* and *Tret* are allowances made in selling goods by weight.

Draft is an allowance on the gross weight in favor of the buyer or importer: it is always deducted before the *Tare.*

Tare is an allowance made to the buyer for the weight of the hogshead, barrel, or bag, &c., containing the commodity sold.

Gross Weight is the whole weight of the goods, together with that of the hogshead, barrel, bag, &c., which contains them.

Suttle is what remains after a *part* of the allowances have been deducted from the gross weight.

Net Weight is what remains after all the deductions are made.

EXAMPLES.

1. What is the net weight of 25 hogsheads of sugar, the gross weight being 66*cwt.* 3*qr.* 14*lb.*; tare 11*lb.* per hogshead?

	cwt.	*qr.*	*lb.*	
	66	3	14	gross.
$25 \times 11 = 275$*lb.* - -	2	3		tare.
Ans.	64	0	14	net.

2. If the tare be 4*lb.* per hundred, what will be the tare on 6*T.* 2*cwt.* 3*qr.* 10*lb.*?

Tare for 6*T.* or 120*cwt.* = 480*lb.*
2*cwt.* = 8
3*qr.* = 3
10*lb.* = $0\frac{2}{5}$
Tare - - - - $491\frac{2}{5}$

Ans. 4*cwt.* 3*qr.* $16\frac{2}{5}$*lb.*

---

201. What are Tare and Tret? What is Draft? What is Tare? What is Gross Weight? What is Suttle? What is Net Weight?

3. What is the tare on 32 boxes of soap, weighing 31550*lb.*, allowing 4*lb.* per box for draft and 12 per cent for tare?

	31550 gross.		31422
32×4=	128 draft.		.12
	31422		3770.64

Ans. 3770.64*lb.*=1*T.* 17*cwt.* 2*qr.* 20*lb.* 10*oz.*+.

4. What will be the cost of 3 hogsheads of tobacco at $9,47 per *cwt.* net, the gross weight being of

	cwt.	*qr.*	*lb.*		*lb.*
No. 1 - -	9	3	25 - -	tare	146
" 2 - -	10	2	12 - -	"	150
" 3 - -	11	1	25 - -	"	158

Ans. $261,1826.

5. At £1 5*s.* per *cwt.* net; tare 4*lb.* per *cwt.*: what will be the cost of 4 hogsheads of sugar weighing gross,

	cwt.	*qr.*	*lb.*
No. 1 - - -	10	3	6
" 2 - - -	12	5	2
" 3 - - -	13	1	10
" 4 - - -	11	2	7
	49	0	00 gross.
Tare 4*lb.* per *cwt.*	1	3	21
	47	0	4 net.

Ans. £58 16*s.*

6. At 21 cents per *lb.*, what will be the cost of 5*hhd.* of coffee weighing in gross,

	cwt.	*qr.*	*lb.*		*lb.*
No. 1 - -	5	2	14 - -	tare	94
" 2 - -	9	1	20 - -	"	100
" 3 - -	6	2	22 - -	"	88
" 4 - -	7	2	25 - -	"	89
" 5 - -	8	0	13 - -	"	100

Ans. $713,58.

7. At £7 5*s.* per *cwt.* net, how much will 20*hhd.* of sugar come to, each weighing gross 8*cwt.* 0*qr.* 5*lb.*; tare 12*lb.* per *cwt.*? *Ans.* £927 3*s.* 7½*d.*

SIMPLE INTEREST.

202. Interest is an allowance made for the use of money that is borrowed.

For example, if I borrow $100 of Mr. Wilson for one year, and agree to pay him $6 for the use of it, the $6 is called the *interest* of $100 for one year, and at the end of the time Mr. Wilson should receive back his $100 together with the $6 interest, making the sum of $106.

The money on which interest is paid, is called the *Principal.*

The money paid for the use of the principal, is called the *Interest.*

The principal and interest, taken together, are called the *Amount.*

In the above example,

$100 is the principal,
$ 6 is the interest, and
$106 is the amount.

The interest of $100 for one year, determines the rate of interest, or rate per cent. In the example above, the rate of interest is 6 per cent, or $6 for the use of the hundred. Had $8 been paid for the use of the $100, the rate of interest would have been 8 per cent: or had $3 only been paid, the rate of interest would have been 3 per cent.

Legal interest is the rate of interest established by law. In the New England States, and indeed in most of the other states, the legal interest is 6 per cent per annum, that is, 6 per cent by the year. In New York, however, it is 7, and in Alabama 8 per cent.

---

202. What is Interest? What is the money called on which interest is paid? What is the money called which is paid for the use of the principal? What is the amount? What determines the rate of interest? What is legal interest? What is meant by per annum? How much is the interest per annum in most of the states? What is it in New York? In Alabama?

CASE I.

203. To find the interest on any given principal for one or more years.

The interest of each dollar, for a single year, will be so many hundredths of itself as are expressed by the rate of interest. Thus, if the rate of interest be 4 per cent, each dollar will produce annually an interest of .04 of a dollar, or 4 cents: if the rate be 5 per cent, it will produce .05 of a dollar, or 5 cents: if 6 per cent, .06, or 6 cents, &c.

Hence, to find the interest on any given sum for one or more years,

Multiply the principal by the decimal fraction which expresses the rate of interest, and the product so arising by the number of years. Or,

Multiply the decimal fraction which expresses the rate of interest by the number of years, and then multiply the principal by this product.

EXAMPLES.

1. What is the interest on $1960 for four years, at 7 per cent per annum?

The rate of interest being 7 per cent, each dollar will produce .07 of a dollar, or 7 cents, in one year: hence, $137,20 will be the interest on the sum for one year, and $548,80 for 4 years.

OPERATION.

$1960	
.07	
$137,20	int. for 1 year.
4	number of years.
$548,80	*Ans.*

---

203. What will be the interest of one dollar for one year? What will express decimally the interest on one dollar for one year at 4 per cent? What will express it at 5 per cent? At 6? At 7? At 8? How do you find the interest on any sum for one or more years? What w[illegible] multiplier when the rate of interest is 4 per cent, and the [illegible] *years?* When the rate is 6 per cent and the time 5 *years? When the rate is* 8 per cent and the time 3 years?

2. What is the interest on $78,457 dollars for three years, at 5 per cent per annum?

Since there are three places of decimals in the multiplicand and two in the multiplier, there will be five in the product (ART. 169.). Observe *that the two first, counting from the comma to the right,* are cents, the third mills, the fourth tenths of mills, &c.

OPERATION.

```
              78,457
.05×3=           .15
              392285
              78457
Ans.    $11,76855
```

3. What is the interest on $365,874 for one year, at 5½ per cent?

We first find the interest at 5 per cent, and then the interest for ½ per cent: the sum is the interest at 5½ per cent.

OPERATION.

```
$365,874
    .05½
18,29370
 1,82937  ½ per cent.
$20,12307 Ans.
```

4. What is the interest on $650 for one year at 6 per cent? *Ans.* $39,00.

5. What is the interest on $950 for four years at 7 per cent per annum? *Ans.* ——

6. What is the interest on $3675 for three years at 7 per cent per annum? *Ans.* $771,75.

7. What is the interest on $459 for five years at 8 per cent per annum? *Ans.* $183,60.

8. What is the interest on $375, 27*cts.* 3*m.* for two years at 7 per cent? *Ans.* $52,538+.

9. What is the interest on $211,26 for one year at 4½ per cent per annum? *Ans.* $9,506+.

10. What is the interest on $1576,91 for 3 years at 7 per cent? *Ans.* $331,15+.

11. What is the interest on $957,08 for 6 years at 3½ per cent? *Ans.* ——

12. What is the interest on $375,45 for 7 years at 7 per cent per annum? *Ans.* $183,970+.

13. What is the interest on $4049,87 for 2 years at 5 per cent per annum? *Ans.* $404,98+.

14. What is the interest on $16199,48 for 16 years at $5\frac{1}{4}$ per cent per annum? *Ans.* ——

CASE II.

204. To find the interest for any number of months, at the rate of 6 per cent per annum.

At the rate of 6 per cent per annum, one month produces $\frac{1}{2}$ per cent on the principal; and hence, every two months produces one per cent on the principal. Therefore to find the interest for months,

Divide the number of months by 2 and regard the quotient as hundredths. Then multiply the principal by the decimal so found, and the product will be the interest.

EXAMPLES.

1. What is the interest on $651 for 8 months, at 6 per cent per annum?

The decimal corresponding to 8 months is .04: hence, the interest is $26,04.

OPERATION.

$651	
.04	half the number of months.
$26,04	

2. What is the interest on $614,364 for 9 months, at 6 per cent per annum?

The decimal corresponding to 9 months, is $.04\frac{1}{2}$, and hence the interest is $27,64638.

OPERATION.

$614,364
$.04\frac{1}{2}$
2457456
307182
$27,64638

3. What is the interest on $8975 for ten months at 6 per cent per annum? *Ans.* $448,75.

204. At the rate of 6 per cent, what will be the interest on any principal for one month? What time will produce one per cent? How do you find the interest on any principal for any number of months? What is the multiplier for 4 months? What for 6 months? What for 7? What for 8? For 9? What for 10? For 11? What for 12?

4. What is the interest on $8753,65 for 14 months at 6 per cent per annum? *Ans.* $612,7555.

5. What is the interest on $37596,42 for sixteen months at 6 per cent per annum? *Ans.* ——

6. What is the interest on $3976,85 for nine months at 6 per cent per annum? *Ans.* $178,95825.

7. What is the interest on $17507,30 for 14 months, at 6 per cent? *Ans.* $1225,511.

8. What is the interest on $75192,84 for 16 months at 6 per cent? *Ans.* ——

9. What is the interest on $15907,40 for 27 months at 6 per cent? *Ans.* $2147,499.

10. What is the interest on $84377,91 for 7 months at 6 per cent? *Ans.* $2953,226+.

11. What is the interest on $31573,25 for 10 months, at 6 per cent? *Ans.* $1578,662+.

CASE III.

205. To find the interest at 6 per cent per annum for any number of days.

In computing interest the month is reckoned at 30 days. Hence, 60 days, which make two months, will give an interest of one per cent on the principal, and consequently, 6 days will give an interest of one mill on the dollar, or one thousandth of the principal. If, therefore, the days be divided by 6, the quotient will show how many thousandths of the principal must be taken on account of the days. Hence, to find the interest for any number of days less than 60,

Divide the days by 6, and multiply the principal by the quotient, considered as thousandths.

---

205. In computing interest for days, at what is the month reckoned? How many days give one per cent? What part of the principal is one per cent? How many days will give one thousandth of the principal? How will you find how many thousandths of the principal must be taken for the days? How do you find the interest for days? What is the multiplier for 6 days? For 9 days? For 10 days? For 15 days? For 20 days? For 25 days?

EXAMPLES.

1. What is the interest on $297,047 for 28 days, at 6 per cent per annum?

We find that the 28 days give $4\frac{2}{3}$ thousandths. We multiply the principal by .004, and then add $\frac{2}{3}$ of the principal multiplied by one thousandth for the fractional part.

OPERATION.

	$297,047
$28 \div 6 = 4\frac{2}{3}$	$.004\frac{2}{3}$
	1188188
Add $\frac{1}{3}$	99015
" $\frac{1}{3}$	99015
	$1,386218

206. To avoid the fractions which sometimes appear in the multipliers, we may, if we please, first multiply the principal by the number of days, and then divide the product by 6, which will give the same quotient as found above. Hence, to find the interest for any number of days,

Multiply the principal by the number of days, divide the product by 6, and then point off in the quotient three more places for decimals than there are decimal places in the given principal.

2. What is the interest on $657,87 for 13 days at 6 per cent per annum?

We first multiply the given principal by 13; we then divide the product by 6; and since there are two places of decimals in the principal, we point off five in the quotient.

OPERATION.

$657,87
13
197361
65787
6)855231
$1,42538

NOTE. Let each of the following examples be worked by both methods; though, when the days exceed 60, the second method is preferable.

3. What is the interest on $575,72 for 29 days?

Ans. $2,78+.

4. What is the interest on $195,19 for 7 days?

---

206. How may the interest for days be found by the second method?

5. What is the interest on $897,04 for 27 days? *Ans.* $4,0366+.

6. What is the interest on $378,53 for 18 days? *Ans.* $1,135+.

7. What is the interest on $885,62 for 25 days? *Ans.* $3,69+.

8. What is the interest on $3756,25 for 17 days?

9. What is the interest on $981,90 for 70 days? *Ans.* $11,455+.

10. What is the interest on $11268,75 for 17 days? *Ans.* $31,928+.

11. What is the interest on $4428,10 for 165 days? *Ans.* $121,772+.

12. What is the interest on $975,95 for 14 days?

207. The above method of computing interest for days, is the one in general use. It, however, considers the year as made up of 360 instead of 365 days; and hence the result is too large by 5 of the 365 parts into which the interest found may be divided. Hence, the interest found will be too large by its $\frac{1}{73}$ part, by which it must be diminished when entire accuracy is desired.

CASE IV.

208. To find the interest at 6 per cent per annum for years, months, and days.

Find the interest for the years by Case I., for the months by Case II., and for the days by Case III.; then add the several results together, and their sum will be the answer sought. Or,

Form a single multiplier for the years, months, and days, and then multiply the principal by it.

---

207. How many days does the above method give to the year? Is the result obtained too great or too small? By how much is it too great? How will you find the exact interest?

208. How do you find the interest at 6 per cent per annum for years, months, and days? What is the multiplier for 1 year 4 months and 12 days? What for 2 years 8 months and 18 days? For 3 years 10 months and 24 days?

EXAMPLES.

1. What is the interest on $1597,27 at 6 per cent, for 3 years 9 months and 11 days?

1st METHOD.

	$1597,27	$1597,27	$1,59727 for 6*da.*
$.06\times3=$	.18	$.04\frac{1}{2}$	,79863 for 3*da.*
	1277816	638908	,53242 for 2*da.*
	159727	$79863\frac{1}{2}$	$2,92832 for 11*da.*
	$287,5086	$\$71,8771\frac{1}{2}$	

Interest for	3 years	$287,5086
" "	9 months	71,8771+
" "	11 days	2,9283+
	Total interest	$362,3140+

2d METHOD.

Multiplier for	3 years $=.06\times3=$	.18.
" "	9 months	$=.045.$
" "	11 days $=\frac{11}{6}$	$=.001\frac{5}{6}.$
	Entire multiplier	$0.226\frac{5}{6}.$

Then, $\$1597,27\times0.226\frac{5}{6}=\$362,3140+$.

2. What is the interest of $11759,10 at 6 per cent for 9 years 11 months and 16 days? *Ans.* $7028,02+.

3. What is the interest on $9787 for 12 years and 1 day? *Ans.* $7048,27+.

4. What is the interest of $87601,29 for 1 year 1 month and 1 day? *Ans.* ——

5. What is the interest of $806,90 for 1 year and 10 months at 6 per cent per annum? *Ans.* $88,75+.

6. What is the interest of $450,75 for 4 years and 7 months at 6 per cent per annum? *Ans.* $123,95+.

7. What is the interest of $443,50 for 7 years 2 months and 12 days at 6 per cent per annum? *Ans.* ——

8. What will be the total amount of $649,22 after 10 years and 10 months at an interest of 6 per cent?
Ans. $1071,21+.

9. What will be the interest of $1330,50 for 14 years 4 months and 24 days? *Ans.* ——

CASE V.

209. To find the interest when there are months and days, and the rate of interest is greater or less than 6 per cent.

Find the interest at 6 per cent. Then add to it or subtract from it such a part of the interest so found as the given rate exceeds or falls short of 6 per cent per annum.

EXAMPLES.

1. What is the interest on $179,25 at 7 per cent per annum for 3 years and 4 months?

Multiplier for 3 years$=.06\times3=$	.18
" " 4 months $=$	.02
Entire multiplier	.20

Hence, $179,25 $\times$ .20 $=$ $35,8500 interest at 6 per cent.
Add $\frac{1}{6}$ 5,9750
$41,8250 interest at 7 per cent.

2. What is the interest on $974,50 for 9 years 6 months and 18 days, at 4 per cent per annum?

Multiplier for 9 years at 6 per cent$=9\times.06=$	.54
" " 6 months $=$	.03
" " 18 days$=18\div6=3$ $=$	.003
Entire multiplier	.573

Hence, $974,50 $\times$ .573 $=$ $558,3885
Subtract one-third - - - 186,1295
Int. at 4 per cent $372,2590

3. What is the interest of $987,99, at 5 per cent, for 5 years 2 months and 9 days? *Ans.* $256,46+.

4. What is the interest on $437,21, at 3 per cent, for 9 years and 9 months? *Ans.* $127,88+.

5. What is the interest of $15000 for 8 months at 7 per cent per annum? *Ans.* $700.

6. What is the interest of $400 for 21 days at 5 per cent per annum? *Ans.* $1,16+.

---

209. How do you find the interest when there are months and *days*, and the rate greater than 6 per cent? How do you find the interest when it is less?

7. What is the interest of $876,48, at 7 per cent, for 4 years 9 months and 14 days? *Ans.* ——

8. What will be the total amount of $1119,48 after 2 years and a half, at an interest of 7 per cent per annum? *Ans.* $1315,389.

9. What is the interest on $532,41 for 3 years and 3 months at $4\frac{1}{2}$ per cent per annum? *Ans.* $77,86+.

10. What is the interest on $8375,27, at 5 per cent per annum, for 5 years 5 months and 5 days?

11. What is the interest of $8759,27, at 6 per cent per annum, for 1 year 6 months and 9 days? *Ans.* $801,473+.

12. What is the interest, at $6\frac{1}{2}$ per cent per annum, on $7569,11, for 3 years 4 months and 18 days? *Ans.* $1664,573+.

210. In computing interest, it is often very convenient to find the interest for the months by considering them as aliquot parts of a year, and the interest for the days by considering them as aliquot parts of a month.

EXAMPLES.

1. What is the interest of $806,90 for one year 10 months and 10 days at 6 per cent?

$806,90			
.06			
6)$48,4140	int. for 1 year.	$8,069	
2)8,069	int. for 2 months.	5	
3)4,034+	int. for 1 month.	$40,345	int. for 10 mo.
1,344+	int. for 10 days.		

Interest for 1 year, - -	$48,4140
" " 10 months -	40,345
" " 10 days - -	1,344+
Total interest	$90,103+

210. Explain the second method of computing interest for months and days. What part of a year are 3 months? Four months? Six? Eight? Nine? What part of a month are 6 days? Five days? Ten days?

2. What is the interest of $200 for 10 years 3 months and 6 days at 7 per cent?

```
   $200
    .07
4)14,00  int. for 1 year.          $14,00
 3)3,50  int. for 3 months.            10
 5)1,16+int. for 1 month.         $140,00 for 10 years.
    ,23+int. for 6 days.
        $140,00  interest for 10 years.
           3,50  interest for 3 months.
            ,23+interest for 6 days.
   Ans. $143,73+
```

3. What is the interest of $132,26 for 1 year 4 months and 10 days, at 6 per cent per annum? *Ans*. $10,80+.

4. What is the interest of $25,50 for 1 year 9 months and 12 days, at 6 per cent? *Ans*. $2,72+.

5. What is the interest of $347,25 for 1 year 1 month and 6 days, at 4 per cent per annum? Also, at 5 per cent? At 5½ per cent? At 6 per cent? At 7 per cent? At 7½ per cent? At 8 per cent? At 8½ per cent? And at 9 per cent? *Ans*. ——

6. What is the interest, at 6 per cent per annum, on $48,32, for 1 year 1 month and 15 days? *Ans*. $3,26+.

7. What is the interest, at 8 per cent per annum, on $675,87, for 3 years 6 months and 6 days? *Ans*. $190,14+.

8. What is the interest, at 7 per cent, on $587,25, for 5 years 5 months and 5 days? *Ans*. $223,23+.

9. What is the interest on $67589,20 for 3 years 9 months and 12 days, at 5 per cent per annum? *Ans*. $12785,62+.

CASE VI.

211. When the sum on which the interest is to be cast is in pounds, shillings, and pence.

211. How do you determine the interest when the sum is in [illegible]*ds, shillings, and pence?*

RULE.

I. *Reduce the shillings and pence to the decimal of a pound.*

II. *Then find the interest as though the sum were dollars and cents; after which reduce the decimal part of the answer to shillings and pence.*

EXAMPLES.

1. What is the interest, at 6 per cent, of £27 15*s*. 9*d*. for 2 years?

We first find the interest for one year. We then multiply by 2, which gives the interest for two years. We then reduce to pounds, shillings, and pence.

OPERATION.

$$
\begin{array}{r}
£27\ 15s.\ 9d. = £27.7875 \\
.06 \\
\hline
1.667250 \\
2 \\
\hline
£3.334500 \\
20 \\
\hline
6.690000 \\
12 \\
\hline
8.280000 \\
4 \\
\hline
1.120000
\end{array}
$$

Ans. £3 6*s*. 8¼*d*.+

2. What is the interest on £67 19*s*. 6*d*., at 6 per cent, for 3 years 8 months 16 days? *Ans*. £15 2*s*. 8½*d*+.

3. What is the interest on £127 15*s*. 4*d*., at 6 per cent, for 3 years and 3 months? *Ans*. £24 18*s*. 3¼*d*.+.

4. What is the interest of £107 16*s*. 10*d*., at 6 per cent, for 3 years 6 months and 6 days? *Ans*. ——

5. What will £279 13*s*. 8*d*. amount to in 3 years and a half, at 5¼ per cent per annum?

Ans. £331 1*s*. 6*d*.+.

6. What is the interest of £514 10*s*. 2*d*. for 3 years and a half, at 4 per cent? *Ans*. £72 0*s*. 7¼*d*.+.

7. What is the interest of £523 11*s*. 6*d*. for 3 years and a half, at 6 per cent? *Ans*. ——

8. What is the interest on £255 10*s*. 8*d*., at 6 per cent per annum, for 6*yr*. 6*mo*.? Ans. £99 13s. 1¾d.

APPLICATIONS.

212. In computing the interest on notes, observe that the day on which a note is dated and the day on which it falls due, are not both reckoned in determining the time, *but one of them is always excluded.* Thus, a note dated on the first of May and falling due on the 16th of June, will bear interest but one month and 15 days.

Calculate the interest on the following notes:

$127,50 New York, January 1st, 1847.

1. For value received I promise to pay on the 10th day of June next, to Wm. Johnson or order, the sum of one hundred and twenty-seven dollars and fifty cents with interest from date, at 7 per cent. *John Liberal.*

Ans. $131,435+.

$306 Rochester, January 1st, 1843.

2. For value received I promise to pay on the 4th of July, 1845, to Wm. Johnson or order, three hundred and six dollars with interest at 6 per cent from the 1st of March, 1843. *John Liberal.*

Ans. $348,993.

$1040 Harrisburg, July 3d, 1847.

3. Six months after date, I promise to pay to C. Jones or order, one thousand and forty dollars with interest from the 1st of January last, at 7 per cent.

Joseph Springs.

Ans. $1113,204+.

$612 Baltimore, January 1st, 1833.

4. For value received I promise to pay on the 4th of July, 1835, to Wm. Johnson or order, six hundred and twelve dollars with interest at 6 per cent from the 1st of March, 1833. *John Liberal.*

Ans. $697,986.

---

212. What days named in a note are reckoned, and what excluded, in reckoning time? If a note is dated on the 1st and payable on the 15th of a month, how many days will interest run?

PARTIAL PAYMENTS.

213. We shall now give the rule established in New York, (see Johnson's Chancery Reports, vol. i. page 17,) for computing the interest on a bond or note, when partial payments have been made. The same rule is also adopted in Massachusetts, and in most of the other states.

RULE.

I. *Compute the interest on the principal to the time of the first payment, and if the payment exceed this interest, add the interest to the principal and from the sum subtract the payment: the remainder forms a new principal.*

II. *But if the payment is less than the interest, take no notice of it until other payments are made, which in all, shall exceed the interest computed to the time of the last payment: then add the interest, so computed, to the principal, and from the sum subtract the sum of the payments: the remainder will form a new principal on which interest is to be computed as before.*

EXAMPLES.

$349,99 8 Buffalo, May 1st, 1826.

1. For value received I promise to pay James Wilson or order, three hundred and forty-nine dollars ninety-nine cents and eight mills, with interest at 6 per cent.

James Paywell.

On this note were endorsed the following payments:

Dec. 25th, 1826	Received	$49,998
July 10th, 1827	"	$ 4,998
Sept. 1st, 1828	"	$15,008
June 14th, 1829	"	$99,999

What was due April 15th, 1830?

Principal on int. from May 1st, 1826, . .	$349,998
Interest to Dec. 25th, 1826, time of first payment, 7 months 24 days	13,649+
Amount	$363,647

213. What is the rule in regard to partial payments?

Payment Dec. 25th, exceeding interest then due		$ 49,998
Remainder for a new principal		$313,649
Interest of $313,649 from Dec. 25th, 1826, to June 14th, 1829, 2 years 5 months 20 days		$ 46,524+
Amount		$360,173
Payment, July 10th, 1827, less than interest then due . .	$ 4,998	
Payment, Sept. 1st, 1828 . . .	15,008	
Their sum, less than interest then due	$20,006	
Payment, June 14th, 1829 . . .	99,999	
Their sum exceeds the interest then due .		$120,005
Remainder for a new principal, June 14th, 1829		$240,168
Interest of $240,168 from June 14th, 1829, to April 15th, 1830, 10 months 1 day .		12,048
Total due, April 15th, 1830 .		$252,216+

$3469,32 New York, Feb. 6th, 1825.

2. For value received, I promise to pay William Jenks, or order, three thousand four hundred and sixty-nine dollars and thirty-two cents, with interest from date, at 6 per cent. *Bill Spendthrift.*

On this note were endorsed the following payments:—
May 16th, 1828, received $ 545,76.
May 16th, 1830, " $1276,00.
Feb. 1st, 1831, " $2074,72.

What remained due August 11th, 1832?

Ans. $861,018+.

3. A's note of $635,84 was dated Sept. 5, 1817, on which were endorsed the following payments, viz.:— Nov. 13th, 1819, $416,08; May 10th, 1820, $152,00: *what* was due March 1st, 1821, the interest being 6 per *cent?* *Ans.* $168,01+.

TABLE

Showing the number of shillings in a dollar in each State, and the rate of interest: also, the value of a dollar expressed in parts of a pound, which is found by dividing the number of pence in a dollar by the number in a pound.

	STATES.	Number of shillings to the dollar.	Value of the dollar in pounds.	Legal rate of interest.
1	Maine	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
2	N. Hampshire	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
3	Vermont	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
4	Massachusetts	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
5	Rhode Island	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
6	Connecticut	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
7	New York	8 shillings	$\$1=£\frac{96}{240}=£\frac{2}{5}$	7 per cent.
8	Ohio	8 shillings	$\$1=£\frac{96}{240}=£\frac{2}{5}$	6 per cent.
9	New Jersey	7*s.* 6*d.*	$\$1=£\frac{90}{240}=£\frac{3}{8}$	6 per cent.
10	Pennsylvania	7*s.* 6*d.*	$\$1=£\frac{90}{240}=£\frac{3}{8}$	6 per cent.
11	Delaware	7*s.* 6*d.*	$\$1=£\frac{90}{240}=£\frac{3}{8}$	6 per cent.
12	Maryland	7*s.* 6*d.*	$\$1=£\frac{90}{240}=£\frac{3}{8}$	6 per cent.
13	Michigan	8 shillings	$\$1=£\frac{96}{240}=£\frac{2}{5}$	6 per cent.
14	Indiana	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
15	Illinois	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
16	Missouri	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
17	Virginia	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
18	Kentucky	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
19	Tennessee	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
20	N. Carolina	10 shillings	$\$1=£\frac{120}{240}=£\frac{1}{2}$	6 per cent.
21	S. Carolina	4*s.* 8*d.*	$\$1=£\frac{56}{240}=£\frac{7}{30}$	7 per cent.
22	Georgia	4*s.* 8*d.*	$\$1=£\frac{56}{240}=£\frac{7}{30}$	7 per cent.
23	Alabama	Fed. money		8 per cent.
24	Mississippi	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	6 per cent.
25	Louisiana	Fed. money		6 per cent.
26	Arkansas	Fed. money		6 per cent.
27	Florida	Fed. money		6 per cent.
28	Texas	6 shillings	$\$1=£\frac{72}{240}=£\frac{3}{10}$	8 per cent.
29	Canada	5 shillings	$\$1=£\frac{60}{240}=£\frac{1}{4}$	6 per cent.
30	*English*	4*s.* $1\frac{71}{121}$*d.*	$\$1=£\frac{1}{4.84}$	

COINS AND CURRENCIES.

214. Coins are pieces of metal, of gold, silver, or copper, of fixed values, and impressed with a public stamp prescribed by the country where they are made. These are called specie, and are generally declared to be a legal tender in payment of debts. The Constitution of the United States provides, that gold and silver only shall be a legal tender.

The coins of a country and those of foreign countries having a value established by law, together with bank notes redeemable in specie, make up the *Currency*.

The value of the English pound, or sovereign, is $4,84; and hence, the value of the English shilling is 24 cents and 2 mills.

It has already been shown (Art. 65), that Federal Money is the currency of the United States; the pound, however, is occasionally used.

There are two principal reductions:

1st. To change any sum expressed in Federal money into pounds shillings and pence.

Multiply the sum in dollars cents and mills, by the value of $1 expressed in the fraction of a pound, and the product will be the corresponding value in pounds and the decimal of a pound.

2d. To change any sum expressed in pounds shillings and pence, into Federal money.

Reduce the shillings and pence to the decimal of a pound by Art. 178, *and annex the decimal to the entire pounds. Then multiply by the fraction with its terms inverted, which expresses the value of $1 in terms of a pound, and the product will be dollars cents and mills.*

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214. What are coins? What are they called? What is declared in regard to them? What is provided by the Constitution of the United States? What do you understand by Currency? What is the value of the English pound? What of the English shilling? How do you change Federal money to pounds shillings and pence? How do you change pounds shillings and pence into Federal money?

EXAMPLES.

1. What is the value of $375,87, in pounds shillings and pence, New York currency?

We first multiply by $\frac{2}{5}$, and then reduce the decimal of a pound to shillings and pence.

OPERATION.

$375,87 \times \frac{2}{5} = £150.348$
$= £150\ 6s.\ 11\frac{1}{2}d. +$

2. What is the value of £127 18*s*. 6*d*., in Federal money, if the currency be 6 shillings to the dollar?

We first reduce the shillings and pence to the fraction of a £, and then multiply by the fraction of a dollar with its terms inverted.

OPERATION.

£127 18*s*. 6*d*. = 127.925
$127.925 \times \frac{10}{3} = \$426,416+$.

3. What is the value of $2863,75 in pounds shillings and pence, Pennsylvania currency? *Ans.* ——

4. What is the value of £459 3*s*. 6*d*., Georgia currency, in dollars and cents? *Ans.* $1967,892+.

5. What is the value of $9763,28, in pounds shillings and pence, North Carolina currency?

Ans. £4881 12*s*. $9\frac{1}{2}d.$

6. What is the value, in dollars and cents, of £637 18*s*. 8*d*., Nova Scotia currency? *Ans.* $2551,733.

7. Reduce $102,85 to the several currencies.

Ans.
$102,85 = £21 5*s*. English Money.
$102,85 = £25 14*s*. 3*d*. Canada Currency.
$102,85 = £30 17*s*. 1*d*.+ New England Cur.
$102,85 = £41 2*s*. $9\frac{1}{2}d.$+ New York Currency.
$102,85 = £38 11*s*. $4\frac{1}{2}d.$ Pennsylvania Cur.
$102,85 = £23 19*s*. $11\frac{1}{2}d.$ Georgia Currency.

8. Reduce $250 to the several currencies.

Ans.
$250 = £51 13*s*. $0\frac{1}{4}d.$ English Money.
$250 = £62 10*s*. Canada Currency.
$250 = £75 New England Currency.
$250 = £100 New York Currency.
$250 = £93 15*s*. Pennsylvania Currency.
$250 = £58 6*s*. $7\frac{3}{4}d.$ Georgia Currency.

COMPOUND INTEREST.

215. Compound Interest is when the interest on a sum of money becoming due, and not being paid, is added to the principal, and the interest then calculated on this amount, as on a new principal. Hence,

Calculate the interest to the time at which it becomes due: then add it to the principal and calculate the interest on the amount as on a new principal: add the interest again to the principal and calculate the interest as before: do the same for all the times at which payments of interest become due: from the last result subtract the principal, and the remainder will be the compound interest.

EXAMPLES.

1. What will be the compound interest, at 7 per cent, of $3750 for 4 years, the interest being added yearly?

	$3750,000	principal for 1st year.
$3750×.07=	262,500	interest for 1st year.
	4012,500	principal for 2d "
$4012,50×.07=	280,875	interest for 2d "
	4293,375	principal for 3d "
$4293,375×.07=	300,536+	interest for 3d "
	4593,911+	principal for 4th "
$4593,911×.07=	321,573+	interest for 4th "
	4915,484+	amount at 4 years.
1st principal	3750,000	
Amount of interest	$1165,484+	

2. If the interest be computed annually, what will be the interest on $100 for three years, at 6 per cent?
Ans. $19,101+.

3. What will be the compound interest on $295,37, at 6 per cent, for two years, the interest being added annually? *Ans.* $36,50+

215. What is Compound Interest? How do you find the compound interest on any sum?

STOCKS AND CORPORATIONS.

216. Stock is a general name for the money contributed by individuals for the establishment of banks and manufacturing companies, and the individuals who contribute the money are called *Stockholders.*

217. The individuals so associated are called, in their collective capacity, a *Corporation;* and the law which defines their rights and powers, is called the Charter of the Bank or Company.

218. The amount of money paid in by the stockholders to carry on the business of the corporation, is called the *Capital.* The capital is generally divided into a certain number of equal parts called *shares*, and the written evidences of ownership of such shares, are called *certificates of stock.*

219. When the General Government or any of the states borrows money for public purposes, an evidence is given to the lender in the form of a bond, bearing a given interest. Such bonds, when given by the United States, are called United States Stock; and when given by any one of the states, are called State Stocks.

220. The nominal or *par* value of a stock is its original cost; that is, the amount named in the certificate or bond. The *market value* is what it will bring when sold. If the market value is above the par value, the stock is said to be at a premium, or *above par;* but if the market value is below the par value, it is then said to be at a discount, or *below par.*

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216. What is stock? What are individuals called who own the stock?

217. What are they called in their associated capacity? What is the law which incorporates them?

218. What is the amount of money paid in by the stockholders called? How is the capital generally divided? What is the evidence of ownership called?

219. What is United States stock? What are state stocks?

220. What is the nominal or par value of a stock? What is the market value? What do you understand by a stock's being at a premium? What by its being at a discount?

COMMISSION AND BROKERAGE.

221. A person who buys or sells goods for another, receiving therefor a certain rate per cent, is called a factor or commission merchant; and the percentage on any purchase or sale, is called the *commission.*

222. Dealers in money or stocks are called Brokers, and the amount of their commissions on any purchase or sale, is called the brokerage. The commission for goods or moneys is generally a certain per cent or rate per hundred on the moneys paid out or received, and the amount may be determined by the rules of simple interest.

The commission for the purchase and sale of goods varies from 2½ to 12½ per cent, and under some circumstances even higher rates are paid. The brokerage on the purchase and sale of stocks in Wall-street, in the city of New York, is generally one-fourth per cent on the par value of the stock.

EXAMPLES.

1. What is the commission on $4396 at 6 per cent?

We here find the commission, as in simple interest, by multiplying by the decimal which expresses the rate per cent.

OPERATION.

$4396
.06
$263,76

Ans. $263,76.

2. A factor sells 60 bales of cotton at $425 per bale, and is to receive 2½ per cent commission: how much must he pay over to his principal? *Ans.* $24862,50.

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221. What is the business of a commission merchant?

222. What is the business of a broker? How is the commission on goods and moneys generally estimated? What is the general commission on the purchase and sale of goods? How may it be determined? What is the customary brokerage on the purchase and sale of stocks?

3. A sent to B, a broker, $3825 to be invested in stock: B is to receive 2 per cent on the amount paid for the stock: what was the value of the stock purchased?

As B is to receive 2 per cent, it follows that $102 of A's money will purchase but $100 of stock: hence, 100+the commission, is to 100, as the given sum to the value of the stock which it will purchase. Hence, to find the value of the stock purchased,

OPERATION.

```
100
  2
---
102 : 100 : : 3825 : Ans.
                100
        102)382500(3750
            306
            ---
             765
             714
             ---
              510
              510
              ---
          Ans. $3750.
```

Multiply the amount to be invested by 100 *and divide the product by* 100 *plus the brokerage.*

PROOF.

Amount paid - - - - - - - - -	$3750
Brokerage on $3750 at 2 per cent=	75
Total sum - -	$3825

4. I have $5000 to be laid out in stocks which are 15 per cent below par: how much will it purchase at the par or nominal value?

It is plain that every 85 dollars will purchase stock of the par value of $100: hence,

$85 : $100 : : $5000 : *Ans.*

Therefore, to find how much will be purchased at the par value, when the stock is below par,

Multiply the sum to be invested by 100 *and divide the product by* 100 *less the discount.*

5. Messrs. P, W, and K buy 200 shares of United States stock for Mr. A. They pay $197 per share, and are to receive one-fourth per cent on the money they advance: how much must A pay them for the stock?

Ans. $39498,50.

6. A factor receives \$708,75, and is directed to purchase iron at \$45 per ton: he is to receive 5 per cent on the money paid: how much iron can he purchase? *Ans.* 15 *tons.*

7. Messrs. P, W, and K receive \$28750 to be invested in stock. They charge $2\frac{1}{2}$ per cent commission on the amount paid: what is the value of the stock purchased? *Ans.* \$28048,78+.

8. The par value or first cost of 167 shares of bank stock was \$200 per share: what is the present value, if the stock is at a premium of 25 per cent, that is, 25 per cent above par? *Ans.* ——

9. What would be the value of the stock named in the last example, if it were at a discount of 10 per cent? *Ans.* \$30060.

10. One hundred shares of United States Bank stock are worth $18\frac{1}{2}$ per cent premium: the par value being \$200 per share, what is the value of the stock? *Ans.* \$23700.

11. A bank fails, and has in circulation bills to the amount of \$267581. It can pay $9\frac{1}{2}$ per cent: how much money is there on hand? *Ans.* ——

12. Sixty-nine shares of bank stock, of which the par value is \$125, is at a discount of 8 per cent: what is its value? *Ans.* \$7935.

13. My commission merchant sells goods to the amount of \$1000, on which I allow him a commission of 2 per cent, and as he pays over before the money becomes due, I allow him $1\frac{1}{2}$ per cent: how much am I to receive? *Ans.* \$965,30.

14. My broker receives from me \$2000 to be laid out in stocks: what will be the value of my stocks after allowing him $2\frac{1}{2}$ per cent commission? *Ans.* ——

15. I sold \$6910,80 worth of goods for a merchant at a commission of $2\frac{1}{2}$ per cent: how much ought I to pay over to my principal? *Ans.* \$6738,03.

16. I remitted to my agent \$7380 to lay out in the purchase of iron. He takes $3\frac{1}{2}$ per cent on the whole sum for his commission, and then buys iron at 95 dollars per ton: how much does he purchase? *Ans.* ——

LOSS AND GAIN.

223. Loss and Gain is a rule by which merchants discover the amount lost or gained in the purchase and sale of goods. It also instructs them how much to increase or diminish the price of their goods, so as to make or lose so much per cent.

EXAMPLES.

1. Bought a piece of cloth containing 75*yd.* at $5,25 per yard, and sold it at $5,75 per yard: how much was gained in the trade?

We first find the profit on a single yard, and then the profit on the 75 yards.

OPERATION.

$5,75 price of 1 yard.
$5,25 cost of 1 yard.
50*cts.* profit on 1 yard.

yd.		*yd.*		*cts.*		
1	:	75	: :	50	:	*Ans.*
				75		
				$37,50		

Ans. $37,50.

2. Bought a piece of calico containing 50*yd.* at 2*s.* 6*d.* per yard: what must it be sold for per yard to gain £1 0*s.* 10*d.*?

50*yd.* at 2*s.* 6*d.* = £6 5*s.*
Profit = £1 0*s.* 10*d.*
It must sell for £7 5*s.* 10*d.*
50)£7 5*s.* 10*d.*(2*s.* 11*d.*

Ans. 2*s.* 11*d.*

3. Bought a hogshead of brandy at $1,25 per gallon, and sold it for $78: was there a loss or gain?

4. A merchant purchased 3275 bushels of wheat for which he paid $3517,10, but finding it damaged is willing to lose 10 per cent: what must it sell for per bushel?

223. What is the rule of loss and gain?

224. In the sale of goods, knowing the per cent of gain, and the amount received, to find the principal or cost.

I sold a parcel of goods for $195,50, on which I made 15 per cent: what did they cost me?

It is evident that the cost added to 15 hundredths of the cost will be equal to what the goods brought, viz., $195,50. If we call the cost 1, then 1 plus $\frac{15}{100}$ of the cost will be equal to what they bring: that is,

$$1+\frac{15}{100}=\frac{115}{100}=\$195,50;$$

or, cost equals $\$195,50\times100\div115=\$170$.

Hence, to find the cost,

Multiply the amount by 100 *and divide the product by* 100 *plus the per cent of gain, and the quotient will be the cost.*

225. When there is a loss, we have the following method:

If I sell a parcel of goods for $170, by which I lose 15 per cent, what did they cost?

It is evident that the cost, less 15 per cent, that is, less 15 hundredths of the cost, is equal to $170. Hence, 85 hundredths of the cost is equal to $170; and consequently, the cost is equal to

$$\$170\times100\div85=\$200 \text{ cost.}$$

Hence, to find the cost when there is a loss,

Multiply the amount received by 100 *and divide the product by the difference between* 100 *and the per cent lost, and the quotient will be the cost.*

EXAMPLES.

1. Bought a quantity of wine at $1,25 per gallon, but it proves to be bad and am obliged to sell it at 20 per cent less than I gave: how much must I sell it for per gallon? *Ans.* $1 *per gallon.*

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224. Knowing the per cent of gain and the amount received, how do you find the cost?

225. Knowing the per cent and the amount lost, how do you find the cost?

2. A farmer sells 125 bushels of corn for 75*cts.* per bushel; the purchaser sells it at an advance of 20 per cent: how much did he receive for the corn?

3. A merchant buys one tun of wine for which he pays $725, and wishes to sell it by the hogshead at an advance of 15 per cent: what must he charge per hogshead? *Ans.* $208,43+.

4. A merchant buys 158 yards of calico for which he pays 20 cents per yard; one-half is so damaged that he is obliged to sell it at a loss of 6 per cent; the remainder he sells at an advance of 19 per cent: how much did he gain? *Ans.* $2,05+.

5. If I buy coffee at 16 cents and sell it at 20 cents, how much do I make per cent on the money paid?

Ans. 25 *per cent.*

6. If I buy tea at 4*s.* per pound and sell it at 4*s.* 9*d.* per pound, how much should I gain on a purchase of £100? *Ans.* ——

7. A merchant bought 650 pounds of cheese at 10 cents per pound, and sold it at 12 cents per pound: how much did he gain on the whole, and how much per cent on the money laid out? *Ans.* { whole gain $13,00; gain 20 *per cent.*

8. Bought cloth at $1,25 per yard, which proving bad, I wish to sell it at a loss of 18 per cent: how much must I ask per yard? *Ans.* ——

9. Bought 50 gallons of molasses at 75 cents a gallon, 10 gallons of which leaked out. At what price per gallon must the remainder be sold that I may clear 10 per cent on the cost? *Ans.* $1,031¼.

10. Bought a cow for $30 cash, and sold her for $35 at a credit of 8 months: reckoning the interest at 6 per cent, how much did I gain? *Ans.* ——

11. Bought 67 yards of cloth for $112, but 19 yards being spoiled, I am willing to lose 5 per cent: how much must I sell it for per yard? *Ans.* $2,216⅔.

12. Bought 67 yards of cloth for $112, but a number of yards being spoiled, I sell the remainder at $2,216⅔ per yard, and lose 5 per cent: how many yards were spoiled? *Ans.* ——

BANKING.

226. Banks are corporations created by law for the purpose of receiving deposites, loaning money, and furnishing a paper circulation represented by specie.

The notes made by a bank circulate as money, because they are payable in specie on presentation at the bank. They are called *bank notes*, or *bank bills.*

227. The note of an individual, or as it is generally called, a promissory note or note of hand, is a positive engagement, in writing, to pay a given sum at a time specified, and to a person named in the note, or to his order, or sometimes to the bearer at large.

FORMS OF NOTES.

No. 1. *Negotiable Note.*

$25,50. Providence, May 1, 1846.

For value received I promise to pay on demand, to Abel Bond, or order, twenty-five dollars and fifty cents.

REUBEN HOLMES.

No 2. *Note Payable to Bearer.*

$875,39. St. Louis, May 1, 1845.

For value received I promise to pay, six months after date, to John Johns, or bearer, eight hundred and seventy-five dollars and thirty-nine cents.

PIERCE PENNY.

No. 3. *Note by two Persons.*

$659,27. Buffalo, June 2, 1846.

For value received we, jointly and severally, promise to pay to Richard Ricks, or order, on demand, six hundred and fifty-nine dollars and twenty-seven cents.

ENOS ALLAN.
JOHN ALLAN.

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226. What are banks? Why do the notes of a bank circulate as money? What are they called?

227. What is a promissory note?

No. 4. *Note Payable at a Bank.*

$20,25. Chicago, May 7, 1846.

Sixty days after date, I promise to pay John Anderson, or order, at the Bank of Commerce in the city of New York, twenty dollars and twenty-five cents, for value received.

JESSE STOKES.

REMARKS RELATING TO NOTES.

1. The person who signs a note, is called the *drawer* or *maker* of the note; thus Reuben Holmes is the drawer of Note No. 1.

2. The person who has the rightful possession of a note, is called the *holder* of the note.

3. A note is said to be *negotiable* when it is made payable to A B, or order, who is called the payee (see No. 1). Now, if Abel Bond, to whom this note is made payable, writes his name on the back of it, he is said to *endorse* the note, and he is called the endorser; and when the note becomes due, the holder must first demand payment of the maker, Reuben Holmes; and if he declines paying it, the holder may then require payment of Abel Bond, the endorser.

4. If the note is made payable to A B, or bearer, then the drawer alone is responsible, and he must pay to any person who holds the note.

5. The time at which a note is to be paid should always be named, but if no time is specified, the drawer must pay when required to do so, and the note will draw interest after the payment is demanded.

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1. What is the person called who signs a note? 2. What is the person called who owns it? 3. When is a note said to be negotiable? What is the person called to whom a note is made payable? When the payee writes his name on the back, what is he said to do? What is he then called? 4. If a note is made payable to A B, who is responsible for its payment? 5. If no time is specified, when is a note to be paid? 6. Will a note draw interest after it falls due, if not stated in the note? 7. If the rate of interest named in a note is higher than the legal rate, can the amount of the note be collected? 8. When are the banks in New York not allowed to charge over 6 per cent? 9. If two persons jointly and severally give a note, of whom may it be collected? 10. What words should be put in every note? 11. If a note is made payable on a fixed day and in a specified article, and is not paid, what may be done?

6. When a note, payable at a future day, becomes due, and is not paid, it will draw interest, though no mention is made of interest.

7. In each of the States there is a *rate* of interest established by law, which is called the legal interest, and when no rate is specified, the note will always draw legal interest. If a rate *higher* than legal interest be taken, the drawer, in most of the States, is not bound to pay the note.

8. In the State of New York, although the legal interest is 7 per cent, yet the banks are not allowed to charge over 6 per cent, unless the notes have over 63 days to run.

9. If two persons jointly and severally give their note, (see No. 3,) it may be collected of either of them.

10. The words "For value received," should be expressed in every note.

11. When a note is given, payable on a fixed day, and in a specific article, as in wheat or rye, payment must be offered at the specified time, and if it is not, the holder can demand the value in money.

228. By mercantile usage a note does not really fall due until the expiration of 3 days after the time mentioned on its face. The three additional days are called *days of grace*.

When the last day of grace happens to be a Sunday, or a holiday, such as New Year's or the 4th of July, the note must be paid the day before; that is, on the second day of grace.

BANK DISCOUNT.

229. Bank Discount is the charge made by a bank for the payment of money on a note before it becomes due. By the custom of banks, this discount is the interest on the amount named in a note, to be paid in advance, and calculated from the time the note is discounted to the time when it falls due, in which time

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228. How long is the time for the payment of a note extended by mercantile usage? What are these days called? When the last day of grace falls on a Sunday, or holiday, when must the note be paid?

229. What is bank discount? How is it estimated? How is it estimated by the custom of banks? What is the face of a note? What is the present value of a note?

the three days of grace are always included. The amount named in a note is called the face of it.

The PRESENT VALUE of a note is the difference between the face of the note and the discount.

230. There are two kinds of notes discounted at banks: 1st. Notes given by one individual to another for property actually sold—these are called *business notes*, or *business paper*. 2d. Notes made for the purpose of borrowing money, which are called *accommodation notes*, or *accommodation paper*. Notes of the first class are much preferred by the banks, as more likely to be paid when they fall due, or in mercantile phrase, "when they come to maturity."

To find the bank discount on a note,

Add 3 days to the time which the note has to run before it becomes due, and calculate the interest for this time at the given rate per cent.

EXAMPLES.

1. What is the bank discount of a note of $1000 payable in 60 days, at 6 per cent interest? This note will have 63 days to run. *Ans.* ——

2. A merchant sold a cargo of cotton for $15720, for which he receives a note at 6 months: how much money will he receive at a bank for this note, discounting it at 6 per cent interest? *Ans.* $15240,54.

3. What is the bank discount on a note of $556,27 payable in 60 days, discounted at 6 per cent per annum? *Ans.* $5,840+.

4. A has a note against B for $3456, payable in three months; he gets it discounted at 7 per cent interest: how much does he receive? *Ans.* $3393,504.

5. What is the bank discount on a note of $367,47, having 1 year, 1 month, and 13 days to run, as shown by the face of the note, discounted at 7 per cent? *Ans.* $29,0097+.

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230. How many kinds of notes are discounted at banks? What distinguishes one kind from the other, and what are they called? Which kind is preferred? How do you find the bank discount on a note?

6. For value received I promise to pay to John Jones, four months from the 17th of July next, six thousand five hundred and seventy-nine dollars and 15 cents. What will be the discount on this, if discounted on the 1st of August, at 6 per cent per annum? *Ans.* ——

231. It is often necessary to make a note, of which the present value shall be a given amount. For example, if I wish to receive at bank the sum of two hundred dollars, for what amount must I give my note payable in three months?*

If we calculate the interest on one dollar for the time, which will be 3 months added to the 3 days of grace, and at the same rate per cent, this will be the bank discount on $1 payable in 3 months; and if this discount be subtracted from one dollar, the remainder will be the present value of one dollar, to be paid at the end of 3 months. Hence,

Pres. val. of $1 : pres. val. of note : : $1 : amt. of note.

Therefore, to find the face of a note, due at a future time, and bearing a given interest, that shall have a known present value,

Find the present value of $1 for the same time and at the same rate of interest, by which divide the present value of the note, and the quotient will be the face of the note.

EXAMPLES.

1. For what sum must a note be drawn at 3 months, so that when discounted at a bank, at 6 per cent, the amount received shall be $500?

Interest on $1 for the time, 3*mo.* and 3*da.* = $0,0155, which taken from $1, gives present value of $1 = 0,9845; then $500 ÷ 0,9845 = $507,872+ = face of note.

* The rule founded on the above well-known principle was, it is believed, first published by Roswell C. Smith, in his New Arithmetic.

231. What is often necessary in bank business? What will be the present value of one dollar due in 3 months? How will you find the face of a note, of a given present value, that shall be payable at a future time?

PROOF.

Bank interest on $507,872 for 3 months, including 3 days of grace, at 6 per cent =7,872, which being taken from the face of the note, leaves $500 for its present value.

2. For what sum must a note be drawn, at 7 per cent, payable on its face in 1 year 6 months and 14 days, so that when discounted at bank it shall produce $307,27?

Ans. $344,59+.

3. A note is to be drawn having on its face 8 months and 12 days to run, and to bear an interest of 7 per cent, so that it will pay a debt of $5450: what is the amount?

Ans. $5734,32+.

4. What sum, 6 months and 9 days from July 18th, 1846, drawing an interest of 6 per cent, will pay a debt of $674,89 at bank, on the 1st of August, 1846?

Ans. $695,64+.

5. Mr. Johnson has Mr. Squires' note for $874,57, having 4 months to run, from July 13th, and bearing an interest of 5 per cent. On the 1st of October he wishes to pay a debt at bank of $750,25, and gives the note in payment: how much must he receive back from the bank? *Ans.* $118,85+.

6. What must be the amount of a note discounted at 6 per cent, having 4 months and 7 days to run, to pay a debt of $1475,50? *Ans.* ——

7. Mr. Jones, on the 1st of June, desires to pay a debt at bank by a note dated May 16th, having 6 months to run and drawing 7 per cent interest: for what amount must the note be drawn, the debt being $1683,75?

Ans. $1740,61+.

8. What amount at the end of one year, with grace, interest at 5 per cent, will pay $1004,20 at bank?

Ans. $1057,51+.

9. Mr. Wilson is indebted at the bank in the sum of $367,464, which he wishes to pay by a note at 4 months with interest at 7 per cent: for what amount must the note be drawn? *Ans.* ——

DISCOUNT.

232. If I give my note to Mr. Wilson for $106, payable in one year, the true present value of the note will be less than $106 by the interest on its *present value* for one year; that is, its true present value will be $100.

The true present value of a note is that sum which being put at interest until the note becomes due, would increase to an amount equal to the face of the note. Thus, $100 is the true present value of the note to Mr. Wilson.

The discount is the difference between the face of a note and its true present value. Thus, $6 is the discount on the note to Mr. Wilson.

To find the true present value of a note due at a future time, find the interest of $1 for the same time; then,

$1 + its interest : $1 : : given sum : its present value.

Hence, to find the present value of any sum,

Add one dollar to its interest for the given time and divide the given amount by this number, and the quotient will be the present value.

EXAMPLES.

1. What is the present value of a note for $1828,75, due in one year, without grace, and bearing an interest of $4\frac{1}{2}$ per cent per annum?

$1 + its interest for the given time = $1,045:

Hence, $1828,75 ÷ $1,045 = $1750 the present value.

PROOF.

Int. on $1750 for 1 year at $4\frac{1}{2}$ per cent	= $78,75
Add principal - - - - - - - - - -	1750
Amount - -	$1828,75

232. What is the true present value of a note? What is the true discount? How do you find the true present value of a note due at *future time*?

2. A note of $1651,50 is due in 11 months, without grace, but the person to whom it is payable sells it with the discount off at 7 per cent: how much shall he receive? *Ans.* $1551,918+.

3. How much ought Mr. Ready to pay in cash for his note of £36, due 15 months hence, without grace, it being discounted at 5 per cent? *Ans.* £33 17*s.* $7\frac{3}{4}$*d.*+.

233. When payments are to be made at different times, *find the present value of the sums separately, and their sum will be the present value of the note.*

4. What is the present value of a note for $10500, on which $900 are to be paid in six months; $2700 in one year; $3900 in eighteen months; and the residue at the expiration of two years, all without grace, the rate of interest being 6 per cent per annum? *Ans.* ——

5. What is the discount of £4500, one-half payable in 6 months and the other half at the expiration of a year, without grace, at 7 per cent per annum? *Ans.* £223 5*s.* 8*d.*+.

6. What is the present value of $5760, one-half payable in 3 months, one-third in 6 months, and the rest in 9 months, without grace, at 6 per cent per annum? *Ans.* $5620,175+.

7. Mr. A gives his note to B for $720, one-half payable in 4 months and the other half in 8 months, without grace: what is the present value of said note, discount at 5 per cent per annum? *Ans.* $702,485+.

8. What is the present value of £825 payable as follows: one-half in 3 months, one-third in 6 months, and the rest in 9 months, without grace, the discount being 6 per cent per annum? *Ans.* £804 19*s.* 5*d.*+.

9. Bought goods for £750 ready money, and sold them for £900 payable by a note at 6 months, without grace: now, if I discount the note at 6 per cent per annum, will I make or lose? *Ans.* ——

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233. When payments are made at different times, how do you find the true present value?

10. What is the present value of $4000 payable in 9 months, without grace, discount $4\frac{1}{2}$ per cent per annum? *Ans.* $3869,407+.

11. How much corn must I carry to a miller that I may receive a bushel of meal, $\frac{1}{16}$ being allowed for toll and waste? *Ans.* 1*bu.* $2\frac{2}{15}$*qt.*

12. Mr. Johnson has a note against Mr. Williams for $2146,50, dated August 17th, 1838, which becomes due Jan. 11th, 1839: if the note is discounted at 6 per cent, what ready money must be paid for it September 25th, 1838? *Ans.* $2109,236+.

13. C owes D $3456, to be paid October 27th, 1842: C wishes to pay on the 24th of August, 1838, to which D consents: how much ought D to receive, interest at 6 per cent? *Ans.* $2763,694+.

14. What is the present value of a note of $4800, due 4 years hence, without grace, the interest being computed at 5 per cent per annum? *Ans.* ——

15. A man having a horse for sale, offered it for $225 cash in hand, or 230 at 9 months, without grace; the buyer chose the latter: did the seller lose or make by his offer, supposing money to be worth 7 per cent?
Ans. He lost $6,473+.

INSURANCE.

234. Insurance is an agreement, generally in writing, by which an individual or company bind themselves to exempt the owners of certain property, such as ships, goods, houses, &c., from loss or hazard.

The written agreement made by the parties, is called the *policy*.

The amount paid by him who owns the property to those who insure it, as a compensation for their risk, is called the *premium*. The premium is generally so much

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234. What is insurance? What is the written agreement called? What is the amount paid for the insurance called? How are the premiums generally estimated? How are they found?

per cent on the property insured, and is found by the rules of simple interest.

EXAMPLES.

1. What would be the premium for the insurance of a house valued at $8754 against loss by fire for 1 year, at $\frac{1}{2}$ per cent?

By multiplying by .01, we have the insurance at 1 per cent - - - - - } 87,54

The half, is the insurance at half per cent $43,77.

2. What would be the premium for insuring a ship and cargo, valued at $37500, from New York to Liverpool, at $3\frac{1}{2}$ per cent? *Ans.* ——

3. What would be the insurance on a ship valued at $47520 at $\frac{1}{2}$ per cent: also at $\frac{1}{3}$ per cent?
Ans. $237,60.—$158,40.

4. What would be the insurance on a house valued at $14000 at $1\frac{1}{2}$ per cent? Also at $\frac{3}{4}$ per cent? At $\frac{1}{2}$ per cent? At $\frac{1}{3}$ per cent? At $\frac{1}{4}$ per cent?
Ans. $210.—$105.—$70.—$46,66+.—$35.

5. What is the insurance on a store and goods valued at $27000 at $2\frac{1}{4}$ per cent? At 2 per cent? At $1\frac{1}{2}$ per cent? At $\frac{3}{4}$ per cent? At $\frac{1}{2}$ per cent? At $\frac{1}{4}$ per cent? At $\frac{1}{5}$ per cent? At $\frac{1}{6}$ per cent? *Ans.* ——

6. What is the premium of insurance on $9870, at 14 per cent? *Ans.* $1381,80.

7. A merchant wishes to insure on a vessel and cargo at sea, valued at $28800: what will be the premium at $1\frac{3}{4}$ per cent? *Ans.* ——

8. What is the premium on $750 at $1\frac{3}{4}$ per cent?
Ans. $13,12$\frac{1}{2}$.

9. What is the premium on $8750 at $3\frac{1}{2}$ per cent?
Ans. $306,25.

10. A merchant owns three-fourths of a ship valued at $24000, and insures his interest at $2\frac{1}{2}$ per cent: what does he pay for his policy? *Ans.* $450.

11. A merchant learns that his vessel and cargo, valued at $36000, have been injured to the amount of $12000: he effects an insurance on the remainder at $5\frac{1}{2}$ per cent: what premium does he pay? *Ans.* ——

ASSESSING TAXES.

235. A tax is a certain sum required to be paid by the inhabitants of a town, county, or state, for the support of government. It is generally collected from each individual, in proportion to the amount of his property.

In some states, however, every white male citizen over the age of twenty-one years is required to pay a certain tax. This tax is called a poll-tax; and each person so taxed is called a *poll.*

236. In assessing taxes, the first thing to be done is to make a complete inventory of all the property in the town on which the tax is to be laid. If there is a poll-tax, make a full list of the polls and multiply the number by the tax on each poll, and subtract the product from the whole tax to be raised by the town: the remainder will be the amount to be raised on the property. Having done this, *divide the whole tax to be raised by the amount of taxable property, and the quotient will be the tax on* $1. Then multiply this quotient by the inventory of each individual, and the product will be the tax on his property.

EXAMPLES.

1. A certain town is to be taxed $4280; the property on which the tax is to be levied is valued at $1000000. Now there are 200 polls, each taxed $1,40. The property of A is valued at $2800, and he pays 4 polls,
B's at $2400, pays 4 polls, E's at $7242, pays 4 polls,
C's at $2530, pays 2 " F's at $1651, pays 6 "
D's at $2250, pays 6 " G's at $1600,80, pays 4 "
What will be the tax on one dollar, and what will be A's tax, and also that of each on the list?

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235. What is a tax? How is it generally collected? What is a poll-tax?

236. What is the first thing to be done in assessing a tax? If there is a poll-tax, how do you find the amount? How then do you find the per cent of tax to be levied on a dollar? How do you then find the amount to be levied on each individual?

First, \$1,40 × 200 = \$280 amount of poll-tax.
\$4280 − \$280 = \$4000 amount to be levied on property.
Then, \$4000 ÷ \$1000000 = 4 mills on \$1.

Now, to find the tax of each, as A's, for example,

A's inventory - - -	\$2800
	,004
	11,20
4 polls at \$1,40 each -	5,60
A's whole tax - - -	\$16,80

In the same manner the tax of each person in the township may be found.

237. Having found the per cent, or the amount to be raised on each dollar, form a table showing the amount which certain sums would produce at the same rate per cent. Thus, after having found, as in the last example, that four mills are to be raised on every dollar, we can, by multiplying in succession by the numbers 1, 2, 3, 4, 5, 6, 7, 8, &c., form the following

TABLE.

\$		\$	\$		\$	\$		\$
1	gives	0,004	20	gives	0,080	300	gives	1,200
2	"	0,008	30	"	0,120	400	"	1,600
3	"	0,012	40	"	0,160	500	"	2,000
4	"	0,016	50	"	0,200	600	"	2,400
5	"	0,020	60	"	0,240	700	"	2,800
6	"	0,024	70	"	0,280	800	"	3,200
7	"	0,028	80	"	0,320	900	"	3,600
8	"	0,032	90	"	0,360	1000	"	4,000
9	"	0,036	100	"	0,400	2000	"	8,000
10	"	0,040	200	"	0,800	3000	"	12,000

This table shows the amount to be raised on each sum in the columns under \$'s.

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237. How do you form an assessment table?

1. To find the amount of B's tax from this table.

B's tax on $2000	- - is	-	$8,000
B's tax on 400	- - is	-	$1,600
B's tax on 4 polls, at $1,40		-	$5,600
B's total tax	- - is	-	$15,200

2. To find the amount of C's tax from the table.

C's tax on $2000	- - is	-	$8,000
C's tax on 500	- - is	-	$2,000
C's tax on 30	- - is	-	$0,120
C's tax on 2 polls	- - is	-	$2,800
C's total tax	- - is	-	$12,920

In a similar manner, we might find the taxes to be paid by D, E, &c.

EQUATION OF PAYMENTS.

238. I owe Mr. Wilson $2 to be paid in 6 months; $3 to be paid in 8 months; and $1 to be paid in 12 months. I wish to pay his entire dues at a single payment, to be made at such a time, that neither he nor I shall lose interest: at what time must the payment be made?

The method of finding the mean time of payment of several sums due at different times, is called *Equation of Payments.**

Taking the example above,

Int. of $2 for 6*mo.*=int. of $1 for	12*mo.*	$2\times 6=12$
" of $3 for 8*mo.*=int. of $1 for	24*mo.*	$3\times 8=24$
" of $1 for 12*mo.*=int. of $1 for	12*mo.*	$1\times 12=12$
$6	48	48

The interest on all the sums, to the times of payment,

* The mean time of payment is sometimes found by first finding the *present* value of each payment; but the rule here given has the sanction of the best authorities in this country and England.

238. What is Equation of Payments? What is the sum of the products, which arise from multiplying each payment by the time to which it becomes due, equal to? How do you find the mean time of payment?

is equal to the interest of $1 for 48 months. But 48 is equal to the sum of all the products which arise from multiplying each sum by the time at which it becomes due: hence, the sum of the products is equal to the time which would be necessary for $1 to produce the same interest as would be produced by all the sums.

Now, if $1 will produce a certain interest in 48 months, in what time will $6 (or the sum of the payments) produce the same interest? The time is obviously found by dividing 48 (the sum of the products) by $6, (the sum of the payments.)

Hence, to find the mean time,

Multiply each payment by the time before it becomes due, and divide the sum of the products by the sum of the payments: the quotient will be the mean time.

EXAMPLES.

1. B owes A $600: $200 is to be paid in two months, $200 in four months, and $200 in six months: what is the mean time for the payment of the whole?

We here multiply each sum by the time at which it becomes due, and divide the sum of the products by the sum of the payments.

OPERATION.

$200 \times 2 = 400$
$200 \times 4 = 800$
$200 \times 6 = 1200$
$6|00 \quad)24|00$
4

Ans. 4 *months.*

2. A merchant owes $600, of which $100 is to be paid in 4 months, $200 in 10 months, and the remainder in 16 months: if he pays the whole at once, at what time must he make the payment? *Ans.* ——

3. A merchant owes $600 to be paid in 12 months, $800 to be paid in 6 months, and $900 to be paid in 9 months: what is the equated time of payment?

Ans. 8*mo.* $22\frac{4}{23}$*da.*

4. A owes B $600; one-third is to be paid in 6 months, one-fourth in 8 months, and the remainder in 12 months: what is the mean time of payment? *Ans.* 9 *months.*

5. A merchant has due him $300 to be paid in 60 days, $500 to be paid in 120 days, and $750 to be paid in 180 days: what is the equated time for the payment of the whole? *Ans.* 137$\frac{13}{31}$ *days.*

6. A merchant has due him $1500; one-sixth is to be paid in 2 months, one-third in 3 months, and the rest in 6 months: what is the equated time for the payment of the whole? *Ans.* 4$\frac{1}{3}$ *months.*

239. If one of the payments is due on the day from which the equated time is reckoned, its corresponding product will be nothing, but the payment must still be added in finding the sum of the payments.

7. I owe $1000 to be paid on the 1st of January, $1500 on the 1st of February, $3000 on the 1st of March, and $4000 on the 15th of April: reckoning from the 1st of January, and calling February 28 days, on what day must the money be paid?

Ans. Payment in 67$\frac{6}{19}$ *days, or on the* 9*th March.*

240. In finding the equated time of payments for several sums, due at different times, any day may be assumed as the one from which we reckon. Thus, if I owe Mr. Wilson $100 to be paid on the 15th of July, $200 on the 15th of August, and $300 on the ninth of September, and we require the mean time of a single payment, it would be most convenient to estimate from the first of July.

From 1st of July to 1st payment 14 days
" " " to 2d payment 45 days
" " " to 2d payment 70 days.

Then, by rule given above, we have,

$$\begin{array}{l} 100 \times 14 = 1400 \\ 200 \times 45 = 9000 \\ 300 \times 70 = 21000 \\ \overline{600} \quad 6|00)314|00 \\ \hline \qquad\qquad 52\frac{1}{3} \end{array}$$

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239. When you reckon the time from the date at which the first payment becomes due, do you include the first payment?

240. Is it material from what day the time for equated payment be estimated?

Hence, the amount will fall due in $52\frac{1}{3}$ days from the 1st of July; that is, on the 22d day of August.

But we may, if we please, demand at what time the payment would be due from the 1st of June.

From June 1st	to 1st payment	44 days
" " "	to 2d payment	75 days
" " "	to 3d payment	100 days.

Thus,

$$\begin{array}{l} 100\times \ 44= \ 4400 \\ 200\times \ 75=15000 \\ 300\times 100=30000 \\ \hline 600 \quad 6|00)494|00 \\ \qquad\qquad\quad 82\frac{1}{3} \end{array}$$

Hence, the payment becomes due in $82\frac{1}{3}$ days from June 1st, or on the 22d of August—the same as before.

Any day may, therefore, be taken as the one from which the mean time is estimated.

8. Mr. Jones purchased of Mr. Wilson, on a credit of six months, goods to the following amounts:

15th of January,	a bill of	$3750,
10th of February,	a bill of	3000,
6th of March,	a bill of	2400,
8th of June,	a bill of	2250.

He wishes, on the 1st of July, to give his note for the amount: at what time must it be made payable?

9. Mr. Gilbert bought $4000 worth of goods: he was to pay $1600 in five months, $1200 in six months, and the remainder in eight months: what will be the time of credit, if he pays the whole amount at a single payment?

Ans. 6*mo.* 6*da.*

10. A merchant bought several lots of goods, as follows:

A bill of	$650,	June 6th,
A bill of	890,	July 8th,
A bill of	7940,	August 1st.

Now, if the credit is 6 months, at what time will the whole become due? *Ans.* Jan. 25th.

PARTNERSHIP OR FELLOWSHIP.

241. Partnership or Fellowship is the joining together of several persons in trade, with an agreement to share the losses and profits according to the amount which each one puts into the partnership. The money employed is called the *Capital Stock*.

The gain or loss to be shared is called the *Dividend*.

It is plain that the whole stock which suffers the gain or loss, must be to the gain or loss, as the stock of any individual to his part of the gain or loss. Hence,

As the whole stock is to each man's share, so is the whole gain or loss to each man's share of the gain or loss.

PROOF.

Add all the separate profits or shares together; their sum should be equal to the gross profit or stock.

EXAMPLES.

1. A and B buy certain merchandise amounting to £160, of which A pays £90, and B £70: they gain by the purchase £32: what is each one's share of the profits?

A - - £90
B - - £70

$$\pounds 160 : \begin{Bmatrix} 90 \\ 70 \end{Bmatrix} :: \pounds 32 : \begin{Bmatrix} \pounds 18 \text{ A's share.} \\ \pounds 14 \text{ B's share.} \end{Bmatrix}$$

2. A and B have a joint stock of $2100, of which A owns $1800 and B $300: they gain in a year $1000: what is each one's share of the profits?

Ans. A's=$857,14+; B's=$142,85+.

3. A, B, C, and D have £20,000 in trade: at the end of a year their profits amount to £16,000: what is each one's share, supposing A to receive £50 and D £30 out of the profits for extra services?

Ans. { A's=£4030; B's=£3980; C's=£3980; D's=£4010.

241. What is Partnership or Fellowship? What is the gain or *loss called*? What is the rule for finding each one's share?

4. Five persons, A, B, C, D, and E have to share between them an estate of $10,000: A is to have one-fourth; B one-eighth; C one-sixth; D one-eighth; and E what is left: what will be the share of each?

Ans. A's=$2500; B's=$1250; C's=$1666,66+; D's=$1250; E's=$3333,34.

DOUBLE FELLOWSHIP.

242. When several persons who are joined together in trade employ their capital for different periods of time, the partnership is called *Double Fellowship.*

For example, suppose A puts $100 in trade for 5 years, B $200 for 2 years, and C $300 for 1 year: this would make a case of double fellowship.

Now it is plain that there are two circumstances which should determine each one's share of the profits: 1*st, The amount of capital he puts in; and* 2*dly, The time which it is continued in the business.*

Hence, each man's share should be proportioned to the capital he puts in, multiplied by the time it is continued in trade. Therefore, to find each share,

Multiply each man's stock by the time he continues it in trade; then say, as the sum of the products is to each particular product, so is the whole gain or loss to each man's share of the gain or loss.

EXAMPLES.

1. A and B enter into partnership: A puts in £840 for 4 months, and B puts in £650 for 6 months; they gain £300: what is each one's share of the profits?

A's stock £840 × 4 = 3360
B's stock £650 × 6 = 3900

$$\pounds 7260 : \begin{cases} 3360 \\ 3900 \end{cases} :: \pounds 300 : \begin{cases} \pounds 138\ 16s.\ 10d. \\ \pounds 161\ 3s.\ 1d. \end{cases}$$

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242. What is Double Fellowship? What two circumstances determine each one's share of the profits? Give the rule for finding each one's share.

2. A put in trade £50 for 4 months, and B £60 for 5 months: they gained £24: how is it to be divided between them? *Ans.* { A's share, £9 12*s*. B's " £14 8*s*.

3. C and D hold a pasture together, for which they pay £54: C pastures 23 horses for 27 days, and D 21 horses for 39 days: how much of the rent ought each one to pay? *Ans.* C £23 5*s*. 9*d*. D £30 14*s*. 3*d*.

4. Four traders form a company: A puts in $400 for 5 months; B $600 for 7 months; C $960 for 8 months; D $1200 for 9 months. In the course of trade they lost $750: how much falls to the share of each? *Ans.* ——

CUSTOM HOUSE BUSINESS.

243. Persons who bring goods or merchandise into the United States, from foreign countries, are required to land them at particular places or ports, called Ports of Entry, and to pay a certain amount on their value, called a *Duty*. This duty is imposed by the General Government, and must be the same on the same articles of merchandise, in every part of the United States.

Besides the duties on merchandise, vessels employed in commerce are required, by law, to pay certain sums for the privilege of entering the ports. These sums are large or small, in proportion to the size or tonnage of vessels. The moneys arising from duties and tonnage, are called *revenues*.

244. The revenues of the country are under the general direction of the Secretary of the Treasury, and to secure their faithful collection, the government has appointed various officers at each port of entry or place where goods may be landed.

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243. What is a port of entry? What is a duty? By whom are duties imposed? What charges are vessels required to pay? What are the moneys arising from duties and tonnage called?

244. Under whose direction are the revenues of the country?

245. The office established by the government at any port of entry, is called a *Custom House*, and the officers attached to it are called Custom House Officers.

246. All duties levied by law on goods imported into the United States, are collected at the various custom houses, and are of two kinds, *Specific* and *Ad valorem*.

A *specific* duty is a certain sum on a particular kind of goods named; as so much per square yard on cotton or woollen cloths, so much per ton weight on iron, or so much per gallon on molasses.

An *ad valorem* duty is such a per cent on the actual cost of the goods in the country from which they are imported. Thus, an ad valorem duty of 15 per cent on English cloths, is a duty of 15 per cent on the cost of cloths imported from England.

247. The laws of Congress provide, that the cargoes of all vessels freighted with foreign goods or merchandise, shall be weighed or gauged by the custom house officers at the port to which they are consigned. As duties are only to be paid on the articles, and not on the boxes, casks, and bags which contain them, certain deductions are made from the weights and measures, called *Allowances*.

Gross Weight is the whole weight of the goods, together with that of the hogshead, barrel, box, bag, &c., which contains them.

Draft is an allowance from the gross weight on account of waste, where there is not actual tare.

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245. What is a custom house? What are the officers attached to it called?

246. Where are the duties collected? How many kinds are there, and what are they called? What is a specific duty? An ad valorem duty?

247. What do the laws of Congress direct in relation to foreign goods? Why are deductions made from their weight? What are these deductions called? What is gross weight? What is draft? What is the greatest draft allowed? What is tare? What are the different kinds of tare? What allowances are made on liquors?

	lb.		*lb.*
On	112	it is	1,
From	112 to 224	"	2,
"	224 to 336	"	3,
"	336 to 1120	"	4,
"	1120 to 2016	"	7,
Above	2016 any weight	"	9;

consequently, 9*lb.* is the greatest draft allowed.

Tare is an allowance made for the weight of the boxes, barrels, or bags containing the commodity, and is of three kinds. 1st. Legal tare, or such as is established by law; 2d. Customary tare, or such as is established by the custom among merchants; and 3d. Actual tare, or such as is found by removing the goods and actually weighing the boxes or casks in which they are contained.

On liquors in casks, *customary tare* is sometimes allowed on the supposition that the cask is not full, or what is called its *actual wants;* and then an allowance of 5 per cent for leakage.

A tare of 10 per cent is allowed on porter, ale, and beer, in bottles, on account of breakage, and 5 per cent on all other liquors in bottles. At the custom house, bottles of the common size are estimated to contain $2\frac{1}{4}$ gallons the dozen. For tables of Tare and Duty, see Ogden on the Tariff of 1842.

EXAMPLES.

1. What will be the duty on 125 cartons of ribbons, each containing 48 pieces, and each piece weighing 3*oz.* net, and paying a duty of $2,50 per *lb.*? *Ans.* $2812,5.

2. What will be the duty on 225 bags of coffee, each weighing gross 160*lb.*, invoiced at 6 cents per *lb.*; 2 per cent being the legal rate of tare, and 20 per cent the duty? *Ans.* $418,068.

3. What duty must be paid on 275 dozen bottles of claret, estimated to contain $2\frac{1}{4}$ gallons per dozen, 5 per cent being allowed for breakage, and the duty being 35 cents per gallon? *Ans.* $351,45+.

4. A merchant imports 175 cases of indigo, each case weighing 196*lb.* gross: 15 per cent is the customary rate of tare, and the duty 5 cents per *lb.* What duty must he pay on the whole? *Ans.* $1442,875.

5. What is the tare and duty on 75 casks of Epsom salts, each weighing gross 2*cwt.* 2*qr.* 27*lb.*, and invoiced at $1\frac{7}{8}$ cents per *lb.*, the customary tare being 11 per cent, and the rate of duty 20 per cent? *Ans.* ——

FORMS RELATING TO BUSINESS IN GENERAL.

FORMS OF ORDERS.

Messrs. M. James & Co.

Please pay John Thompson, or order, five hundred dollars, and place the same to my account, for value received. Peter Worthy.

Wilmington, N. C., June 1, 1846.

Mr. Joseph Rich,

Please pay, for value received, the bearer, sixty-one dollars and twenty cents, in goods from your store, and charge the same to the account of your

Obedient Servant,

John Parsons.

Savannah, Ga., July 1, 1846.

FORMS OF RECEIPTS.

Receipt for Money on Account.

Received, Natchez, June 2d, 1845, of John Ward, sixty dollars on account.

$60,00 John P. Fay

Receipt for Money on a Note.

Received, Nashville, June 5, 1846, of Leonard Walsh, six hundred and forty dollars, on his note for one thousand dollars, dated New York, January 1, 1845.

$640,00 J. N. Weeks.

A BOND FOR ONE PERSON, WITH A CONDITION.

KNOW ALL MEN BY THESE PRESENTS, That, *I James Wilson of the City of Hartford and State of Connecticut, am* held and firmly bound unto *John Pickens of the Town of Waterbury, County of New Haven and State of Connecticut*, in the sum *of Eighty dollars* lawful money of the United States of America, to be paid to the said *John Pickens, his* executors, administrators, or assigns: for which payment well and truly to be made *I* bind *myself, my* heirs, executors, and administrators, firmly by these presents. Sealed with *my* Seal. Dated the *Ninth day* of *March* one thousand eight hundred and *thirty-eight.*

THE CONDITION of the above obligation is such, that if the above bounden *James Wilson, his* heirs, executors, or administrators, shall well and truly pay or cause to be paid, unto the above-named *John Pickens, his* executors, administrators, or assigns, the just and full sum of

[*Here insert the condition.*]

then the above obligation to be void, otherwise to remain in full force and virtue.

Sealed and delivered in
the presence of

John Frost,
Joseph Wiggins, } *James Wilson.* L.S.

Note. The part in Italic to be filled up according to circumstance.

If there is no condition to the bond, then all to be omitted after and including the words "THE CONDITION, &c."

A BOND FOR TWO PERSONS, WITH A CONDITION.

KNOW ALL MEN BY THESE PRESENTS, That, We *James Wilson and Thomas Ash of the City of Hartford and State of Connecticut*, are held and firmly bound unto *John Pickens of the Town of Waterbury, County of New Haven and State of Connecticut*, in the sum *of Eighty dollars* lawful money of the United States of America, to be paid to the said *John Pickens, his* executors or assigns: for which payment well and truly to be made *We* bind *ourselves, our* heirs, executors, and administrators, firmly by these presents. Sealed with *our* Seal. Dated the *Ninth day* of *March* one thousand eight hundred and *thirty-eight.*

THE CONDITION of the above obligation is such, that if the above bounden *James Wilson and Thomas Ash, their* heirs, executors, or administrators, shall well and truly pay or cause to be paid, unto the above-named *John Pickens, his* executors, administrators, or assigns, the just and full sum of

[*Here insert the condition.*]

then the above obligation to be void, otherwise to remain in full force and virtue.

Sealed and delivered in
the presence of

John Frost,	*James Wilson,*	L.S.
Joseph Wiggins,	*Thomas Ash.*	L.S.

NOTE. The part in Italic to be filled up according to circumstance.

If there is no condition to the bond, then all to be omitted after and including the words "THE CONDITION, &c."

ALLIGATION MEDIAL.

248. A merchant mixes 8*lb.* of tea, worth 75*cts.* per pound, with 16*lb.* worth $1,02 per *lb*: what is the value of the mixture per pound?

The manner of finding the price of this mixture is called *Alligation Medial.* Hence,

ALLIGATION MEDIAL *teaches the method of finding the price of a mixture when the simples of which it is composed, and their prices, are known.*

In the example above, the simples 8*lb.* and 16*lb.*, and also their prices per pound, 75*cts.* and $1,02, are known.

8*lb.* of tea at 75*cts.* per *lb.* - - - - - - -	6,00
16*lb.* " " $1,02 per *lb.* - - - - - - -	16,32
24 sum of simples. Total cost	$22,32

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248. What is Alligation Medial? How do you find the price of the mixture?

Now, if the entire cost of the mixture, which is \$22,32, be divided by 24, the number of *lbs.* or sum of the simples, the quotient 93*cts.* will be the price per pound. Hence, to find the price of the mixture,

OPERATION.

```
24)22,32(93cts.
   216
   ---
    72
    72
```

Divide the entire cost of the whole mixture by the sum of the simples, and the quotient will be the price of the mixture.

EXAMPLES.

1. A farmer mixes 30 bushels of wheat worth 5*s.* per bushel, with 72 bushels of rye at 3*s.* per bushel, and with 60 bushels of barley worth 2*s.* per bushel: what is the value of a bushel of the mixture?

30	bushels of wheat at 5*s.* - -	150*s.*
72	" " rye at 3*s.* - - -	216*s.*
60	" " barley at 2*s.* - -	120*s.*
162		162)486(3*s.*
		486

Ans. 3*s.*

2. A wine merchant mixes 15 gallons of wine at \$1 per gallon with 25 gallons of brandy worth 75 cents per gallon: what is the value of a gallon of the compound?
Ans. 84*cts.*+.

3. A grocer mixes 40 gallons of whiskey worth 31*cts.* per gallon with 3 gallons of water, which costs nothing: what is the value of a gallon of the mixture?
Ans. $28\frac{36}{43}$*cts.*

4. A goldsmith melts together 2*lb.* of gold of 22 carats fine, 6*oz.* of 20 carats fine, and 6*oz.* of 16 carats fine: what is the fineness of the mixture? *Ans.* $20\frac{2}{3}$ *carats.*

5. On a certain day the mercury in the thermometer was observed to average the following heights: from 6 in the morning to 9, 64°; from 9 to 12, 74°; from 12 to 3, 84°; and from 3 to 6, 70°: what was the mean temperature of the day? Ans. 73°.

ALLIGATION ALTERNATE.

249. A farmer would mix oats worth 3*s*. per bushel with wheat worth 9*s*. per bushel, so that the mixture shall be worth 5*s*. per bushel: what proportion must be taken of each sort?

The method of finding how much of each sort must be taken is called *Alligation Alternate*. Hence,

Alligation Alternate *teaches the method of finding what proportion must be taken of several simples, whose prices are known, to form a compound of a given price.*

Alligation Alternate is the reverse of Alligation Medial, and may be proved by it.

For a first example, let us take the one above stated. If oats worth 3*s*. per bushel be mixed with wheat worth 9*s*., how much must be taken of each sort that the compound may be worth 5*s*. per bushel?

If the price of the mixture were 6*s*., half the sum of the prices of the simples, it is plain that it would be necessary to take just as much oats as wheat.

5	3	—	4 Oats.
	9	—	2 Wheat.

But since the price of the mixture is *nearer* to the price of the oats than to that of the wheat, less wheat will be required in the mixture than oats.

Having set down the prices of the simples under each other, and linked them together, we next set 5*s*., the price of the mixture, on the left. We then take the difference between 9 and 5 and place it opposite 3, the price of the oats, and also the difference between 5 and 3, and place it opposite 9, the price of the wheat. The difference standing opposite each kind shows how much of that kind is to be taken. In the present example, the mixture will consist of 4 bushels of oats and 2 of wheat; and any other quantities bearing the same proportion to

249. What is Alligation Alternate? How do you prove Alligation Alternate?

each other, such as 8 and 4, 20 and 10, &c., will give a mixture of the same value.

PROOF BY ALLIGATION MEDIAL.

4 bushels of oats at 3*s*. - -	12*s*.
2 bushels of wheat at 9*s*. - -	18*s*.
6	6)30
	Ans. 5*s*.

CASE I.

250. To find the proportion in which several simples of given prices must be mixed together, that the compound may be worth a given price.

I. *Set down the prices of the simples under each other, in the order of their values, beginning with the lowest.*

II. *Link the least price with the greatest, and the one next to the least with the one next to the greatest, and so on, until the price of each simple which is less than the price of the mixture is linked with one or more that is greater; and every one that is greater with one or more that is less.*

III. *Write the difference between the price of the mixture and that of each of the simples opposite that price with which the particular simple is linked; then the difference standing opposite any one price, or the sum of the differences when there is more than one, will express the quantity to be taken of that price.*

EXAMPLES.

1. A merchant would mix wines worth 16*s*., 18*s*., and 22*s*. per gallon in such a way, that the mixture may be worth 20*s*. per gallon: how much must be taken of each sort?

20	16	2 at 16*s*.
	18	2 at 18*s*.
	22	4+2=6 at 22*s*.

Ans. { 2*gal*. at 16*s*., 2 at 18*s*., and 6 at 22*s*.: or any other quantities bearing the proportion of 2, 2, and 6.

250. How do you find the proportions so that the compound may be of a given price?

2. What proportions of coffee at 16*cts.*, 20*cts.*, and 28*cts.* per *lb.* must be mixed together so that the compound shall be worth 24*cts.* per *lb.*?

Ans. In the proportion of 4*lb.* at 16*cts.*, 4*lb.* at 20*cts.*, and 12*lb.* at 28*cts.*

3. A goldsmith has gold of 16, of 18, of 23, and of 24 carats fine: what part must be taken of each so that the mixture shall be 21 carats fine?

Ans. 3 of 16, 2 of 18, 3 of 23, and 5 of 24.

4. What portion of brandy at 14*s.* per gallon, of old Madeira at 24*s.* per gallon, of new Madeira at 21*s.* per gallon, and of brandy at 10*s.* per gallon, must be mixed together so that the mixture shall be worth 18*s.* per gallon?

Ans. 6*gal.* at 10*s.*, 3 at 14*s.*, 4 at 21*s.*, and 8*gal.* at 24*s.*

CASE II.

251. When a given quantity of one of the simples is to be taken.

I. *Find the proportional quantities of the simples as in Case I.*

II. *Then say, as the number opposite the simple whose quantity is given, is to the given quantity, so is either proportional quantity to the part of its simple to be taken.*

EXAMPLES.

1. How much wine at 5*s.*, at 5*s.* 6*d.*, and 6*s.* per gallon must be mixed with 4 gallons at 4*s.* per gallon, so that the mixture shall be worth 5*s.* 4*d.* per gallon?

64	48	8	simple whose quantity is known.
	60	2	proportional quantities.
	66	4	
	72	16	

Then 8 : 4 :: 2 : 1
8 : 4 :: 4 : 2
8 : 4 :: 16 : 8

Ans. 1*gal.* at 5*s.*, 2 at 5*s.* 6*d.*, and 8 at 6*s.*

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251. How do you find the proportion when the quantity of one of the simples is given?

PROOF BY ALLIGATION MEDIAL.

4*gal.*	at	4*s.*	per gallon	-	192*d.*
1	"	5*s.*	"	- -	60
2	"	5*s.* 6*d.*	"	- -	132
8	"	6*s.*	"	- -	576
15					15)960(64*d.* price of mixture

2. A farmer would mix 14 bushels of wheat, at $1,20 per bushel, with rye at 72*cts.*, barley at 48*cts.*, and oats at 36*cts.*: how much must be taken of each sort to make the mixture worth 64 cents per bushel?

Ans. { 14*bu.* of wheat; 8*bu.* of rye; 4*bu.* of barley; and 28*bu.* of oats.

3. There is a mixture made of wheat at 4*s.* per bushel, rye at 3*s.*, barley at 2*s.*, with 12 bushels of oats at 18*d.* per bushel: how much is taken of each sort when the mixture is worth 3*s.* 6*d.*?

Ans. { 96*bu.* of wheat; 12*bu.* of rye; 12*bu.* of barley; and 12*bu.* of oats.

4. A distiller would mix 40*gal.* of French brandy at 12*s.* per gallon, with English at 7*s.* and spirits at 4*s.* per gallon: what quantity must be taken of each sort, that the mixture may be afforded at 8*s.* per gallon?

Ans. { 40*gal.* French; 32*gal.* English; and 32*gal.* of spirits.

CASE III.

252. When the quantity of the compound is given as well as the price.

I. *Find the proportional quantities as in Case I.*

II. *Then say, as the sum of the proportional quantities, is to the given quantity, so is each proportional quantity, to the part to be taken of each.*

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252. How do you determine the proportion when the quantity of the compound is given as well as the price?

EXAMPLES.

1. A grocer has four sorts of sugar, worth 12*d.*, 10*d.*, 6*d.*, and 4*d.* per pound; he would make a mixture of 144*lb.* worth 8*d.* per pound: what quantity must be taken of each sort?

8	4	4	12 : 144 :: 4 : 48
	6	2	12 : 144 :: 2 : 24
	10	2	12 : 144 :: 2 : 24
	12	4	12 : 144 :: 4 : 48

Sum of the proportional parts 12

Ans. { 48*lb.* at 4*d.*; 24*lb.* at 6*d.*; 24*lb.* at 10*d.*; and 48*lb.* at 12*d.*

PROOF BY ALLIGATION MEDIAL.

48*lb.* at 4*d.*	- - -	192*d.*
24*lb.* at 6*d.*	- - -	144*d.*
24*lb.* at 10*d.*	- - -	240*d.*
48*lb.* at 12*d.*	- - -	576*d.*
144		144)1152(8*d.*

Hence, the average cost is 8*d.*

2. A grocer having four sorts of tea, worth 5*s.*, 6*s.*, 8*s.*; and 9*s.* per *lb.*, wishes a mixture of 87*lb.* worth 7*s.* per *lb.*: how much must be taken of each sort?

Ans. { 29*lb.* at 5*s.*; 14½*lb.* at 6*s.*; 14½*lb.* at 8*s.*; and 29*lb.* at 9*s.*

3. A vintner has four sorts of wine, viz.: white wine at 4*s.* per gallon, Flemish at 6*s.* per gallon, Malaga at 8*s.* per gallon, and Canary at 10*s.* per gallon: he would make a mixture of 60 gallons to be worth 5*s.* per gallon: what quantity must be taken of each?

Ans. { 45*gal.* of white wine; 5*gal.* of Flemish; 5*gal.* of Malaga; and 5*gal.* of Canary.

4. A silversmith has four sorts of gold, viz.: of 24 carats fine, of 22 carats fine, of 20 carats fine, and of 15 carats fine: he would make a mixture of 42*oz.* of 17 carats fine: how much must be taken of each sort?

Ans. { 4 of 24; 4 of 22; 4 of 20; and 30 of 15 carats fine.

DUODECIMALS.

253. Duodecimals are denominate fractions in which 1 foot is the unit that is divided.

The unit 1 foot is first supposed to be divided into 12 equal parts, called inches or primes, and marked ′.

Each of these parts is supposed to be again divided into 12 equal parts, called seconds, and marked ″.

Each second is divided, in like manner, into 12 equal parts, called thirds, and marked ‴.

This division of the foot gives

1′ inch or prime - - - $= \frac{1}{12}$ of a foot.
1″ second is $= \frac{1}{12}$ of $\frac{1}{12}$ - $= \frac{1}{144}$ of a foot.
1‴ third is $= \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12} = \frac{1}{1728}$ of a foot.

Hence, in duodecimals, the divisions of the foot increase from the lower denominations to the higher, according to the scale of twelves.

254. Duodecimals are added and subtracted like other denominate numbers, 12 of a lesser denomination making one of a greater, as in the following

TABLE.

12‴	make	1″	second.
12″	"	1′	inch or prime.
12′	"	1	foot.

EXAMPLES.

1. In 185′, how many feet? *Ans.* 15*ft.* 5′.
2. In 250″, how many feet and inches? *Ans.* 1*ft.* 8′ 10″.
3. In 4367‴, how many feet? *Ans.* 2*ft.* 6′ 3″ 11‴.

EXAMPLES IN ADDITION AND SUBTRACTION.

1. What is the sum of 3*ft.* 6′ 3″ 2‴ and 2*ft.* 1′ 10″ 11‴? *Ans.* 5*ft.* 8′ 2″ 1‴.

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253. In Duodecimals, what is the unit that is divided? How is it divided? How are these parts again divided? What are the parts called?

254. How are duodecimals added and subtracted? How many of one denomination make 1 of the next greater?

2. What is the sum of 8*ft.* 9′ 7″ and 6*ft.* 7′ 3″ 4‴?
Ans. 15*ft.* 4′ 10″ 4‴.

3. What is the difference between 9*ft.* 3′ 5″ 6‴ and 7*ft.* 3′ 6″ 7‴? *Ans.* ——

4. What is the difference between 40*ft.* 6′ 6″ and 29*ft.* 7‴? *Ans.* 11*ft.* 6′ 5″ 5‴.

MULTIPLICATION OF DUODECIMALS.

255. It is known that feet multiplied by feet give square feet in the product. It is now required to show what fractions of the square foot will arise from multiplying feet by the divisions of the foot, and the divisions of the foot by each other.

EXAMPLES.

1. Multiply 6*ft.* 7′ 8″ by 2*ft.* 9′.

OPERATION.

	ft.		
	6	7′	8″
	2	9′	
2 × 8″ =		1′	4″
2 × 7′ =	1	2′	
2 × 6 =	12		
9′ × 8″ =			6″
9′ × 7′ =		5′	3″
9′ × 6 =	4	6′	
Prod.	18	3′	1″

Set down the multiplier under the multiplicand, so that feet shall fall under feet, and the corresponding divisions under each other. It is found most convenient to begin with the highest denomination of the multiplier, and multiply it by the lowest denomination of the multiplicand. Recollecting that 7′ expresses $\frac{7}{12}$ of a foot, and that 8″ expresses $\frac{8}{12}$ of $\frac{1}{12}$ of a foot, we see that 2 × 8″ will give 16-twelfths of twelfths of a square foot; that is, one-twelfth and four-twelfths of onc-twelfth, or 4″. The 2 feet multiplied by 7′ give 14-twelfths of a square foot; that is, 1 square foot and two-twelfths, or 2′. The feet multiplied by 6 give 12 square feet.

Again, 9 inches or $\frac{9}{12}$ of a foot multiplied by 8-twelfths of $\frac{1}{12}$ of a foot, will give 72 twelfths of twelfths of

255. In multiplication, how do you set down the multiplier? Where do you begin to multiply? How do you carry from one denomination to another?

twelfths of a square foot, which are equal to six-twelfths of twelfths, or to 6″. Then 9′×7′ gives 63 twelfths of twelfths of a square foot, equal to 5′ and 3″: and 9′×6 gives 4 square feet and 6′.

256. Hence we see,

1st. *That feet multiplied by feet give square feet in the product.*

2d. *That feet multiplied by inches give twelfths of square feet in the product.*

3d. *That inches multiplied by inches give twelfths of twelfths of square feet in the product.*

4th. *That inches multiplied by seconds give twelfths of twelfths of twelfths of square feet in the product.*

2. Multiply 9*ft.* 4*in.* by 8*ft.* 3*in.*

OPERATION.

9	4′		
8	3′		
74	8′		
2	4′	0″	
77	0′	0″	*Ans.*

Beginning with the 8 feet, we say, 8 times 4 are 32′, which is equal to 2 feet 8′: set down the 8′. Then say, 8 times 9 are 72 and 2 to carry are 74 feet: then multiplying by 3′ we say, 3 times 4′ are 12″, equal to 1 inch: set down 0 in the second's place: then 3 times 9 are 27 and 1 to carry make 28′, equal to 2*ft.* 4′. Therefore the entire product is equal to 77*ft.*

3. How many solid feet in a stick of timber which is 25*ft.* 6*in.* long, 2*ft.* 7*in.* broad, and 3*ft.* 3*in.* thick?

4. Multiply 9*ft.* 2*in.* by 9*ft.* 6*in.* *Ans.* 87*ft.* 1′.

5. Multiply 24*ft.* 10*in.* by 6*ft.* 8*in.* *Ans.* ——

6. Multiply 70*ft.* 9*in.* by 12*ft.* 3*in.*

Ans. 866*ft.* 8′ 3″.

7. How many cords and cord feet in a pile of wood 24 feet long, 4 feet wide, and 3*ft.* 6*in.* high?

Ans. 2 *cords* and 5 *cord feet.*

NOTE. It must be recollected that 16 solid feet make one cord foot (ART. 88).

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256. Repeat the four principles. How many solid feet make a cord foot?

INVOLUTION.

257. If a number be multiplied by itself, the product is called the *second power*, or *square* of that number. Thus, $4\times4=16$: the number 16 is the second power or square of 4.

If a number be multiplied by itself, and the product arising be again multiplied by the number, the second product is called the 3d *power*, or *cube* of the number. Thus, $3\times3\times3=27$: the number 27 is the 3d *power*, or *cube* of 3.

The term *power* designates the product arising from multiplying a number by itself a certain number of times, and the number multiplied is called the *root*.

Thus, in the first example above, 4 is the root, and 16 the square or 2d power of 4.

In the second example, 3 is the root, and 27 the 3d power or cube of 3. The first power of a number is the number itself.

258. *Involution is the process of finding the powers of numbers.*

The number which designates the power to which the root is to be raised, is called the *index* or *exponent* of the power. It is generally written on the right, and a little above the root. Thus, 4^2 expresses the 2d power of 4, or that 4 is to be multiplied by itself once: hence,

$$4^2=4\times4=16.$$

For the same reason 3^3 denotes that 3 is to be raised to the 3d power, or cubed: hence,

$3^3=3\times3\times3=27$: we may therefore write

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257. If a number be multiplied by itself once, what is the product called? If it be multiplied by itself twice, what is the product called? What does the term power mean? What is the root? What is the first power of a number?

258. What is Involution? What is the number called which designates the power? Where is it written? What is the exponent of the square of a number? Of the cube? Of the fourth power? How do you raise a number to any power?

$$4 = 4 \text{ the 1st power of } 4.$$
$$4^2 = 4\times4 = 16 \text{ the 2d power of } 4.$$
$$4^3 = 4\times4\times4 = 64 \text{ the 3d power of } 4.$$
$$4^4 = 4\times4\times4\times4 = 256 \text{ the 4th power of } 4.$$
$$4^5 = 4\times4\times4\times4\times4 = 1024 \text{ the 5th power of } 4.$$
&c., &c., &c.

Hence, to raise a number to any power,

Multiply the number continually by itself as many times less 1 as there are units in the exponent, and the last product will be the power sought.

EXAMPLES.

1. What is the 3d power of 125?
   Ans. $125\times125\times125=1953125$.
2. What is the cube of 7? *Ans.* 343.
3. What is the square of 60? *Ans.* 3600.
4. What is the 4th power of 5? *Ans.* 625.
5. What is the 5th power of 9? *Ans.* 59049.
6. What is the cube of 1? *Ans.* 1.

EVOLUTION.

259. We have seen that Involution teaches how to find the power when the root is given. Evolution is the reverse of Involution: it teaches how to find the root when the power is known. The root is that number which being multiplied by itself a certain number of times, will produce the given power.

The square root of a number is that number which being multiplied by itself once, will produce the given number.

The cube root of a number is that number which being multiplied by itself twice, will produce the given number.

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259. What is Evolution? What does it teach? What is the root of a number? What is the square root of a number? What is the cube root of a number? Make the sign denoting the square root. How do you denote the cube root?

For example, 6 is the square root of 36, because $6 \times 6 = 36$; and 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$. The sign $\sqrt{\ }$ placed before a number denotes that its square root is to be extracted. Thus, $\sqrt{36} = 6$. The sign $\sqrt{\ }$ is called the radical sign, or the sign of the square root.

When we wish to express that the cube root is to be extracted, we place the figure 3 over the sign of the square root: thus, $\sqrt[3]{8} = 2$ and $\sqrt[3]{64} = 4$, and 3 is called the index of the root.

EXTRACTION OF THE SQUARE ROOT.

260. To extract the square root of a number, is to find a number which being multiplied by itself once, will produce the given number. Thus,

$$\sqrt{4} = 2; \quad \text{for } 2 \times 2 = 4;$$
$$\sqrt{9} = 3; \quad \text{for } 3 \times 3 = 9.$$

Before proceeding to explain the rule for extracting the square root, let us first see how the squares of numbers are formed.

The first ten numbers are

1,	2,	3,	4,	5,	6,	7,	8,	9,	10	Roots.
1	4	9	16	25	36	49	64	81	100	Squares.

The numbers in the second line are the squares of those in the first; and the numbers in the first line are the *square roots* of the corresponding numbers of the second.

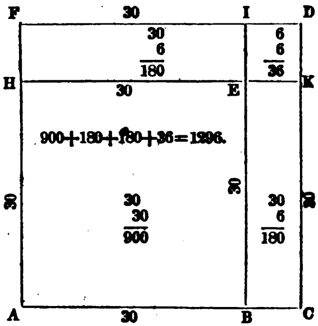
Now, it is evident that, *the square of a number expressed by a single figure will not contain any figure of a*

260. What is required when we wish to extract the square root of a number? What is the greatest square of a single figure? What is the highest order of units that can be derived from the square of a single figure? How many perfect squares are there among the numbers less than one hundred? What is the square of a number expressed by two figures equal to? In what places of figures will the square of the tens be found? In what places will the product of the tens by the units be found?

higher order than tens; and also, that if a number contains three figures, its root must contain tens and units.

The numbers 1, 4, 9, &c., of the second line, are called *perfect squares*, because they have exact roots.

Let us now see how the square of any number may be formed, say the number 36. This number is made up of 3 tens or 30, and 6 units.



Let the line AB represent the 3 tens or 30, and BC the six units.

Let AD be a square on AC, and AE a square on the ten's line AB.

Then ED will be a square on the unit line 6, and the rectangle EF will be the product of HE which is equal to the ten's line, by IE which is equal to the unit line. Also, the rectangle BK will be the product of EB which is equal to the ten's line, by the unit line BC. But the whole square on AC is made up of the square AE, the two rectangles FE and EC, and the square ED. Hence,

The square of two figures is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

Let it now be required to extract the square root of 1296.

Since the number contains more than two places, its root will contain tens and units. But as the square of one ten is one hundred, it follows that the square of the tens of the required root, must be found in the two figures on the left of 96. Hence, we point off the number into periods of two figures each.

We next find the greatest square contained in 12, which is 3 tens or 30.

```
12 96(36
 9
66)396
   396
```

We then square 3 tens which gives 9 hundred, and then place 9 under the hundred's place, and subtract.

This takes away the square AE and leaves the two rectangles FE and BK, together with the square ED on the unit line.

Now, since tens multiplied by units will give at least tens in the product, it follows that the area of the two rectangles FE and EC must be expressed by the figures of the given number at the left of the unit's place 6, which figures may also express a part of the square ED.

If, then, we divide the figures 39, at the left of 6, by twice the tens, that is, by twice AB or BE, the quotient will be BC or EK, the unit of the root.

Then, placing BC or 6, in the root, and also in the divisor, and then multiplying the whole divisor 66 by 6, we obtain for a product the two rectangles FE and CE, together with the square ED.

Hence, the square root of 1296 is 36; or, in other words, 36 is the side of a square whose area is 1296.

CASE I.

261. To extract the square root of a whole number,

I. *Point off the given number into periods of two figures each, counted from the right, by setting a dot over the place of units, another over the place of hundreds, and so on.*

II. *Find the greatest square in the first period on the left, and place its root on the right after the manner of a quotient in division. Subtract the square of the root from the first period, and to the remainder bring down the second period for a dividend.*

III. *Double the root already found and place it on the left for a divisor. Seek how many times the divisor is contained in the dividend, exclusive of the right-hand figure,*

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261. What is the first step in extracting the square root of numbers? What the second? What the third? What the fourth? What the fifth? Give the entire rule.

and place the figure in the root and also at the right of the divisor.

IV. *Multiply the divisor, thus augmented, by the last figure of the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend. But if the product should exceed the dividend, diminish the last figure of the root.*

V. *Double the whole root already found, for a new divisor, and continue the operation as before, until all the periods are brought down.*

EXAMPLES.

1. What is the square root of 263169?

OPERATION.

```
     2̇6 3̇1 6̇9(513
     25
101)131
    101
1023)3069
     3069
```

We first place a dot over the 9, making the right-hand period 69. We then put a dot over the 1 and also over the 6, making three periods.

The greatest perfect square in 26, is 25, the root of which is 5. Placing 5 in the root, subtracting its square from 26, and bringing down the next period 31, we have 131 for a dividend, and by doubling the root we have 10 for a divisor. Now 10 is contained in 13, 1 time. Place 1 both in the root and in the divisor: then multiply 101 by 1; subtract the product and bring down the next period.

We must now double the whole root 51 for a new divisor, or we may take the first divisor after having doubled the last figure 1; then dividing we obtain 3, the third figure of the root.

262. There will be as many figures in the root as there are periods in the given number.

If the given number has not an exact root, there will be a remainder after all the periods are brought down,

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262. How many figures will there be in the root? If the given number has not an exact root, what may be done?

in which case ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

2. What is the square root of 36729?

In this example there are two periods of decimals, which give two places of decimals in the root.

OPERATION.

```
     3 67 29(191.64+.
     1
  29)267
     261
   381)629
       381
  3826)24800
        22956
 38324)184400
        153296
         31104 Rem.
```

3. What is the square root of 106929? *Ans.* 327.

4. What is the square root of 2268741? *Ans.* 1506.23+.

5. What is the square root of 7596796? *Ans.* ——

6. What is the square root of 36372961? *Ans.* ——

7. What is the square root of 22071204? *Ans.* 4698.

CASE II.

263. To extract the square root of a decimal fraction,

I. *Annex one cipher, if necessary, so that the number of decimal places shall be even.*

II. *Point off the decimals into periods of two figures each, by putting a point over the place of hundredths, a second over the place of ten thousandths, &c.: then extract the root as in whole numbers, recollecting that the number of decimal places in the root must be equal to the number of periods in the given decimal.*

---

263. How do you extract the square root of a decimal fraction? When there is a decimal and a whole number joined together, will the same rule apply?

EXAMPLES.

1. What is the square root of .5?

We first annex one cipher to make even decimal places. We then extract the root of the first period, to which we annex ciphers, forming new periods.

OPERATION.

```
       .50(.707+
        49
   140)100
        000
  1407)10000
        9849
         151 Rem.
```

NOTE. When there is a decimal and a whole number joined together the same rule will apply.

2. What is the square root of 3271.4207? *A.* 57.19+.
3. What is the square root of 4795.25731?
4. What is the square root of 4.372594? *A.* 2.091+.
5. What is the square root of .00032754? *Ans.* ——
6. What is the square root of .00103041? *A.* .0321.
7. What is the square root of 4.426816? *Ans.* ——

CASE III.

264. To extract the square root of a vulgar fraction,

I. *Reduce mixed numbers to improper fractions, and compound and complex fractions to simple ones, and then reduce the fraction to its lowest terms.*

II. *Extract the square root of the numerator and denominator separately, if they have exact roots; but when they have not, reduce the fraction to a decimal and extract the root as in Case II.*

EXAMPLES.

1. What is the square root of $\frac{2304}{5184}$? *Ans.* $\frac{2}{3}$.
2. What is the square root of $\frac{2704}{4225}$? *Ans.* ——
3. What is the square root of $\frac{9216}{12544}$? *Ans.* $\frac{6}{7}$.
4. What is the square root of $\frac{275}{341}$? *Ans.* .89802+.
5. What is the square root of $\frac{357}{476}$? *Ans.* .86602+.
6. What is the square root of $\frac{478}{549}$? *Ans.* .93309+.

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264. How do you extract the square root of a vulgar fraction?

EXTRACTION OF THE CUBE ROOT.

265. To extract the cube root of a number is to find a *second* number which being multiplied into itself twice, shall produce the given number.

Thus, 2 is the cube root of 8; for, $2\times2\times2=8$: and 3 is the cube root of 27; for, $3\times3\times3=27$.

Roots	1,	2,	3,	4,	5,	6,	7,	8,	9.
Cubes	1	8	27	64	125	216	343	512	729.

From which we see, that the cube of units will not give a higher order than hundreds. We may also remark, that the cube of one ten or 10, is 1000: and the cube of 9 tens or 90, 729000; and hence, *the cube of tens will not give a lower denomination than thousands, nor a higher denomination than hundreds of thousands.* Hence also, if a number contains more than three figures its cube root will contain more than one; if the number contains more than six figures the root will contain more than two; and so on, every three figures from the right giving one additional place in the root, and the figures which remain at the left hand, although less than three, will also give one place in the root.

Let us now see how the cube of any number, as 16, is formed. Sixteen is composed of 1 ten and 6 units, and may be written 10+6. Now to find the cube of 16 or of 10+6, we must multiply the number by itself twice.

To do this we place the numbers thus			10+	6
			10+	6
Product by the units - - - -			60+	36
Product by the tens - - -		100+	60	
Square of 16 - - - -		100+	120+	36
Multiply again by 16 - - - -			10+	6
Product by the units - - -		600+	720+	216
Product by the tens -	1000+	1200+	360	
Cube of 16 - -	1000+	1800+	1080+	216

265. What is required when we are to extract the cube root of a number?

1. By examining the composition of this number it will be found that the first part 1000 is the cube of the tens; that is,

$$10\times10\times10=1000.$$

2. The second part 1800 is equal to three times the square of the tens multiplied by the units; that is,

$$3\times(10)^2\times6=3\times100\times6=1800.$$

3. The third part 1080 is equal to three times the square of the units multiplied by the tens; that is,

$$3\times6^2\times10=3\times36\times10=1080.$$

4. The fourth part is equal to the cube of the units; that is,

$$6^3=6\times6\times6=216.$$

Let it now be required to extract the cube root of the number 4096.

OPERATION.

$$\begin{array}{l} \dot{4}\ 09\dot{6}(16 \\ 1 \\ 1^2\times3=3)3\ 0\ \ (9\text{-}8\text{-}7\text{-}6 \\ 16^3=4\ 096. \end{array}$$

Since the number contains more than three figures, we know that the root will contain at least units and tens.

Separating the three right-hand figures from the 4, we know that the cube of the tens will be found in the 4. Now, 1 is the greatest cube in 4.

Hence, we place the root 1 on the right, and this is the tens of the required root. We then cube 1 and subtract the result from 4, and to the remainder we bring down the first figure 0 of the next period.

Now, we have seen that the second part of the cube of 16, viz., 1800, being three times the square of the tens multiplied by the units, will have no significant figure of a less denomination than hundreds, and consequently will make up a part of the 30 hundreds above. But this 30 hundreds also contains all the hundreds which come from the 3d and 4th parts of the cube of 16. If this were not the case, the 30 hundreds divided by three times the square of the tens would give the unit figure exactly.

Forming a divisor of three times the square of the tens, we find the quotient to be ten; but this we know to be too large. Placing 9 in the root and cubing 19, we find the result to be 6859. Then trying 8 we find the cube of 18 still too large; but when we take 6 we find the exact number. Hence, the cube root of 4096 is 16.

CASE I.

266. To extract the cube root of a whole number,

I. *Point off the given number into periods of three figures each, by placing a dot over the place of units, a second over the place of thousands, and so on to the left: the left-hand period will often contain less than three places of figures.*

II. *Seek the greatest cube in the first period, and set its root on the right after the manner of a quotient in division. Subtract the cube of this figure from the first period, and to the remainder bring down the first figure of the next period, and call this number the dividend.*

III. *Take three times the square of the root just found for a divisor and see how often it is contained in the dividend, and place the quotient for a second figure of the root. Then cube the figures of the root thus found, and if their cube be greater than the first two periods of the given number, diminish the last figure, but if it be less, subtract it from the first two periods, and to the remainder bring down the first figure of the next period, for a new dividend.*

IV. *Take three times the square of the whole root for a new divisor, and seek how often it is contained in the new dividend: the quotient will be the third figure of the root. Cube the whole root and subtract the result from the first three periods of the given number, and proceed in a similar way for all the periods.*

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266. How do you extract the cube root of a whole number?

EXAMPLES.

1. What is the cube root of 99252847?

	99 252 847(463	
	$4^3=64$	
	$4^2\times3=48$)352	dividend.
First two periods - - -	99 252	
$(46)^3=46\times46\times46=$	97 336	
$3\times(46)^2=6348$)	19168	2d dividend.
The first three periods -	99 252 847	
$(463)^3$	=99 252 847	

Ans. 463.

2. What is the cube root of 389017? *Ans.* 73.
3. What is the cube root of 5735339? *Ans.* ——
4. What is the cube root of 32461759? *Ans.* 319.
5. What is the cube root of 84604519? *Ans.* ——
6. What is the cube root of 259694072? *Ans.* 638.
7. What is the cube root of 48228544? *Ans.* 364.
8. What is the cube root of 27054036008? *Ans.* 3002.

CASE II.

267. To extract the cube root of a decimal fraction,

Annex ciphers to the decimals, if necessary, so that it shall consist of 3, 6, 9, *&c., places. Then put the first point over the place of thousandths, the second over the place of millionths, and so on over every third place to the right; after which extract the root as in whole numbers.*

NOTE 1. There will be as many decimal places in the root as there are periods in the given number.

NOTE 2. The same rule applies when the given number is composed of a whole number and a decimal.

NOTE 3. If in extracting the root of a number there is

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267. How do you extract the cube root of a decimal fraction? How many decimal places will there be in the root? Will the same rule apply when there is a whole number and a decimal? In extracting the root, if there is a remainder, what may be done?

a remainder after all the periods have been brought down, periods of ciphers may be annexed by considering them as decimals.

EXAMPLES.

1. What is the cube root of .157464? *Ans.* .54.
2. What is the cube root of .870983875? *Ans.* ——
3. What is the cube root of 12.977875? *Ans.* 2.35.
4. What is the cube root of .751089429? *Ans.* ——
5. What is the cube root of .353393243? *Ans.* ——
6. What is the cube root of 3.408862625? *Ans.* 1.505.
7. What is the cube root of 27.708101576? *Ans.* 3.026.

CASE III.

268. To extract the cube root of a vulgar fraction,

I. *Reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms.*

II. *Then extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal, and extract the root as in the last Case.*

EXAMPLES.

1. What is the cube root of $\frac{250}{686}$? *Ans.* $\frac{5}{7}$.
2. What is the cube root of $12\frac{19}{27}$? *Ans.* $2\frac{1}{3}$.
3. What is the cube root of $31\frac{15}{343}$? *Ans.* ——
4. What is the cube root of $\frac{324}{1500}$? *Ans.* $\frac{3}{5}$.
5. What is the cube root of $\frac{4}{7}$? *Ans.* .829+.
6. What is the cube root of $\frac{5}{9}$? *Ans.* ——
7. What is the cube root of $\frac{2}{3}$? *Ans.* .873+.

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268. *How do you extract the cube root of a vulgar fraction?*

ARITHMETICAL PROGRESSION.

269. If we take any number, as 2, we can, by the continued addition of any other number, as 3, form a *series* of numbers: thus,

2, 5, 8, 11, 14, 17, 20, 23, &c.,

in which each number is formed by the addition of 3 to the preceding number.

This series of numbers may also be formed by subtracting 3 continually from the larger number: thus,

23, 20, 17, 14, 11, 8, 5, 2.

A series of numbers formed in either way is called an *Arithmetical Series*, or an *Arithmetical Progression;* and the number which is added or subtracted is called the *common difference.*

When the series is formed by the continued addition of the common difference, it is called an *ascending* series; and when it is formed by the subtraction of the common difference, it is called a *descending* series; thus,

2, 5, 8, 11, 14, 17, 20, 23, is an ascending series.
23, 20, 17, 14, 11, 8, 5, 2, is a descending series.

The several numbers are called *terms* of the progression; the first and last terms are called the *extremes*, and the intermediate terms are called the *means*.

270. In every arithmetical progression there are five things which are considered, any three of which being given or known, the remaining two can be determined. They are,

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269. How do you form an Arithmetical Series? What is the common difference? What is an ascending series? What a descending series? What are the several numbers called? What are the first and last terms called? What are the intermediate terms called?

270. In every arithmetical progression how many things are considered? What are they?

1st, the first term;
2d, the last term;
3d, the common difference;
4th, the number of terms;
5th, the sum of all the terms.

271. By considering the manner in which the ascending progression is formed, we see that the 2d term is obtained by adding the common difference to the 1st term; the 3d, by adding the common difference to the 2d; the 4th, by adding the common difference to the 3d, and so on; *the number of additions being* 1 *less than the number of terms found.*

But instead of making the additions, we may multiply the common difference by the number of additions, that is, by 1 less than the number of terms, and add the first term to the product. Hence, we have

CASE I.

Having given the first term, the common difference, and the number of terms, to find the last term.

Multiply the common difference by 1 *less than the number of terms, and to the product add the first term.*

EXAMPLES.

1. The first term is 3, the common difference 2, and the number of terms 19: what is the last term?

We multiply the number of terms less 1, by the common difference 2, and then add the first term.

OPERATION.

18	number of terms less 1.
2	common difference.
36	
3	1st term.
39	last term.

2. A man bought 50 yards of cloth, for which he was to pay 6 cents for the 1st yard, 9 cents for the 2d, 12 cents for the 3d, and so on increasing by the common difference 3: how much did he pay for the last yard?

Ans. $1,53.

---

271. How do you find the last term when the first term and common difference are known?

3. A man puts out $100 at simple interest, at 7 per cent.; at the end of the 1st year it will have increased to $107, at the end of the 2d year to $114, and so on, increasing $7 each year: what will be the amount at the end of 16 years? *Ans.* $205.

272. Since the last term of an arithmetical progression is equal to the first term added to the product of the common difference by 1 less than the number of terms, it follows, that the difference of the extremes will be equal to this product, and that the common difference will be equal to this product divided by 1 less than the number of terms. Hence, we have

CASE II.

Having given the two extremes and the number of terms of an arithmetical progression, to find the common difference.

Subtract the less extreme from the greater and divide the remainder by 1 less than the number of terms: the quotient will be the common difference.

EXAMPLES.

1. The extremes are 4 and 104, and the number of terms 26: what is the common difference?

We subtract the less extreme from the greater and divide the difference by one less than the number of terms.

OPERATION.

$$\begin{array}{r} 104 \\ 4 \\ \hline 26-1=25)100(4 \end{array}$$

2. A man has 8 sons, the youngest is 4 years old and the eldest 32, their ages increase in arithmetical progression: what is the common difference of their ages?

$32-4=28$: then $8-1=7)28(4$. *Ans.* 4.

3. A man is to travel from New York to a certain place in 12 days; to go 3 miles the first day, increasing every day by the same number of miles; so that the last day's journey may be 58 miles: required the daily increase. *Ans.* 5 *miles*

---

272. How do you find the common difference, when you know the two extremes and number of terms?

273. If we take any arithmetical series, as

3	5	7	9	11	13	15	17	19, &c.	
19	17	15	13	11	9	7	5	3	by reversing the order of the terms.
22	22	22	22	22	22	22	22	22	

Here we see that the sum of the terms of these two series is equal to 22, the sum of the extremes, multiplied by the number of terms; and consequently, the sum of either series is equal to the sum of the two extremes multiplied by half the number of terms; hence, we have

CASE III.

To find the sum of all the terms of an arithmetical progression,

Add the extremes together and multiply their sum by half the number of terms: the product will be sum of the series.

EXAMPLES.

1. The extremes are 2 and 100, and the number of terms 22: what is the sum of the series?

We first add together the two extremes, and then multiply by half the number of terms.

OPERATION.

2	1st term
100	last term
102	sum of extremes
11	half the number of terms
1122	sum of series.

2. How many strokes does the hammer of a clock strike in 12 hours? *Ans.* 78.

3. The first term of a series is 2, the common difference 4, and the number of terms 9, what is the last term and sum of the series? *Ans.* last term 34, sum 162.

4. If 100 eggs are placed in a right line, exactly one yard from each other, and the first one yard from a basket: what distance will a man travel who gathers them up singly, and places them in the basket?

Ans. 5 *miles*, 1300 *yards.*

273. How do you find the sum of an arithmetical series?

GEOMETRICAL PROGRESSION.

274. If we take any number, as 3, and multiply it continually by any other number, as 2, we form a series of numbers: thus,

3 6 12 24 48 96 192, &c.,

in which each number is formed by multiplying the number before it by 2.

This series may also be formed by dividing continually the largest number 192 by 2. Thus,

192 96 48 24 12 6 3.

A series formed in either way, is called a Geometrical Series, or a Geometrical Progression, and the number by which we continually multiply or divide, is called the *common ratio.*

When the series is formed by multiplying continually by the common ratio, it is called an *ascending series;* and when it is formed by dividing continually by the common ratio, it is called a *descending series.* Thus,

3 6 12 24 48 96 192 is an ascending series.
192 96 48 24 12 6 3 is a descending series.

The several number are called *terms* of the progression.

The first and last terms are called the *extremes,* and the intermediate terms are called the *means.*

275. In every Geometrical, as well as in every Arithmetical Progression, there are five things which are considered, any three of which being given or known, the remaining two can be determined. They are,

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274. How do you form a Geometrical Progression? What is the common ratio? What is an ascending series? What is a descending series? What are the several numbers called? What are the first and last terms called? What are the intermediate terms called?

275. In every geometrical progression, how many things are considered? What are they? How many must be known before the remaining ones can be found? What is any term equal to? How do you find the last term?

1st, the first term,
2d, the last term,
3d, the common ratio,
4th, the number of terms,
5th, the sum of all the terms.

By considering the manner in which the ascending progression is formed, we see that the second term is obtained by multiplying the first term by the common ratio; the 3d term by multiplying this product by the common ratio, and so on, the number of multiplications being one less than the number of terms. Thus,

$3=3$ 1st term,
$3\times2=6$ 2d term,
$3\times2\times2=12$ 3d term,
$3\times2\times2\times2=24$ 4th term, &c. for the other terms.

But $2\times2=2^2$, $2\times2\times2=2^3$, and $2\times2\times2\times2=2^4$.

Therefore, any term of the progression is equal to the first term multiplied by the ratio raised to a power 1 less than the number of the term.

CASE I.

Having given the first term, the common ratio, and the number of terms, to find the last term,

Raise the ratio to a power whose exponent is one less than the number of terms, and then multiply the power by the first term: the product will be the last term.

EXAMPLES.

1. The first term is 3 and the ratio 2: what is the 6th term?

$$2\times2\times2\times2\times2=2^5=32$$

3 1st term.

Ans. 96

2. A man purchased 12 pears: he was to pay 1 farthing for the 1st, 2 farthings for the 2d, 4 for the 3d, and so on doubling each time: what did he pay for the last? Ans. £2 2s. 8d.

3. A gentleman dying left nine sons, and bequeathed his estate in the following manner: to his executors £50; his youngest son to have twice as much as the executors, and each son to have double the amount of the son next younger: what was the eldest son's portion? *Ans.* £25600.

4. A man bought 12 yards of cloth, giving 3 cents for the 1st yard, 6 for the 2d, 12 for the 3d, &c.: what did he pay for the last yard? *Ans.* $61,44.

CASE II.

276. Having given the ratio and the two extremes to find the sum of the series.

Subtract the less extreme from the greater, divide the remainder by 1 less than the ratio, and to the quotient add the greater extreme: the sum will be the sum of the series.

EXAMPLES.

1. The first term is 3, the ratio 2, and last term 192: what is the sum of the series?

$192-3=189$ difference of the extremes,

$2-1=1$) 189 (189; then $189+192=381$ *Ans.*

2. A gentleman married his daughter on New Year's day, and gave her husband 1*s.* towards her portion, and was to double it on the first day of every month during the year: what was her portion? *Ans.* £204 15*s.*

3. A man bought 10 bushels of wheat on the condition that he should pay 1 cent for the 1st bushel, 3 for the 2d, 9 for the 3d, and so on to the last: what did he pay for the last bushel, and for the 10 bushels?

Ans. Last bushel $196,83, total cost $295,24.

4. A man has 6 children; to the 1st he gives $150, to the 2d $300, to the 3d $600, and so on, to each twice as much as the last: how much did the eldest receive, and what was the amount received by them all?

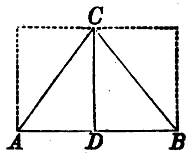
Ans. Eldest $4800, total $9450.

---

276. How do you find the sum of the series?

MENSURATION.

277. A triangle is a figure bounded by three straight lines. Thus, BAC is a triangle.



The three lines BA, AC, BC, are called *sides:* and the three corners, B, A, and C, are called *angles*. The side AB is called the *base*.

When a line like CD is drawn making the angle CDA equal to the angle CDB, then CD is said to be perpendicular to AB, and CD is called the *altitude* of the triangle. Each triangle CAD or CDB is called a right-angled triangle. The side BC, or the side AC, opposite the right angle, is called the *hypothenuse*.

The area or content of a triangle is equal to half the product of its base by its altitude.

EXAMPLES.

1. The base, AB, of a triangle is 40 yards, and the perpendicular, CD, 20 yards: what is the area?

We first multiply the base by the altitude, and the product is square yards, which we divide by 2 for the area.

OPERATION.

$$\begin{array}{r} 40 \\ 20 \\ \hline 2)800 \\ \hline 400 \end{array}$$

Ans. 400 *square yards.*

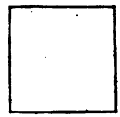
2. In a triangular field the base is 40 chains, and the perpendicular 15 chains: how much does it contain? (Art. 87). *Ans.* 30 *acres.*

---

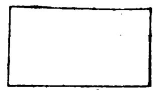
277. What is a triangle? What is the base? What the altitude? What is a right-angled triangle? Which side is the hypothenuse? What is the area of a triangle equal to?

8. There is a triangular field, of which the base is 35 rods and the perpendicular 26 rods: what is its content? *Ans.* 2*A*. 3*R*. 15*P*.

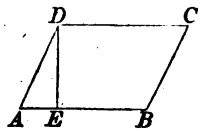
278. A square is a figure having four equal sides, and all its angles right angles.



279. A rectangle is a four-sided figure like a square, in which the sides are perpendicular to each other, but the adjacent sides are not equal.



280. A parallelogram is a four-sided figure which has its opposite sides equal and parallel, but its angles not right angles. The line DE, perpendicular to the base, is called the altitude.



281. To find the area of a square, rectangle, or parallelogram,

Multiply the base by the perpendicular height, and the product will be the area.

EXAMPLES.

1. What is the area of a square field of which the sides are each 33.08 chains? *Ans.* 109*A*. 1*R*. 28*P*.+.
2. What is the area of a square piece of land of which the sides are 27 chains? *Ans.* ——
3. What is the area of a square piece of land of which the sides are 25 rods each? *Ans.* 3*A*. 3*R*. 25*P*.

---

278. What is a square?
279. What is a rectangle?
280. What is a parallelogram?
281. How do you find the area of a square, rectangle, or parallelogram?

4. What is the content of a rectangular field, the length of which is 40 rods and the breadth 20 rods?

Ans. 5 *acres.*

5. What is the content of a field 40 rods square?

Ans. 10 *acres.*

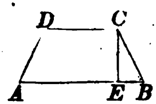
6. What is the content of a rectangular field 15 chains long and 5 chains broad? *Ans.* ——

7. What is the content of a field 27 chains long and 9 rods broad? *Ans.* 6*A.* 0*R.* 12*P.*

8. The base of a parallelogram is 271 yards, and the perpendicular height 360 feet: what is the area?

Ans. 32520 *square yards.*

282. A trapezoid is a four-sided figure ABCD, having two of its sides, AB, DC, parallel. The perpendicular CE is called the altitude.



283. To find the area of a trapezoid,

Multiply the sum of the two parallel sides by the altitude, and divide the product by 2, the quotient will be the area.

EXAMPLES.

1. Required the area of the trapezoid ABCD, having given

AB=321.51*ft.*, DC=214.24*ft.*, and CE=171.16*ft.*

We first find the sum of the sides, and then multiply it by the perpendicular height, after which, we divide the product by 2, for the area.

OPERATION.

$321.51+214.24=535.75=$ sum of parallel sides.

Then,

$535.75\times171.16=91698.97$

and, $\frac{91698.97}{2}=45849.485$

$=$the area.

---

282. What is a trapezoid?

283. How do you find the area of a trapezoid?

2. What is the area of a trapezoid, the parallel sides of which are 12.41 and 8.22 chains, and the perpendicular distance between them 5.15 chains?

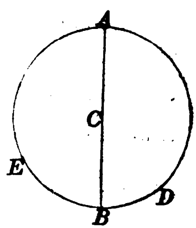
Ans. 5*A*. 1*R*. 9.956*P*.

3. Required the area of a trapezoid whose parallel sides are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet and 5 inches. *Ans.* 230*Sq. ft.* 5′ 7″.

4. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25, and the perpendicular distance between them 10.75 yards. *Ans.* 176.03125*Sq. yd.*

5. What is the area of a trapezoid whose parallel sides are 7.50 chains, and 12.25 chains, and the perpendicular height 15.40 chains? *Ans.* 15*A*. 0*R*. 33.2*P*.

6. What is the content when the parallel sides are 20 and 32 chains, and the perpendicular distance between them 26 chains? *Ans.* 67*A*. 2*R*. 16*P*



284. A circle is a portion of a plane bounded by a curved line, every part of which is equally distant from a certain point within, called the centre.

The curved line AEBD is called the *circumference;* the point C the *centre;* the line AB passing through the centre, a *diameter;* and CB the *radius.*

The circumference AEBD is 3.1416 times greater than the diameter AB. Hence, if the diameter is 1, the circumference will be 3.1416. Hence, also, if the diameter is known, the circumference is found by multiplying 3.1416 by the diameter.

---

284. What is a circle? What is the centre? What is the diameter? What the radius? How many times greater is the circumference than the diameter? How do you find the circumference when the diameter is known?

EXAMPLES.

1. The diameter of a circle is 4, what is the circumference?

The circumference is found by simply multiplying 3.1416 by the diameter.

OPERATION.

```
      3.1416
           4
Ans. 12.5664
```

2. The diameter of a circle is 93, what is the circumference? *Ans.* ——

3. The diameter of a circle is 20, what is the circumference? *Ans.* 62.832.

285. Since the circumference of a circle is 3.1416 times greater than the diameter, it follows that if the circumference is known we may find the diameter by dividing it by 3.1416.

EXAMPLES.

1. What is the diameter of a circle whose circumference is 78.54.

We divide the circumference by 3.1416, the quotient 25 is the diameter.

OPERATION.

```
3.1416)78.5400(25
       62832
       157080
       157080
```

2. What is the diameter of a circle whose circumference is 11652.1944? *Ans.* 3709.

3. What is the diameter of a circle whose circumference is 6850? *Ans.* 2180.41+.

286. To find the area or content of a circle,

Multiply the square of the diameter by the decimal .7854.

---

285. How do you find the diameter when the circumference known?

286. How do you find the area of a circle?

EXAMPLES.

1. What is the area of a circle whose diameter is 6?

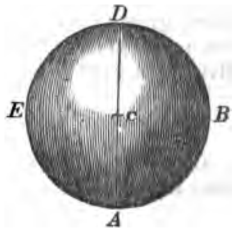
	OPERATION.
We first square the diameter, giving 36, which we then multiply by the decimal .7854: the product is the area of the circle.	$6^2=36$ $.7854\times36=28.2744$ *Ans.* 28.2744

2. What is the area of a circle whose diameter is 10? *Ans.* 78.54.

3. What is the area of a circle whose diameter is 7? *Ans.* ——

4. How many square yards in a circle whose diameter is $3\frac{1}{2}$ feet? *Ans.* 1.069016+.

287. A sphere is a solid terminated by a curved surface, all the points of which are equally distant from a certain point within called the centre. The line AD passing through its centre C is called the diameter of the sphere, and AC its radius.



288. To find the surface of a sphere,

Multiply the square of the diameter by 3.1416.

EXAMPLES.

1. What is the surface of a sphere whose diameter is 12?

	OPERATION.
We simply multiply the decimal 3.1416 by the square of the diameter: the product is the surface.	3.1416 $12^2=144$ *Ans.* 452.3904

287. What is a sphere? What is a diameter? What is a radius?

288. How do you find the surface of a sphere?

2. What is the surface of a sphere whose diameter is 7? *Ans.* 153.9384.

3. Required the number of square inches in the surface of a sphere whose diameter is 2 feet or 24 inches. *Ans.* ——

4. Required the area of the surface of the earth, its mean diameter being 7918.7 miles. *Ans.* 196996571.722104 *sq. miles.*

289. To find the solidity of a sphere,

Multiply the surface by the diameter and divide the product by 6, *the quotient will be the solidity.*

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 12?

We first find the surface by multiplying the square of the diameter by 3.1416. We then multiply the surface by the diameter, and divide the product by 6.

OPERATION.

$$\overline{12}^2 = 144$$
multiply by 3.1416
surface = 452.3904
diameter 12
6)5428.6848
solidity = 904.7808

2. What is the solidity of a sphere whose diameter is 4? *Ans.* 33.5104.

3. What is the solidity of a sphere whose diameter is 14 inches? *Ans.* ——

4. What is the solidity of the earth, its mean diameter being 7918.7 miles? *Ans.* 259992792079.860+.

5. What is the solidity of a sphere whose diameter is 6 feet? *Ans.* 113.0976 *S. ft.*

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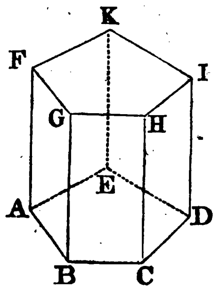
289. How do you find the solidity of a sphere?

290. A prism is a solid whose ends are equal plane figures and whose faces are parallelograms.



The sum of the sides which bound the base is called the perimeter of the base, and the sum of the parallelograms which bound the solid is called the convex surface.

To find the convex surface of a prism,

Multiply the perimeter of the base by the perpendicular height, and the product will be the convex surface.

EXAMPLES.

1. What is the convex surface of a prism whose base is bounded by five equal sides, each of which is 35 feet, the altitude being 26 feet? *Ans.* 4550*Sq. ft.*

2. What is the convex surface when there are eight equal sides, each 15 feet in length, and the altitude is 12 feet? *Ans.* 1440*Sq. ft.*

291. To find the solid content of a prism,

Multiply the area of the base by the altitude, and the product will be the content.

EXAMPLES.

1. What is the content of a square prism, each side of the square which forms the base being 15, and the altitude of the prism 20 feet?

We first find the area of the square which forms the base, and then multiply by the altitude.

OPERATION.

$\overline{15}^2 = 225$

20

Ans. 4500

290. What is a prism? What is the perimeter of its base? What is its convex surface? How do you find the convex surface of a prism?

291. How do you find the solid content of a prism?

2. What is the solid content of a cube each side of which is 24 inches? *Ans.* 13824 *solid in.*

3. How many cubic feet in a block of marble of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches? *Ans.* $21\frac{1}{9}$ *solid ft.*

4. How many gallons of water will a cistern contain whose dimensions are the same as in the last example? *Ans.* $157\frac{213}{231}$ *gal.*

5. Required the solidity of a triangular prism whose height is 10 feet, and area of the base 350? *Ans.* 3500.

292. A cylinder is a round body with circular ends. The line EF is called the axis or altitude, and the circular surface the *convex surface* of the cylinder.

293. To find the convex surface,

Multiply the circumference of the base by the altitude, and the product will be the convex surface.

EXAMPLES.

1. What is the convex surface of a cylinder, the diameter of whose base is 20 and the altitude 50?

We first multiply the diameter by 3.1416, which gives the circumference of the base. Then multiplying by the altitude, we obtain the convex surface.

OPERATION.

```
         3.1416
             20
        -------
        62.8320
             50
        -------
Ans.  3141.6000
```

292. What is a cylinder? What is the axis or altitude? What is the convex surface?

293. How do you find the convex surface?

2. What is the convex surface of a cylinder whose altitude is 14 feet and the circumference of its base 8 feet 4 inches? *Ans.* 116.666+*Sq. ft.*

3. What is the convex surface of a cylinder, the diameter of whose base is 30 inches and altitude 5 feet? *Ans.* 5654.88 *Sq. in.*

4. What is the convex surface of a cylinder, the diameter of whose base is 20 and altitude 50 feet? *Ans.* 3141.6 *Sq. ft.*

294. To find the solidity of a cylinder,

Multiply the area of the base by the altitude: the product will be the solid content.

EXAMPLES.

1. Required the solidity of a cylinder of which the altitude is 12 feet, and the diameter of the base 15 feet.

We first find the area of the base, and then multiply by the altitude: the product is the solidity.

OPERATION.

$15^2 = 225$
.7854
area base 176.7150
12
2120.5800

2. What is the solidity of a cylinder, the diameter of whose base is 20 and the altitude 29? *Ans.* 9110.64.

3. What is the solidity of a cylinder, the diameter of whose base is 12 and the altitude 30? *Ans.* 3392.928.

4. What is the solidity of a cylinder, the diameter of whose base is 16 and altitude 9? *Ans.* ——

5. What is the solidity of a cylinder, the diameter of whose base is 50 and altitude 15? *Ans.* 29452.5.

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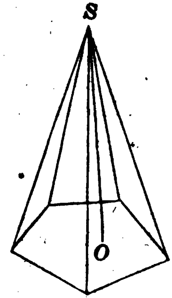
294. How do you find the solidity of a cylinder?



295. A pyramid is a solid formed by several triangular planes united at the same point S, and terminating in the different sides of a plane figure, as ABCDE. The altitude of the pyramid is the line SO, drawn perpendicular to the base.

296. To find the solidity of a pyramid,

***Multiply the area of the base by the altitude, and divide the product by* 3.**

EXAMPLES.

1. Required the solidity of a pyramid, of which the area of the base is 95 and the altitude 15.

We simply multiply the area of the base 95, by the altitude 15, and then divide the product by 3.

OPERATION.

$$\begin{array}{r} 95 \\ 15 \\ \hline 475 \\ 95 \\ \hline 3)1425 \\ \hline \textit{Ans.}\ 475 \end{array}$$

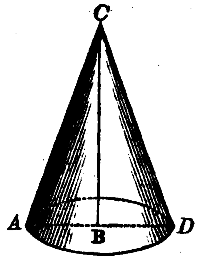
2. What is the solidity of a pyramid, the area of whose base is 260 and the altitude 24? *Ans.* 2080.
3. What is the solidity of a pyramid, the area of whose base is 207 and altitude 18? *Ans.* ——.
4. What is the solidity of a pyramid, the area of whose base is 403 and altitude 30? *Ans.* 4030.
5. What is the solid content of a pyramid, the area of whose base is 270 and altitude 16? *Ans.* 1440.

---

295. What is a pyramid? What is the altitude of a pyramid?
296. *How do you find the solidity of a pyramid?*

6. A pyramid has a rectangular base, the sides of which are 25 and 12; the altitude of the pyramid is 36: what is its solid content? *Ans.* ——

7. A pyramid with a square base, of which each side is 30, has an altitude of 20: what is its solid content? *Ans.* 6000.



297. A cone is a round body with a circular base, tapering off to a point called the *vertex.* The point C is the vertex, and the line CB is called the axis or altitude.

298. To find the solidity of a cone,

Multiply the area of the base by the altitude, and divide the product by 3.

EXAMPLES.

1. Required the solidity of a cone, the diameter of whose base is 5 and the altitude 10.

We first square the diameter and multiply it by .7854 which gives the area of the base. We next multiply by the altitude, and then divide the product by 3.

OPERATION.

$5^2 = 25$

$25 \times .7854 = 19.635$

10

3)196.35

Ans. 65.45

2. What is the solidity of a cone, the diameter of whose base is 18 and the altitude 27? *Ans.* 2290.2264.

3. What is the solid content of a cone, the diameter of whose base is 20 and the altitude 30? *Ans.* ——

297. What is a cone? What is the vertex? What is the axis?
298. How do you find the solidity of a cone?

4. What is the solidity of a cone whose altitude is 27 feet, and the diameter of the base 10 feet? *Ans.* 706.86.

5. What is the solidity of a cone whose altitude is 12 feet, and the diameter of its base 15 feet? *Ans.* 706.86.

GAUGING.

299. The mean diameter of a cask is found by adding to the head diameter, two-thirds of the difference between the bung and head diameters, or if the staves are not much curved, by adding six-tenths. This reduces the cask to a cylinder. Then, to find the solidity, we multiply the square of the mean diameter by the decimal .7854 and the product by the length. This will give the solid content in cubic inches. Then if we divide by 231, we have the content in gallons (ART. 90).

Multiply the length by the square of the mean diameter, then by the decimal .7854, and divide by 231.

OPERATION.

$$l \times d^2 \times \frac{.7854}{231} = l \times d^2 \times .0034$$

If, then, we divide the decimal .7854 by 231, the quotient carried to four places of decimals is .0034, and this decimal multiplied by the square of the mean diameter and by the length of the cask, will give the content in gallons.

300. Hence, for gauging or measuring casks, we have the following

Multiply the length by the square of the mean diameter: then multiply by 34 *and point off four decimal places, and the product will then express gallons and the decimals of a gallon.*

---

299. How do you find the mean diameter of a cask? How then do you find the solid content in cubic inches? How in gallons?

300. What is the rule for gauging casks?

EXAMPLES.

1. How many gallons in a cask whose bung diameter is 36 inches, head diameter 30 inches, and length 50 inches?

We first find the difference of the diameters, of which we take two-thirds and add to the head diameter. We then multiply the square of the mean diameter, the length and 34 together, and point off four decimal places in the product.

OPERATION.

$36-30=6$

$\frac{2}{3}$ of $6=4$

$30+4=34$

$34^2=1156$

$1156\times50\times34=$

196.52*gal.*

2. What is the number of gallons in a cask whose bung diameter is 38 inches, head diameter 32 inches, and length 42 inches? *Ans.* ——

3. How many gallons in a cask whose length is 36 inches, bung diameter 35 inches, and head diameter 30 inches? *Ans.* 136 *gallons.*

4. A water tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes: the tap discharges, at a medium, 40 gallons in 31 minutes. Now, supposing these to be left open, and the water to be turned on at 2 o'clock in the morning; a servant at 5 shuts the tap, and is solicitous to know in what time the tub will be filled in case the water continues to flow.

Ans. The tub will be full at 3*min.* $48\frac{114}{217}$*sec. after* 6.

5. How many gallons in a cask whose length is 40 inches, head diameter 34 inches, and bung diameter 38 inches? *Ans.* ——

PROMISCUOUS QUESTIONS.

1. A merchant bought 13 packages of goods, for which he paid $326: what will 39 packages cost at the same rate? *Ans.* $978.

2. Two merchants, A and B, traded together; A put in £320 for 5 months, and B £460 for 3 months; they gained £100: how much should each one receive?
Ans. A £53 13*s.* $9\frac{135}{149}$*d.*, B £46 6*s.* $2\frac{14}{149}$*d.*

3. If I buy 1000 ells Flemish of linen for £90, what must it be sold for per ell English, to make £10 by the purchase? *Ans.* ——

4. If $\frac{5}{6}$ of a gallon of wine cost $\frac{5}{8}$ of a £, what will $\frac{5}{8}$ of a tun cost? *Ans.* ——

5. If an officer's salary is £48 per annum, how much will he receive in 232 days? *Ans.* £30 10*s.* $2\frac{1}{4}$*d.*+.

6. If a gentleman spends, one day with another, £1 7*s.* $10\frac{1}{2}$*d.*, and at the end of the year has saved £340, what is his yearly income? *Ans.* £848 14*s.* $4\frac{1}{2}$*d.*

7. If 8 cannons expend, in one day, 48 barrels of powder, how much will 24 cannons expend in 22 days? *Ans.* ——

8. What number is that which being multiplied by $\frac{2}{3}$ will produce $\frac{1}{4}$? *Ans.* $\frac{3}{8}$.

9. A person dying divided his property between his widow and his four sons; to his widow he gave $1780, and to each of his sons $1250: he had been $25\frac{1}{2}$ years in business, and had cleared on an average $126 a year: how much had he when he began business? *Ans.* $3567.

10. A besieged garrison consisting of 360 men was provisioned for 6 months, but hearing of no relief at the end of 5 months, dismissed so many of the garrison that the remaining provision lasted 5 months: how many men were sent away? *Ans.* 288.

11. Two persons, A and B, are indebted to C; A owes $2173, which is the least debt, and the difference of the debts is $371: what is B's debt? *Ans.* ——

12. A man had 12 sons, the youngest was 3 years old and the eldest 58, and their ages increased in arithmetical progression: what was the common difference of their ages? *Ans.* 5 *years.*

13. A snail in getting up a pole 20 feet high, was observed to climb up 8 feet every day, but to descend 4 feet every night: in what time did he reach the top of the pole? *Ans.* 4 *days.*

14. What is the difference between twice four and forty, and twice forty-four: also between twice five and fifty, and twice fifty-five? *Ans.* 40 and 50.

15. A lady being asked her age, and not wishing to give a direct answer, said, I have 9 children, and three years elapsed between the birth of each of them; the eldest was born when I was 19 years old, and the youngest is now exactly 19: what was her age?

Ans. 62 *years.*

16. What number added to the 43d part of 4429, will make the sum 240? *Ans.* 137.

17. A man went to sea at 17 years of age; 8 years after he had a son born, who lived 46 years, and died before his father; after which the father lived twice twenty years and died: what was the age of the father?

Ans. ——

18. A brigade of horse, consisting of 384 men, is to be formed into a solid body consisting of 32 men in front: how many ranks will there be? *Ans.* 12.

19. A room 30 feet long, and 18 feet wide, is to be covered with painted cloth $\frac{3}{4}$ of a yard in width: how many yards will cover it? *Ans.* 80.

20. A, B, and C trade together and gain $120, which is to be shared according to each one's stock; A put in $140, B $300, and C $160: what is each man's share?

Ans. A's $28, B's $60, and C's $32.

21. How many planks 15 feet long and 15 inches wide, will floor a barn $60\frac{1}{2}$ feet long and $33\frac{1}{2}$ feet wide?

Ans. ——

22. A person owned $\frac{3}{8}$ of a mine, and sold $\frac{3}{4}$ of his interest for $1710: what was the value of the entire mine? *Ans.* $3800.

23. Two men depart from the same place and travel in different directions; one goes 7 miles and the other 11 miles per day: how far will they be apart at the end of the 12th day? *Ans.* 216 *miles.*

24. The swiftest velocity of a cannon ball, is about 2000 feet in a second of time. In what time, at that rate, would it be in moving from the earth to the sun, admitting the distance to be 95 millions of miles, and the year to contain 365 days 6 hours. *Ans.* $7\frac{12457}{13149}$ *years.*

25. The slow or parade step is 70 paces per minute, at 28 inches each pace: how fast is that per hour?

Ans. ——

26. A wall of 700 yards in length was to be built in 29 days; twelve men were employed on it for 11 days, and only completed 220 yards: how many men must be added to complete the wall in the required time?

Ans. 4.

27. How far will 500 millions of guineas reach, when laid down in a straight line touching one another, supposing each guinea to be an inch in diameter?

Ans. 7891*mi.* 728*yd.* 2*ft.* 8*in.*

28. A gentleman whose annual income is £1500, spends 20 guineas a week: does he save or run in debt, and how much? *Ans.* ——

29. A person bought 160 oranges at 2 for a penny, and 180 more at 3 for a penny; after which he sold them out at the rate of 5 for 2 pence: did he make or lose, and how much? *Ans.* ——

30. My factor sends me word that he has bought goods to the value of £500 13*s.* 6*d.* upon my account: what will his commission come to at $3\frac{1}{2}$ per cent.?

Ans. £17 10*s.* $5\frac{1}{2}$*d.*+.

31. If a quantity of provisions serves 1500 men 12 weeks, at the rate of 20 ounces a day for each man; how many men will the same provisions maintain for 20 weeks, at the rate of 8 ounces a day for each man?

Ans. ——

32. A younger brother received $8400, which was just $\frac{7}{9}$ of his elder brother's fortune: what was the father worth? Ans. $19200.

33. If 20 men can perform a piece of work in 12 days, how many men will accomplish three times as much in one-fifth of the time? *Ans.* 300.

34. Suppose that I have $\frac{3}{16}$ of a ship worth $1200; what part have I left after selling $\frac{2}{5}$ of $\frac{4}{9}$ of my share, and what is it worth?

Ans. $\frac{37}{240}$ left, worth $986,66+.

35. What number is that which being multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{1}{2}$, the product will be 1? *Ans.* $1\frac{1}{4}$.

36. What number is that which being multiplied by three thousandths, the product will be 2637? *Ans.* ——

37. What length must be cut off a board $8\frac{1}{4}$ inches broad to contain a square foot, or as much as 12 inches in length and 12 in breadth? *Ans.* $16\frac{16}{17}$ *inches.*

38. A man exchanged 70 bushels of rye, at $0,92 per bushel, for 40 bushels of wheat, at $1,37$\frac{1}{2}$ per bushel, and received the balance in oats, at $0,40 per bushel: how many bushels of oats did he receive? *Ans.* ——

39. My horse and saddle together are worth $132, and the horse is worth 10 times as much as the saddle: what is the value of the horse? *Ans.* ——

40. Four persons traded together on a capital of $6000, of which A put in $\frac{1}{2}$, B put in $\frac{1}{4}$, C put in $\frac{1}{6}$, and D the rest; at the end of 4 years they had gained $4728: what was each one's share of the gain?

Ans. A's $2364, B's $1182, C's $788, D's $394.

41. A farmer being asked how many sheep he had, answered, that he had them in five fields, in the 1st he had $\frac{1}{4}$ of his flock, in the 2d $\frac{1}{6}$, in the 3d $\frac{1}{8}$, in the 4th $\frac{1}{12}$, and in the 5th 450: how many had he? *Ans.* ——

42. The circumference of the earth is 360 degrees, and each degree is $69\frac{1}{2}$ miles, how long would a man be in travelling round it, who travelled at the rate of 20 miles a day, the year being reckoned at 365 days 6 hours? *Ans.* 3 *years* $155\frac{1}{4}$ *days.*

43. How many bricks 8 inches long and 4 inches wide, will pave a yard that is 100 feet by 50 feet; also a yard that is 50 feet square?

Ans. 22500;—*2d yard* 11250.

44. Sound travels about 1142 feet in a second: now, if the flash of a cannon be seen at the moment it is fired, and the report heard 45 seconds after, what distance would the observer be from the gun?

Ans. 9*mi.* 5*fur.* 34*rd.*+.

45. Two persons depart from the same place, one travels 32, and the other 36 miles a day: if they travel in the same direction, how far will they be apart at the end of 19 days, and how far if they travel in contrary directions?

Ans. { 76 *miles* same direction. 1292 *miles* opposite directions. }

46. In a certain orchard, $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ of them bear peaches, $\frac{1}{6}$ of them plums, 120 of them cherries, and 80 of them pears: how many trees are there in the orchard? *Ans.* ——

47. A person being asked the time, said the time past noon is equal to $\frac{1}{3}$ of the time past midnight: what was the hour? *Ans.* 3 *o'clock.*

48. A circular fish pond is 865 feet in diameter: what is its circumference, and what is its area?

Ans. { circumference 2717.484*ft.* area 587655.915 *sq. ft.* }

49. How many stones 2 feet long, 1 foot wide, and 6 inches thick, will build a wall 12 yards long, 2 yards high, and 4 feet thick? *Ans.* ——

50. A well is to be stoned; of which the diameter is 6 feet 6 inches, the thickness of the wall is to be 1 foot 6 inches, leaving the diameter of the well within the stones 3 feet 6 inches. If the well is 40 feet deep, how many feet of stone will be required? *Ans.* 942.48*ft.*

51. A reservoir of water has two cocks to supply it. The first would fill it in 40 minutes, and the second in 50. It has likewise a discharging cock, by which it may be emptied when full in 25 minutes. Now if all the cocks are opened at once, and the water runs uniformly as we have supposed, how long before the cistern will be filled? *Ans.* ——

52. There is a stone which measures 4 feet 6 inches long, 2 feet 9 inches broad, and 5 feet 4 inches deep: how many solid feet does it contain? *Ans.* ——

53. A ship has a leak by which it would fill and sink in 15 hours, but by means of a pump it could be emptied, if full, in 16 hours. Now, if the pump is worked from the time the leak begins, how long before the ship will sink?

It will fill $\frac{1}{15}$ in an hour; they pump out $\frac{1}{16}$, hence the water gains $\frac{1}{15}-\frac{1}{16}=\frac{1}{240}$ of the ship per hour.

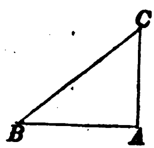
Ans. 240 *hours.*

54. A person dying, worth $5460, left a wife and two children, a son and daughter, absent in a foreign country. He directed that if his son returned, the mother should have one third of the estate, and the son the remainder; but if the daughter returned, she should have one third, and the mother the remainder. Now, it so happened that they both returned; how must the estate be divided to fulfil the father's intentions?

Ans. Daughter $780, *Son* $3120, *Wife* $1560.

55. A cistern containing 60 gallons of water has three unequal cocks for discharging it; the largest will empty it in one hour, the second in two hours, and the third in three: in what time will the cistern be emptied if they all run together? *Ans.* ——

56. A house is 40 feet from the ground to the eaves, and it is required to find the length of a ladder which will reach the eaves, supposing the foot of the ladder cannot be placed nearer to the house than 30 feet.



It is demonstrated in Geometry that in every right-angled triangle, such as BAC, the square of the hypothenuse BC is equal to the sum of the squares of the other sides, AC and AB. That is,

$$BC^2=AC^2+AB^2.$$

If then we extract the square root, we have

$$BC=\sqrt{CA^2+AB^2}.$$

When, therefore, the sides CA, AB are known, we can find the side CB, by first squaring CA and AB, taking the sum and extracting the square root.

Thus, in the example above, we square each of the sides, take the sum, which is 2500, the square root of which is 50. Hence, 50 is the length of the required ladder.

$$40^2=1600$$
$$30^2=\ 900$$
$$2500$$
$$\sqrt{2500}=50$$

57. If a house is 50 feet deep, and the upright which supports the ridge pole is 12 feet high, what will be the length of the rafters? *Ans.* 27.7*ft.*+.

58. When it is 12 o'clock at New York, what is the hour at London, New York being 75° of longitude west of London?

Since the circumference of the earth is supposed to be divided into 360 degrees (ART. 94), and since the sun apparently passes through these 360° every twenty-four hours, it follows that in a single hour it will pass through one twenty-fourth of 360° or 15°. Hence there are

15° of motion in 1 hour of time,
1° of motion in 4 minutes,
1′ of motion in 4 seconds.

If two places, therefore, have different longitudes, they will have different times, and the difference of time will be one hour for every 15° of longitude, or 4 minutes for each degree, and 4 seconds for each minute. It must be observed that the place which is most *easterly* will have the time first, because the sun travels from east to west.

To return then to our question, the difference of longitude between London and New York being 75°, the difference of time will be found in minutes by multiplying 75° by 4, giving 300 minutes, or 5 hours. Now since New York is west of London, the time will be *later* in London: that is, when it is twelve o'clock at New York, it will be five P. M. in London; or when it is twelve at London, it will be seven A. M at New York.

OPERATION.
75°
4
60)300
Ans. 5 *hours.*

59. Boston is 6° 40′ east longitude from the city of

Washington: when it is 6 o'clock P. M. at Washington what is the hour at Boston?

The 6 degrees being multiplied by 4 gives 24 minutes of time, and the 40 minutes being multiplied by 4 gives 160 seconds, or 2 minutes 40 seconds. The sum is 26 minutes 40 seconds, and since Boston is east of Washington the time is later at Boston.

OPERATION.

$6 \times 4 = 24m.$

$40 \times 4 = 160sec. = 2m.\ 40sec.$

$26m.\ 40sec.$

Ans. 26*m.* 40*sec. past* 6.

60. The difference of longitude of two places is 85° 20′: what is the difference of time?

Ans. 5*hr.* 41*m.* 20*sec.*

61. A traveller leaves New Haven at 8 o'clock on Monday morning, and walks towards Albany at the rate of 3 miles an hour. Another traveller sets out from Albany at 4 o'clock on the same evening and walks towards New Haven, at the rate of 4 miles an hour. Now supposing the distance to be 130 miles, whereabout on the road will they meet?

Ans. $69\frac{3}{7}$ *miles from New Haven.*

62. A thief is escaping from an officer. He has 40 miles the start, and travels at the rate of 5 miles an hour, the officer in pursuit travels at the rate of 7 miles an hour: how far must he travel before he overtakes the thief? *Ans.* He travels 20 *hours*, and 140 *miles.*

63. A can do a piece of work alone in 10 days, and B in 13 days: in what time can they do it if they work together? *Ans.* ——

64. The accounts of a certain school are as follows: viz., $\frac{1}{16}$ of the boys learn geometry, $\frac{3}{8}$ learn grammar, $\frac{3}{10}$ learn arithmetic, $\frac{3}{20}$ learn to write, and 9 learn to read: what is the number in each branch?

Ans. { 5 learn geometry, 30 grammar, 24 arithmetic, 12 writing, and 9 reading.

65. If $120 be divided among three persons, A, B, and C, so that when A has $3, B shall have 5 and C 7: how much will each receive?

Ans. A $24, B $40, and C $56.

A PRACTICAL SYSTEM OF BOOK-KEEPING.

Persons transacting business find it necessary to write down the articles bought or sold, together with their prices and the names of the persons with whom the bargains are made.

Book-keeping is the method of recording such transactions in a regular manner. It is divided into two kinds, called Single Entry and Double Entry. The method by Single Entry is the most simple, and answers for all common business. This method we will here explain.

Book-keeping by Single Entry requires two books, a Day-book and a Ledger; and when cash sales are extensive, an additional book is necessary, which is called a Cash-book.

DAY-BOOK.

This book should contain a full history of the business transactions, in the precise order in which they may occur.

The transfer of an account from the Day-book to the Ledger, is called *posting* the account.

Each page of the Day-book should be ruled with two columns on the right hand of the page, one for dollars, and one for cents, and one column on the left hand for entering the page of the Ledger on which the account may be posted.

The Day-book should begin with the name of the owner and his place of residence; and then should follow a full account of the transactions in business in the exact order in which they may have taken place.

The name of the person, or customer, is first written with the term *Dr.* or *Cr.* opposite, according as he becomes a debtor or creditor by the transaction.

Generally, the person who receives is Debtor, and the person who parts with his property is the Creditor.

Thus, if I sell goods to A B, on credit, he becomes my debtor to the amount of the goods, and the goods should be specified particularly in making the charge.

If I buy goods on credit of C D, I enter C D Cr. by the amount of the goods, taking care to specify the goods in the charge.

If I pay money for, or on account of another person, he becomes Dr. to me for the amount paid.

The Day-book and Ledger are generally designated, Day-book A, Day-book B, Ledger A, Ledger B, &c.: for when one book, in the course of business, is filled with charges a new one is taken.

DAY-BOOK A.

Edward P. Nixon, New York, June 1, 1846.

Page 1.

Folio Ledger.	*New York, June 1st,* 1846.		$	cts.
	George Wilson, - - - - -	Dr.		
✓ 1.	To 11 cwt. of sugar at $9 per cwt. -	$99,00		
	To 66 lb. of coffee at 20 cents per lb. -	13,20	112	20.
	Henry Jones, - - - - -	Dr.		
✓ 1.	To balance of former account - -	$159,10		
	To 5 gals. of molasses at 32 cts. per gal.	1,60	160	70.
	2d.			
	Charles Patch, - - - - -	Dr.		
✓ 2.	To Cash, - - - - - - -	$327,09		
	To 1 hogshead of molasses - - -	124,02	451	11.
	3d.			
	Henry Jones, - - - - -	Cr.		
✓ 1.	By Cash, - - - - - - - -		160	70.
	5th.			
	George Wilson, - - - - -	Cr.		
✓ 1.	By Cash - - - - - - -	$100,00		
	By his note of date for - - -	12,20	112	20
	6th.			
	Charles Patch, - - - - -	Dr.		
✓ 2.	To Cash - - - - - - -	$275,10		
	To 1 horse - - - - - -	125,00		
	To 85 lb. of butter at 20 cents per lb. -	17,00	417	10
	Charles Patch, - - - - -	Cr.		
✓ 2.	By Cash - - - - - - -	$400,00		
	By his note of this date, due Aug. 1, 1846	251,11	651	11.
	8th.			
	Jared Newton, - - - - -	Dr.		
✓ 2.	To 1 piece of linen 36 yards - -	$42,50		
	To 3 yds. of broadcloth at $4,50 per yd.	13,50		
	To 46 lb. of nails at 6cts. - - -	2,76	58	76.
	10th.			
	Jared Newton, - - - - -	Cr.		
✓ 2.	By Cash - - - - - - -	$37,50		
	By do. error - - - - - -	21,26	58	76

LEDGER.

The Ledger is a book into which are collected, in a condensed form, all the scattered accounts from the Day-book.

Two pages of the Ledger, facing each other, are generally used in stating an account, in which case each is regarded as half a page; but sometimes a page is divided into two equal parts. The name of the person with whom the account is stated should be written in large letters at the top of the page.

Two columns should be ruled on the right of each half-page of the Ledger, one for dollars and one for cents; there should also be two columns on the left to insert the date of the transaction, and a column for inserting the page of the Day-book from which the account is transferred.

The Debits are entered on the left-hand side of the page, and the Credits on the other side directly opposite. The difference between the debits and credits is always entered under the least sum when the account is closed, and is called the *balance*, as in the account of Charles Patch.

At the top of the left-hand column we enter the year, under which we enter the day of the month on which the transaction took place; and in the column adjoining the column for dollars and cents, we enter the page of the Day-book from which the account is transferred.

When there are several articles charged in the Day-book, we need not specify them all, but may enter them in the Ledger under the general name of "Sundries." Having posted the account, we enter the page of the Ledger to which it has been transferred, in the left-hand column of the Day-book and opposite the account, and make a mark ✓ to show that the account is correctly posted. This we make also against the dollar column of the Ledger.

We begin posting with the account of George Wilson, who stands charged on the Day-book with $112,20. We then open an account with Henry Jones, who stands next in the Day-book, and so with each person named, in his order.

On passing through the Day-book we find George Wilson credited on page 1 by 100 dollars cash, and a note for $12,20. These items we enter in the Ledger, on the credit side of his account, and as the debits and credits are equal, his account is balanced. No erasure should ever be made in the account books. When an error is discovered, if it be in favor of the customer, he should be charged with the amount, and if against him, he should be credited with the amount. In posting the account of Jared Newton, a mistake was made against him of $21,26, which was rectified by crediting him with the amount.

When a charge is entered on the wrong side of the book, as when a person is charged with that for which he ought to have been credited, *twice* the amount must be entered on the other side of the book to make the account right.

Every Ledger should have an Index, where the names of all persons, who have accounts in the Ledger, should be arranged in alphabetical order.

When a Ledger is filled, all the accounts are balanced, and when we transfer the balances to a new Ledger we charge "To balance from Ledger A, page —."

Index to Ledger.

	Folio.		Folio.		Folio.
				P.	
				Patch, Charles	2
J.		N.		W.	
Jones, Henry	1	Newton, Jared	2	Wilson, George	1

LEDGER A.

George Wilson. Page 1.

1846. June	1	To Mdse.	1	112	20	1846. June	5	By sundries,	1	112	20

Henry Jones.

1846. June	1	To Mdse.	1	160	70	1846. June	3	By Cash,	1	160	70

CHARLES PATCH. Page 2.

				$	c.					$	c.
1846.						1846.					
June	2	To sundries,	1	451	11	June	6	By sundries,	1	651	11
"	6	" do.	1	417	10	"	"	" bal. trans. to new ac.		217	10
				868	21					868	21
1846.											
June	6	To bal. from old ac.		217	10						

JARED NEWTON.

				$	c.					$	c.
1846.						1846.					
June	8	To Mdse.	1	58	76	June	10	By Cash,	1	37	50
								" cr. of ac.		21	26
				58	76					58	76

CASH BOOK.

This book records the amount of Cash received and paid out each day.

The CASH is made Dr. to the amount of cash on hand, at the commencement of each day, and to all that is received during the day, and credited with the amounts paid out and with the balance on hand

DR. CASH. CR.

1846.			$	c.	1846.			$	c.
June	1	To Cash on hand,	327	27	June	6	By rent for house,	427	18
"	6	" J. Patrick,	47	15	"	20	" Tho. Tappan,	12	90
"	9	" P. Weeks,	125	09	July	1	" goods bought,	512	10
July	3	" R. Lowndes,	82	12	"	9	" expenses to Boston,	80	13
"	10	" T. Ames,	450	81	"	25	" Cash on hand,		13
			1032	44				1032	44
		Cash on hand,		13					

FARMERS AND MECHANICS' ACCOUNT-BOOK.

The following is a very convenient form for book-keeping, and requires but a single book. It is probably the best form for farmers and mechanics.

J. BELL.	Dr.			J. BELL.	Cr.		
1846.		$	c.	1846.		$	c.
June 1	To 5 cords of wood, at $1,75 per cord,	8	75	July 6	By shoeing horse,	1	00
" 6	To 1 day's work,	1	00	" 10	" mending sleigh,	3	25
July 9	To 4*bu.* of rye, at 62 cents per *bu.*	2	48	" 20	" ironing wagon,	5	12
				Aug. 1	" Cash to balance,	2	86
		12	23			12	23

THE END.

DAVIES'

COURSE OF MATHEMATICS.